7

Bernoulli's Equation and its Applications

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7·1 Introduction

In the previous chapter, we have discussed the motion of liquid particles without taking into consideration any force or energy causing the flow. But in this chapter, we shall discuss the motion of liquids and the forces causing the flow. This topic is also known as Hydrodynamics.

7·2 Energy of a Liquid in Motion

The energy, in general, may be defined as the capacity to do work. Though the energy exists in many forms, yet the following are important from the subject point of view :

1. Potential energy,
2. Kinetic energy, and
3. Pressure energy.

7·3 Potential Energy of a Liquid Particle in Motion

It is energy possessed by a liquid particle by virtue of its position. If a liquid particle is Z metres above the horizontal datum (arbitrarily chosen), the potential energy of the particle will be Z metre-kilogram (briefly written as mkg) per kg of the liquid. The potential head of the liquid, at that point, will be Z metres of the liquid.

7·4 Kinetic Energy of a Liquid Particle in Motion

It is the energy, possessed by a liquid particle, by virtue of its motion or velocity. If a liquid particle is flowing with a mean velocity of v metres per second, then the kinetic energy of the particle will be $\frac{v^2}{2g}$ mkg per kg of the liquid. Velocity head of the liquid, at that velocity, will be $\frac{v^2}{2g}$ metres of the liquid.

7·5 Pressure Energy of a Liquid Particle in Motion

It is the energy, possessed by a liquid particle, by virtue of its existing pressure. If a liquid particle is under a pressure of p kN/m^2 (*i.e.*, kPa), then the pressure energy of the particle will be $\frac{p}{w}$ mkg per kg of the liquid, where w is the specific weight of the liquid. Pressure head of the liquid under that pressure will be $\frac{p}{w}$ metres of the liquid.

7·6 Total Energy of a Liquid Particle in Motion

The total energy of a liquid, in motion, is the sum of its potential energy, kinetic energy and pressure energy, Mathematically total energy,

$$E = Z + \frac{v^2}{2g} + \frac{p}{w} \text{ m of liquid.}$$

Note : As a matter of fact, the units of energy are in N-m (or joule). But, according to the subject point of view, the units of energy are taken in terms of m of the liquid.

7·7 Total Head of a Liquid Particle in Motion

The total head of a liquid particle, in motion, is the sum of its potential head, kinetic head and pressure head. Mathematically, total head,

$$H = Z + \frac{v^2}{2g} + \frac{p}{w} \text{ m of liquid.}$$

Example 7·1. *Water is flowing through a tapered pipe having end diameters of 150 mm and 50 mm respectively. Find the discharge at the larger end and velocity head at the smaller end, if the velocity of water at the larger end is 2 m/s.*

Solution. Given : $d_1 = 150$ mm $= 0{\cdot}15$ m ; $d_2 = 50$ mm $= 0{\cdot}05$ m and $v_1 = 2{\cdot}5$ m/s.

Discharge at the larger end

We know that the cross-sectional area of the pipe at the larger end,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0{\cdot}15)^2 = 17{\cdot}67 \times 10^{-3} \text{ m}^2$$

and discharge at the larger end,

$$Q_1 = a_1 \cdot v_1 = (17{\cdot}67 \times 10^{-3}) \times 2{\cdot}5 = 44{\cdot}2 \times 10^{-3} \text{ m}^3\text{/s}$$

$$= 44{\cdot}2 \text{ litres/s} \quad \textbf{Ans.}$$

Velocity head at the smaller end

We also know that the cross-sectional area of the pipe at the smaller end,

$$a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0{\cdot}05)^2 = 1{\cdot}964 \times 10^{-3} \text{ m}^2$$

Since the discharge through the pipe is continuous, therefore

$$a_1 \cdot v_1 = a_2 \cdot v_2$$

or

$$v_2 = \frac{a_1 \cdot v_1}{a_2} = \frac{(17{\cdot}67 \times 10^{-3}) \times 2{\cdot}5}{1{\cdot}964 \times 10^{-3}} = 22{\cdot}5 \text{ m/s}$$

∴ Velocity head at the smaller end

$$= \frac{v_2^2}{2g} = \frac{(22{\cdot}5)^2}{2 \times 9{\cdot}81} = 25{\cdot}8 \text{ m} \quad \textbf{Ans.}$$

Example 7·2. *A circular pipe of 250 mm diameter carries an oil of specific gravity 0·8 at the rate of 120 litres/s and under a pressure of 2 kPa. Calculate the total energy in metres at a point which is 3 m above the datum line.*

Solution. Given : $d = 250$ mm $= 0{\cdot}25$ m; Specific gravity oil $= 0{\cdot}8$; $Q = 120$ litres/s $= 120 \times 10^{-3}$ m^3/s ; $p =$ kPa $= 20$ kN/m^2 and $Z = 3$ m.

