BERNOULLI'S EQUATION

ENERGY POSSESED BY THE FLUID BY VIRTUE OF ITS MOTION:-

- **POTENTIAL ENERGY:** It is the energy possessed of a liquid by virtue of its position or location in space.
- It is expressed by Potential Head which is denoted by Z(m).
- **PRESSURE ENERGY:** It is the energy possessed of a liquid by virtue of the pressure at which it is maintained. It is measured by Piezometer tube. Pressure head h= p/w (m),
- **KINETIC ENERGY:** It is the energy possessed by the liquid by virtue of motion or velocity. It is expressed as $v^2/2g$(m)
- **BERNOULLI'S THEOREM:** In a steady continuous flow of a frictionless incompressible fluid, the sum of potential energy head, the pressure head and the kinetic head is same at all the points.i.e. sum of the potential head, pressure head and kinetic head is constant.
- $Z_1 + p_1/w + v_1^2/2g = Z_2 + p_2/w + v_2^2/2g$

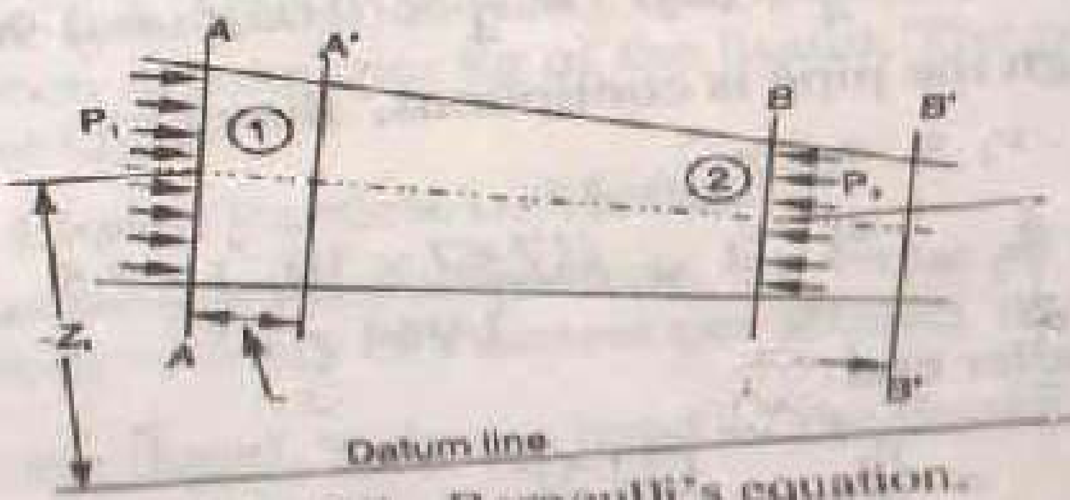


Fig. 7·1. Bernoulli's equation.

Proof of the Bernoulli's Equation.

Consider a perfect incompressible liquid, flowing though a non uniform pipe as shown in the above figure.

Let us consider two sections AA and BB of the pipe. Now let us assume that the pipeline is running full and there is a continuity of flow between the two sections.

Let, Z_1 =Height of AA above datum line,

p_1 =Pressure at AA,

v_1 =Velocity of liquid at AA,

a_1 = Area of pipe at AA, and Z_2,p_2 ,v_2 and a_2 corresponding values at BB.

Let the liquid between the two sections AA and BB move to A^1A^1 , and B^1B^1 thorough a very small lengths dl_1 dl_2 as shown in the figure.

The movement of the liquid between AA and BB is equivalent to the movement of the liquid AA and A^1A^1 to BB and B^1B^1 , the remaining liquid between A^1A^1 and BB being unaffected.

Let W be the weight of the liquid between AA and A^1A^1 .Since the flow is continuous, therefore

$W = w\, a_1\, dl_1 = w\, a_2\, dl_2$

Or, $a_1\, dl_1 = W/w$

Similarly, $a_2\, dl_2 = W/w$

Therefore, $a_1\, dl_1 = a_2\, dl_2$.

Work done by pressure at AA, in moving the liquid to A^1A^1 = Force x Distance **= $p_1\, a_1\, dl_1$**

Similarly, the , work done by pressure at BB in moving the liquid to B^1B^1 **= (-)$p_2\, a_2\, dl_2$**

(-) sign is taken the as the direction of p_2 is opposite to that of p_1.