We know that the area of the pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0{\cdot}25)^2 = 49{\cdot}09 \times 10^{-3} \text{ m}^2$$

and velocity of oil,

$$v = \frac{Q}{a} = \frac{120 \times 10^{-3}}{49{\cdot}09 \times 10^{-3}} = 2{\cdot}44 \text{ m/s}$$

$\therefore$ Total energy $= Z + \dfrac{v^2}{2g} + \dfrac{p}{w} = 3 + \dfrac{(2{\cdot}44)^2}{2 \times 9{\cdot}81} + \dfrac{20}{0{\cdot}8 \times 9{\cdot}81}$ m

$= 3 + 0{\cdot}3 + 2{\cdot}5 = 5{\cdot}8$ m **Ans.**

Example 7·3. *Water is flowing through a pipe of 70 mm diameter under a gauge pressure of 50 kPa, and with a mean velocity of 2·0 m/s. Neglecting friction, determine the total head, if the pipe is 7 metres above the datum line.*

Solution. Given : $d = 70$ mm $= 0{\cdot}07$ m; $p = 50$ kPa $= 50$ kN/m^2; $v = 2$ m/s and $Z = 7$ m.

We know that the total head of water,

$$H = Z + \frac{v^2}{2g} + \frac{p}{w} = 7 + \frac{(2)^2}{2 \times 9{\cdot}81} + \frac{50}{9{\cdot}81} \text{ m}$$

$$= 7 + 0{\cdot}2 + 5{\cdot}1 = 12{\cdot}3 \text{ m} \quad \textbf{Ans.}$$

7·8 Bernoulli's Equation

*It states, "*For a perfect incompressible liquid, flowing in a continuous stream, the total energy of a particle remains the same, while the particle moves from one point to another.*" This statement is based on the assumption that there are no **losses due to friction in the pipe. Mathematically,

$$Z + \frac{v^2}{2g} + \frac{p}{w} = \text{Constant}$$

where Z = Potential energy,

$\dfrac{v^2}{2g}$ = Kinetic energy, and

$\dfrac{p}{w}$ = Pressure energy.

Proof

Consider a perfect incompressible liquid, flowing through a non-uniform pipe as shown in Fig. 7·1.

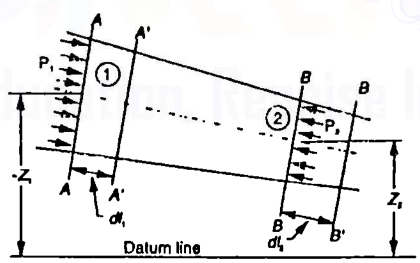

Fig 7·1. Bernoulli's equation.

Let us consider two sections AA and BB of the pipe. Now let us assume that the pipe is running full and there is a continuity of flow between the two sections.

Let Z_1 = Height of AA above the datum,

p_1 = Pressure at AA,

v_1 = Velocity of liquid at AA,

a_1 = Cross-sectional area of the pipe at AA, and

*Bernoulli Daniel was a Swiss engineer, who belonged to a renowned mathematical family and gave this equation in 1738.

** As a matter of fact, there is always some loss of head of the water while flowing through a pipe. For details. please refer to chapter 13.

Z_2, p_2, v_2, a_2 = Corresponding values at BB.

Let the liquid between the two sections AA and BB move to $A'A'$ and $B'B'$ through very small lengths dl_1 and dl_2 as shown in Fig. 7·1. This movement of the liquid between AA and BB is equivalent to the movement of the liquid between AA and $A'A'$ to BB and $B'B'$, the remaining liquid between $A'A'$ and BB being uneffected

Let W be the weight of the liquid between AA and $A'A'$. Since the flow is continuous, therefore

$$W = wa_1 d\,l_1 = wa_2 d\,l_2$$

or $$a_1 \,.\, dl_1 = \frac{W}{w} \qquad ...(i)$$

Similarly $$a_2 \,.\, dl_2 = \frac{W}{w}$$

$\therefore$ $$a_1 \,.\, dl_1 = a_2 \,.\, dl_2 \qquad ...(ii)$$

We know that work done by pressure at AA, in moving the liquid to $A'A'$

$$= \text{Force} \times \text{Distance} = p_1 \,.\, a_1 \,.\, dl_1$$

Similarly, work done by pressure at BB, in moving the liquid to $B'B'$

$$= -p_2 a_2 \,.\, dl_2$$

...(Minus sign is taken as the direction of p_2 is opposite to that of p_1)

$\therefore$ Total work done by the pressure

$$= p_1 a_1 \,.\, dl_1 - p_2 a_2 \,.\, dl_2$$

$$= p_1 \,.\, a_1 dl_1 - p_2 \,.\, a_1 dl_1 \qquad ...(\because a_1 \,.\, dl_1 = a_2 \,.\, d\,l_2)$$

$$= a_1 dl_1 (p_1 - p_2) = \frac{W}{w}(p_1 - p_2) \qquad ...\left(\because a_1 dl_1 = \frac{W}{w}\right)$$

Loss of potential energy $= W(Z_1 - Z_2)$

and again in kinetic energy $$= W\left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right) = \frac{W}{2g}\left(v_2^2 - v_1^2\right)$$

We know that loss of potential energy + Work done by pressu e

= Gain in kinetic energy

$$\therefore \quad W(Z_1 - Z_2) + \frac{W}{w}(p_1 - p_2) = \frac{W}{2g}(v_2^2 - v_1^2)$$

$$(Z_1 - Z_2) + \frac{p_1}{2} - \frac{p_2}{w} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

or $$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}$$

which proves the Bernoulli's equation.

7·9 Euler's Equation for Motion

The *Euler's equation for steady flow of an ideal fluid along a streamline is based on the Newton's Second Law of Motion. The integration of the equation gives Bernoulli's equation in the form of energy per unit weight of the flowing fluid. It is based on the following assumptions :

1. The fluid is non-viscous (*i.e.*, the frictional losses are zero).
2. The fluid is homogeneous and incompressible (*i.e.*, mass density of the fluid is constant).

*Euler Leonhard was a Swiss mathematician and a great learned man of his time.

3. The flow is continuous, steady and along the streamline.
4. The velocity of flow is uniform over the section.
5. No energy or force (except gravity and pressure forces) is involved in the flow.

Consider a steady flow of an ideal fluid along a streamline. Now consider a small element AB of the flowing fluid as shown in Fig. 7·2.

Let dA = Cross-sectional area of the fluid element,

ds = Length of the fluid element,

dW = Weight of the fluid element,

p = Pressure on the element at A,

$p + dp$ = Pressure on the element at B, and

v = Velocity of the fluid element.

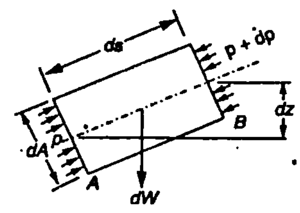

Fig. 7·2. Euler's equation.

We know that the external forces tending to accelerate the fluid element in the direction of the streamline

$$= p \,.\, dA - (p + dp)\, dA$$

$$= -\,dp \,.\, dA \qquad ...(i)$$

We also know that the weight of the fluid element,

$$dW = \rho g \,.\, dA \,.\, ds$$

From the geometry of the figure, we find that the component of the weight of the fluid element in the direction of flow

$$= -\,\rho g \,.\, dA \,.\, ds \cos\theta$$

$$= -\,\rho g \,.\, dA \,.\, ds \left(\frac{dz}{ds}\right) \qquad ...\left(\because \cos\theta = \frac{dz}{ds}\right)$$

$$= -\,\rho g \,.\, dA \,.\, dz \qquad ...(ii)$$

$\therefore$ Mass of the fluid element $= \rho \,.\, dA \,.\, ds$ ...(iii)

We see that the acceleration of the fluid element

$$= \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v \,.\, \frac{dv}{ds} \qquad ...(iv)$$

Now, as per Newton's Second Law of Motion, we know that

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$(-\,dp \,.\, dA) - (\rho g \,.\, dA \,.\, dz) = \rho \,.\, dA \,.\, ds \times v \,.\, \frac{dv}{ds}$$

$$\frac{dp}{\rho} + g \,.\, dz = v \,.\, dv \qquad ...(\text{Dividing both sides by } -\rho dA)$$

or

$$\frac{dp}{\rho} + g \,.\, dz + v \,.\, dv = 0 \qquad ...(v)$$

This is the required Euler's equation for motion and is in the form of a differential equation. Integrating the above equation,

$$\frac{1}{\rho}\int dp + \int g \,.\, dz + \int v \,.\, dv = \text{Constant}$$

$$\frac{p}{\rho} + gZ + \frac{v^2}{2} = \text{Constant}$$

$$p + wZ + \frac{wv^2}{2g} = \text{Constant} \qquad \left(\text{Dividing by } \rho = \frac{w}{g}\right)$$

$$\frac{p}{w} + Z + \frac{v^2}{2g} = \text{Constant} \qquad \text{(Dividing by } w\text{)}$$

or in other words,
$$\frac{p_1}{w} + Z_1 + \frac{v_1^2}{2g} = \frac{p_2}{w} + Z_2 + \frac{v_2^2}{2g}$$

which proves the Bernoulli's equation.