Total work done by the pressure

$= p_1\, a_1\, dl_1 - p_2\, a_2\, dl_2$

$= a_1\, dl_1 (p_1 - p_2)$

$= W/w\, (p_1 - p_2)$.

Loss of Potential Energy= $W(Z_1 - Z_2)$

And again Kinetic Energy = $W(v_2^2/2g - v_1^2/2g) = W(v_2^2 - v_1^2)/2g$

We know loss of Potential Energy + Work done by Pressure = Gain in Kinetic Energy.

$W(Z_1 - Z_2) + W/w\, (p_1 - p_2) = W(v_2^2 - v_1^2)/2g$,

Or, $(Z_1 - Z_2) + (p_1 - p_2)/w = (v_2^2 - v_1^2)/2g$

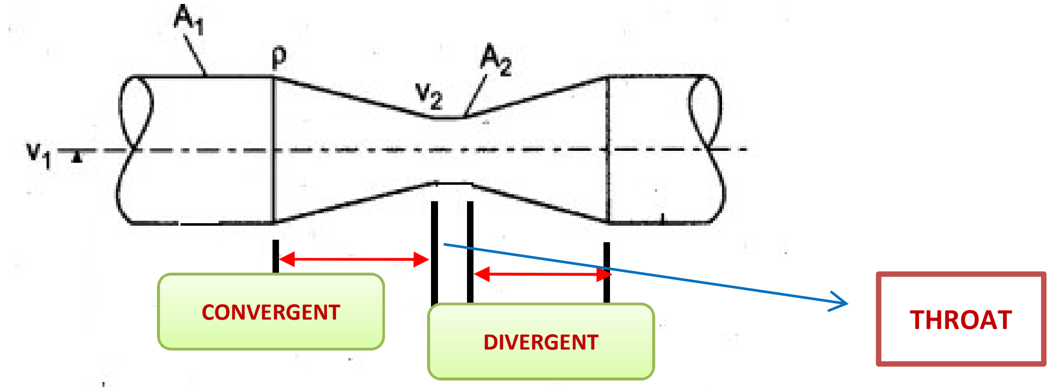
Or, $Z_1 + v_1^2/2g + p_1/w = Z_2 + v_2^2/2g + p_2/w$

Which proves Bernoulli's theorem.

Practical application of Bernoulli's Equation:

1.Veturimeter 2.Orificemeter 3.Pitot tube.

VENTURIMETER



A ventutimeter is an arrangement for finding out the discharge of a liquid flowing in a pipe venturimeter, in its simplest form consist of the following parts:-

Convergent cone, b)Throat & c)Divergent cone.

DISCHARGE THOUGH A VENTURIMETER

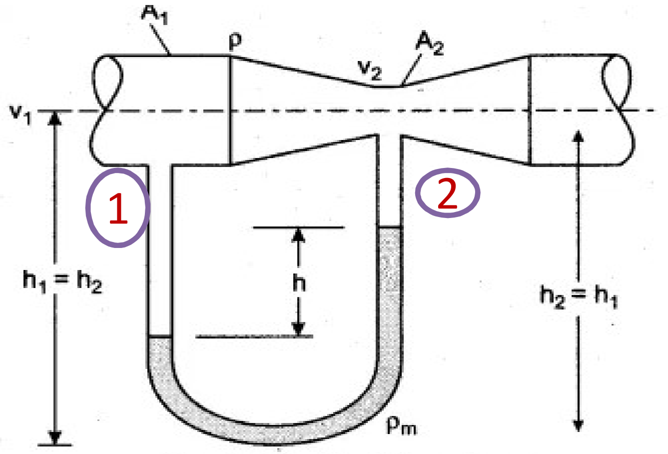


Fig. 11.12 : Venturi meter

Consider a venturimeter though which some liquid is flowing as shown above.

Let p_1 =pressure at section 1,

v_1 =Velocity of water at section 1,

h_1 =Datum head at section 1,

a_1 = Area of venturimeter at section 1, and

p_2, v_2, h_2 and a_2 =corresponding values at section 2.