7·10 Limitations of Bernoulli's Equation

The Bernoulli's theorem or Bernoulli's equation has been derived on certain assumptions, which are rarely possible. Thus the Bernoulli's theorem has the following limitations :

1. The Bernoulli's equation has been derived under the assumption that the velocity of every liquid particle, across any cross-section of a pipe, is uniform. But, in actual practice, it is not so. The velocity of liquid particle in the centre of a pipe is maximum and gradually decreases towards the walls of the pipe due to the pipe friction. Thus, while using the Bernoulli's equation, only the mean velocity of the liquid should be taken into account.
2. The Bernoulli's equation has been derived under the assumption that no external force, except the gravity force, is acting on the liquid. But, in actual practice, it is not so. There are always some external forces (such as pipe friction etc.) acting on the liquid, which effect the flow of the liquid. Thus, while using the Bernoulli's equation, all such external forces should be neglected. But, if some energy is supplied to, or, extracted from the flow, the same should also be taken into account.
3. The Bernoulli's equation has been derived, under the assumption that there is no loss of energy of the liquid particle while flowing. But, in actual practice, it is rarely so. In a turbulent flow, some kinetic energy is converted into heat energy. And in a viscous flow, some energy is lost due to shear forces. Thus, while using Bernoulli's equation, all such losses should be neglected.
4. If the liquid is flowing in a curved path, the energy due to centrifugal force should also be taken into account.

Example 7·4. *The diameter of a pipe changes from 200 mm at a section 5 metres above datum to 50 mm at a section 3 metres above datum. The pressure of water at first section is 500 kPa. If the velocity of flow at the first section is 1 m/s, determine the intensity of pressure at the second section.*

Solution. Given : $d_1 = 200$ mm $= 0{\cdot}2$ m; $Z_1 = 5$ m; $d_2 = 50$ mm $= 0{\cdot}05$ m; $Z_2 = 3$ m; $p = 500$ kPa $= 500$ kN/m^2 and $v_1 = 1$ m/s.

Let v_2 = Velocity of flow at section 2, and

p_2 = Pressure at section 2.

We know that area of the pipe at section 1,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0{\cdot}2)^2 = 31{\cdot}42 \times 10^{-3} \text{ m}^2$$

and area of pipe at section 2,
$$a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0{\cdot}05)^2 = 1{\cdot}964 \times 10^{-3} \text{ m}^2$$

Since the discharge through the pipe is continuous, therefore

$$a_1 \cdot v_1 = a_2 \cdot v_2$$

or
$$v_2 = \frac{a_1 \cdot v_1}{a_2} = \frac{(31{\cdot}42 \times 10^{-3}) \times 1}{(1{\cdot}964 \times 10^{-3})} = 16 \text{ m/s}$$

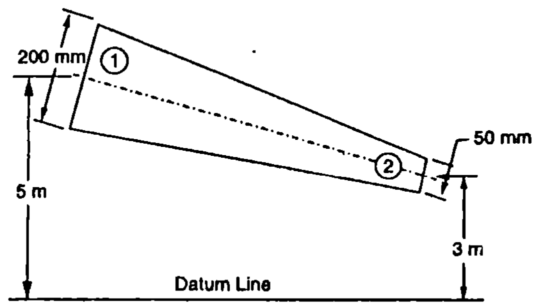

Fig. 7·3

Applying Bernoulli's equation for both the ends of the pipe,

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}$$

$$5 + \frac{(1)^2}{2 \times 9{\cdot}81} + \frac{500}{9{\cdot}81} = 3 + \frac{(16)^2}{2 \times 9{\cdot}81} + \frac{p_2}{9{\cdot}81}$$

$$5 + 0{\cdot}05 + 51 = 3 + 13{\cdot}05 + \frac{p_2}{9{\cdot}81}$$

$$56{\cdot}05 = 16{\cdot}05 + \frac{p_2}{9{\cdot}81}$$

or $$\frac{p_2}{9{\cdot}81} = 56{\cdot}05 - 16{\cdot}05 = 40$$

$\therefore$ $$p_2 = 40 \times 9{\cdot}81 = 392{\cdot}4 \text{ kN/m}^2 = 392{\cdot}4 \text{ kPa} \quad \textbf{Ans.}$$

Example 7·5. *A pipe 300 metres long has a slope of 1 in 100 and tapers from 1 metre diameter at the higher end to 0·5 metre at the lower end. The quantity of water flowing is 900 litres/second. If the pressure at the higher end is 70 kPa, find the pressure at the lower end.*

Solution. Given : l = 300 m; Slope = 1 in 100; d_2 = 1 m; d_1 = 0·5 m; Q = 900 litres/s = 0·9 m^3/s and p_2 = 70 kPa = 70 kN/m^2.

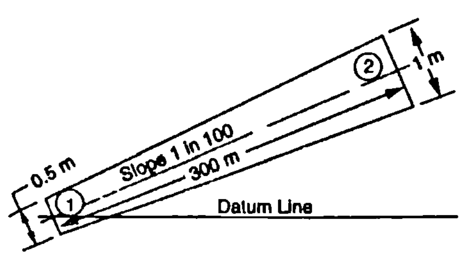

Fig. 7·4.

Let p_1 = Pressure at section 1.

We know that area of the pipe at section 1,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0{\cdot}5)^2 = 0{\cdot}1964 \text{ m}^2$$

and area of the pipe at section 2, $a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (1)^2 = 0{\cdot}7854 \text{ m}^2$

First of all, let us assume the datum line to coincide with the centre of the pipe at section 1 as shown in the figure. Thus $Z_1 = 0$. Now from the geometry of the pipe, we find that the height of higher end from the datum line,

$$Z_2 = \frac{1}{100} \times 300 = 3 \text{ m}$$

We also know that velocity of water at section 1,

$$v_1 = \frac{Q}{a_1} = \frac{0{\cdot}9}{0{\cdot}1964} = 4{\cdot}58 \text{ m/s}$$

and velocity of water at section 2,

$$v_2 = \frac{Q}{a_2} = \frac{0{\cdot}9}{0{\cdot}7854} = 1{\cdot}15 \text{ m/s}$$

Now using Bernoulli's equation for both ends of the pipe,

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}$$

$$0 + \frac{(4{\cdot}58)^2}{2 \times 9{\cdot}81} + \frac{p_1}{9{\cdot}81} = 3 + \frac{(1{\cdot}15)^2}{2 \times 9{\cdot}81} + \frac{70}{9{\cdot}81}$$

$$0 + 1{\cdot}07 + \frac{p_1}{9{\cdot}81} = 3 + 0{\cdot}07 + 7{\cdot}14 = 10{\cdot}21$$

or
$$\frac{p_1}{9{\cdot}81} = 10{\cdot}21 - 1{\cdot}07 = 9{\cdot}14$$

$\therefore$
$$p = 9{\cdot}14 \times 9{\cdot}81 = 89{\cdot}7 \text{ kN/m}^2 = 89{\cdot}7 \text{ kPa} \quad \textbf{Ans.}$$

Example 7·6. *A pipe 5 metre long is inclined at an angle of 15° with the horizontal. The smaller section of the pipe, which is at a lower level, is of 80 mm diameter and the larger section of the pipe is of 240 mm diameter as shown in Fig. 7·5*

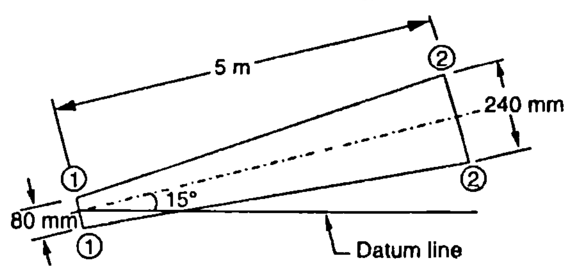

Fig. 7·5.

Determine the difference of pressures between the two sections, if the pipe is uniformly tapering and the velocity of water at the smaller section is 1 m/s.

Solution. Given : l = 5 m; α = 15°; d_1 = 80 mm = 0·08 m; d_2 = 240 mm = 0·24 m and v_1 = 1 m/s.

Let p_1 = Pressure at section 1, and

p_2 = Pressure at section 2.

We know that area of pipe at section 1,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0.08)^2 = 5.027 \times 10^{-3} \text{ m}^2$$

and area of pipe at section 2, $a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0.24)^2 = 45.24 \times 10^{-3} \text{ m}^2$

First of all, let us assume the datum line to coincide with the centre of the pipe at section 1 as shown in the figure. Thus $Z_1 = 0$. Now from the geometry of the pipe, we find that the height of the higher end from the datum line,

$$Z_2 = 0 + 5 \sin \alpha = 5 \sin 15° = 5 \times 0.2588 = 1.294 \text{ m}$$

Since the discharge through the pipe is continuous, therefore

$$a_1 \cdot v_1 = a_2 \cdot v_2$$

or $$v_2 = \frac{a_1 \cdot v_1}{a_2} = \frac{(5.027 \times 10^{-3}) \times 1}{45.24 \times 10^{-3}} = 0.11 \text{ m/s}$$

Applying Bernoulli's equation for both the sections of the pipe,

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}$$

$$0 + \frac{(1)^2}{2 \times 9.81} + \frac{p_1}{9.81} = 1.294 + \frac{(0.11)^2}{2 \times 9.81} + \frac{p_2}{9.81}$$

$$0 + 0.05 + \frac{p_1}{9.81} = 1.294 + 0 + \frac{p_2}{9.81}$$

$$\frac{p_1}{9.81} - \frac{p_2}{9.81} = 1.294 + 0 - 0.05 = 1.244$$

$$\therefore \quad (p_1 - p_2) = 1.244 \times 9.81 = 12.2 \text{ kN/m}^2 = 12.2 \text{ kPa} \quad \textbf{Ans.}$$

Example 7.7. *An oil of sp. gr. 0.8 is flowing upwards through a vertical pipe line, which tapers from 300 mm to 150 mm diameter. A gasoline mercury differential monometer is connected between 300 mm and 150 mm pipe section to measure the rate of flow. The distance between the monometer the tapping is 1 metre and gauge reading is 250 mm of mercury. (a) Find the differential gauge reading in terms of gasoline head. (b) Using Bernoulli's equation and the equation of continuity, find the rate of flow. Neglect friction and other losses between tappings.*

Solution. Given : Specific gravity of gasoline = 0.8; d_1 = 300 mm = 0.3 m; d_2 = 150 mm = 0.15 m; l = 1 m and gauge reading (h) = 250 mm = 0.25 m of mercury.

Differential gauge reading in terms of gasoline head

We know that the gauge reading,

$$\frac{p_1}{W} - \frac{p_2}{w} = 250 \text{ mm (0.25 m) of mercury}$$

$$= 0.25 \times \left(\frac{13.6 - 0.8}{0.8}\right) \text{m of gasoline}$$

$$= 4 \text{ m of gasoline} \quad \textbf{Ans.}$$

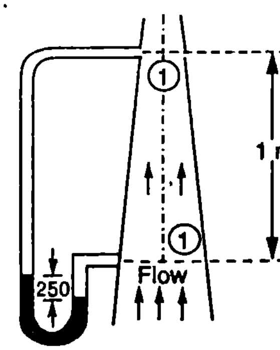

Fig. 7.6.

Rate of flow

Let v_1 = Velocity of oil at section 1, and

v_2 = Velocity of oil at section 2.

We know that the area of pipe at section 1,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0{\cdot}3)^2 = 70{\cdot}69 \times 10^{-3} \text{ m}^2$$

and the area of pipe at section 2,

$$a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0{\cdot}15)^2 = 17{\cdot}67 \times 10^{-3} \text{ m}^2$$

Since the discharge through the pipe is continuous, therefore

$$a_1 \cdot v_1 = a_2 \cdot v_2$$

or

$$v_2 = \frac{a_1 \cdot v_1}{a_2} = \frac{70{\cdot}69 \times 10^{-3} \times v_1}{17{\cdot}67 \times 10^{-3}} = 4v_1$$

Applying Bernoulli's equation for inlet or outlet of the pipe,

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}$$

$$Z_1 + \left(\frac{p_1}{w} - \frac{p_2}{w}\right) = Z_2 + \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$0 + 4 = 1 + \frac{(4v_1)^2}{2g} - \frac{v_1^2}{2g} = 1 + \frac{15v_1^2}{2g}$$

$$\frac{15v_1^2}{2g} = 4 - 1 = 3$$

or

$$v_1 = \sqrt{\frac{3 \times 2 \times 9{\cdot}81}{15}} = 1{\cdot}98 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = a_1 \cdot v_1 = (70{\cdot}69 \times 10^{-3}) \times 1{\cdot}98 = 140 \times 10^{-3} \text{ m}^3\text{/s}$

$= 140$ litres/s **Ans.**

EXERCISE 7·1

1. A uniformly tapering pipe has a 120 mm and 80 mm diameters at its ends. If the velocity of water at the larger end is 2 m/s, find the discharge at the larger end and the velocity head at the smaller end. **[Ans.** 22·62 litres/s; 1·03 m**]**
2. Find the total head of water flowing with a velocity of 8 m/s under a pressure of 80 kPa. The centre line of the pipe is 5 m above the datum line. **[Ans.** 16·41 m**]**
3. A horizontal pipe 100 m long uniformly tapers from 300 mm diameter to 200 mm diameter. What is the pressure head at the smaller end, if the pressure at the larger end is 100 kPa and the pipe is discharging 50 litres of water per second ? **[Ans.** 99·1 kPa**]**
4. Water is flowing through a pipe at the rate of 35 litres/s having diameters 200 mm and 100 mm at sections 1 and 2 respectively. The section 1 is 4 m above the datum and section 2 is 2 m above the datum. Find the pressure at section 2, if the pressure at section 1 is 40 kPa. **[Ans.** 50·3 kPa**]**
5. A 200 m long pipe slopes down at 1 in 100 and tapers from 0·25 m diameter to 0·15 m diameter at the lower end. If the pipe carries 100 litres of oil of specific gravity 0·85, find the pressure at the lower end. The upper end gauge reads 50 kPa. **[Ans.** 54·9 kPa**]**

6. A tapering pipe is used to carry water as shown in Fig. 7·7. The discharge through the pipe was observed to be 170 litres/s.

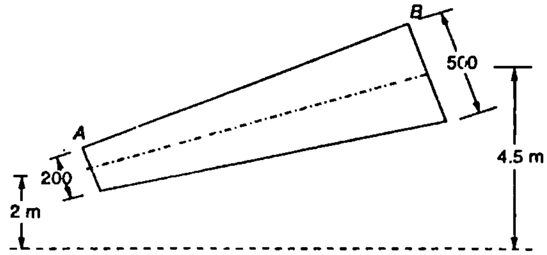

Fig. 7·7. Venturimeter.

If the pressures at A and B are 100 kPa and 75 kPa respectively, determine the direction in which the water will flow through the pipe. [**Ans.** from A to B]

7·11 Practical Applications of Bernoulli's Equation

The Bernoulli's theorem or Bernoulli's equation is the basic equation which has the widest applications in Hydraulics and Applied Hydraulics. Since this equation is applied for the derivation of many formulae, therefore its clear understanding is very essential. Though the Bernoulli's equation has a number of practical applications, yet in this chapter we shall discuss its applications on the following hydraulic devices :

1. Venturimeter. 2. Orificemeter. 3. Pitot tube.

7·12 Venturimeter

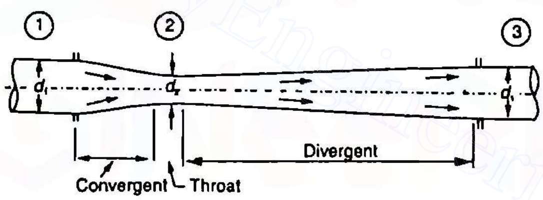

Fig. 7·8

A *venturimeter is an apparatus for finding out the discharge of a liquid flowing in a pipe. A venturimeter, in its simplest form, consists of the following three parts :

(*a*) Convergent cone. (*b*) Throat. (*c*) Divergent cone.

(*a*) ***Convergent cone***

It is a short pipe which converges from a diameter d_1 (diameter of the pipe in which the venturimeter is fitted) to a smaller diameter d_2. The convergent cone is also known as inlet of the venturimeter. The slope of the converging sides is between 1 in 4 or 1 in 5 as shown in Fig. 7·7.

(*b*) ***Throat***

It is a small portion of circular pipe in which the diameter d_2 is kept constant as shown in Fig. 7·7.

(*c*) ***Divergent cone***

It is a pipe, which diverges from a diameter d_2 to a large diameter d_1. The divergent cone is also known as outlet of the venturimeter. The length of the divergent cone is about 3 to 4 times than that of the convergent cone as shown in Fig. 7·7.

*Venturi was an Italian engineer who discussed the phenomenon of pressure reduction at throats in pipes in 1791.

A little consideration will show that the liquid, while flowing through the venturimeter, is accelerated between the sections 1 and 2 (*i.e.*, while flowing through the convergent cone). As a result of the acceleration, the velocity of liquid at section 2 (*i.e.*, at the throat) becomes higher than that at section 1. This increase in velocity results in considerably decreasing the pressure at section 2. If the pressure head at the throat falls below the separation head (which is 2·5 metres of water), then there will be a tendency of separation of the liquid flow. In order to avoid the tendency of separation at throat, there is always a fixed ratio of the diameter of throat and the pipe (*i.e.*, d_2/d_1). This ratio varies from 1/4 to 3/4, but the most suitable value is 1/3 to 1/2.

The liquid, while flowing through the venturimeter, is decelerated (*i.e.*, retarded) between the sections 2 and 3 (*i.e.*, while flowing through the divergent cone). As a result of this retardation, the velocity of liquid decreases which, consequently, increases the pressure. If the pressure is rapidly recovered, then there is every possibility for the stream of liquid to break away from the walls of the metre due to boundary layer effects. In order to avoid the tendency of breaking away the stream of liquid, the divergent cone is made sufficiently longer. Another reason for making the divergent cone longer is to minimise the frictional losses. Due to these reasons, the divergent cone is 3 to 4 times longer than convergent cone as shown in Fig. 7·7.

7·13 Discharge through a Venturimeter

Consider a venturimeter through which some liquid is flowing as shown in Fig. 7·9.

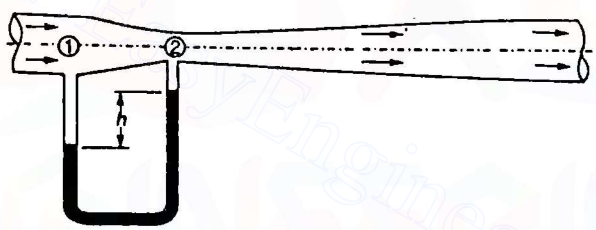

Fig. 7·9. Discharge through a venturimeter.

Let
p_1 = Pressure at section 1,
v_1 = Velocity of water at section 1,
Z_1 = Datum head at section 1,
a_1 = Area of the venturimeter at section 1, and
p_2, v_2, Z_2, a_2 = Corresponding values at section 2.

Applying Bernoulli's equation at sections 1 and 2. *i.e.*,

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w} \qquad ...(i)$$

Let us pass our datum line through the axis of the venturimeter as shown in Fig. 7·8.

Now $Z_1 = 0$ and $Z_2 = 0$.

$$\frac{v_1^2}{2g} + \frac{p_1}{w} = \frac{v_2^2}{2g} + \frac{p_2}{w}$$

or

$$\frac{p_1}{w} - \frac{p_2}{w} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \qquad ...(ii)$$

Since the discharge at sections 1 and 2 is continuous, therefore

$$v_1 = \frac{a_2 v_2}{a_1} \qquad (\because a_1 \cdot v_1 = a_2 \cdot v_2)$$

$$\therefore \qquad v_1^2 = \frac{a_2^2 v_2^2}{a_1^2} \qquad ...(iii)$$

Substituting the above value of v_1^2 in equation (*ii*),

$$\frac{p_1}{w} - \frac{p_2}{w} = \frac{v_2^2}{2g} - \left(\frac{a_2^2}{a_1^2} \times \frac{v_2^2}{2g}\right)$$

$$= \frac{v_2^2}{2g}\left(1 - \frac{a_2^2}{a_1^2}\right) = \frac{v_2^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_1^2}\right)$$

We know that $\frac{p_1}{w} - \frac{p_2}{w}$ is the difference between the pressure heads at sections 1 and 2. When the pipe is horizontal, this difference represents the venturi head and is denoted by h.

$$\text{or} \qquad h = \frac{v_2^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_1^2}\right)$$

$$\text{or} \qquad v_2^2 = 2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right)$$

$$\therefore \qquad v_2 = \sqrt{2gh}\left(\frac{a_1}{\sqrt{a_1^2 - a_2^2}}\right)$$

We know that the discharge through a venturimeter,

$$Q = \text{Coefficient of venturimeter}^* \times a_2 \cdot v_2$$

$$= C \cdot a_2 \cdot v_2 = \frac{C a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

Note : The venturi head (h), in the above equation, has been taken in terms of the liquid head. But, in actual practice, this head is given as the mercury head. In such a case, the mercury head should be converted into the liquid head. If the given liquid is water, the mercury head can be converted into water head by multiplying the mercury head by the specific gravity of mercury, minus the specific gravity of water (*i.e.*, 13·6 – 1 = 12·6).

Sometimes, an oil is being discharged through the venturimeter. In such a case, the venturi head should be taken in terms of oil head, *e.g.*,

$$h = \frac{13{\cdot}6 - w}{w} \times \text{Head of mercury}$$

where 13·6 = Specific gravity of mercury, and
w = Specific weight of the oil.

Example 7·8. *A venturimeter with a 150 mm diameter at inlet and 100 mm at throat is laid with its axis horizontal and is used for measuring the flow of oil specific gravity 0·9. The oil-mercury differential monometer shows a gauge difference of 200 mm. Assume coefficient of the metre as 0·98. Calculate the discharge in litres per minute.*

Solution. Given : d_1 = 150 mm = 0·15 m; d_2 = 100 mm = 0·1 m; Specific gravity of oil = 0·9; h = 200 mm = 0·2 m of mercury and C = 0·98.

* Sometimes it is called as coefficient of discharge. For details, please refer to Art. 9·8.

We know that the area at inlet,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0{\cdot}15)^2 = 17{\cdot}67 \times 10^{-3} \text{ m}^2$$

ıd the area at throat, $$a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0{\cdot}1)^2 = 7{\cdot}854 \times 10^{-3} \text{ m}^2$$

We also know that the difference of pressure head,

$$h = 0{\cdot}2\left(\frac{13{\cdot}6 - 0{\cdot}9}{0{\cdot}9}\right) = 2{\cdot}82 \text{ m of oil}$$

and the discharge through the venturimeter,

$$Q = \frac{C \,.\, a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$= \frac{0{\cdot}98 \times (17{\cdot}67 \times 10^{-3}) \times (7{\cdot}854 \times 10^{-3})}{\sqrt{(17{\cdot}67 \times 10^{-3})^2 - (7{\cdot}854 \times 10^{-3})^2}} \times \sqrt{2 \times 9{\cdot}81 \times 2{\cdot}82} \text{ m}^3\text{/s}$$

$$= \frac{136 \times 10^{-6}}{15{\cdot}83 \times 10^{-3}} \times 7{\cdot}44 = 63{\cdot}9 \times 10^{-3} \text{ m}^3\text{/s} = 63{\cdot}9 \text{ litres/s}$$

$$= 63{\cdot}9 \times 60 = 3834 \text{ litres/min}$$ **Ans.**

Example 7·9. *A venturimetre has an area ratio of 9 to 1, the larger diameter being 300 mm. During the flow, the recorded pressure head in the large section is 6·5 metres and that at the throat 4·25 metres. If the metre coefficient, (C) = 0·99, compute the discharge through the metre.*

Solution. Given : $a_1/a_2 = 9$; $d_1 = 300$ mm $= 0{\cdot}3$ m; Pressure head at large section (h_1) = 6·5 m; Pressure head at smaller section (h_2) = 4·25 m and $C = 0{\cdot}99$.

We know that the area of the large section (inlet),

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0{\cdot}3)^2 = 70{\cdot}69 \times 10^{-3} \text{ m}^2$$

and the area of the smaller section (throat),

$$a_2 = \frac{a_1}{9} = \frac{70{\cdot}69 \times 10^{-3}}{9} = 7{\cdot}854 \times 10^{-3} \text{ m}^2$$

We also know that the difference of pressure heads,

$$h = h_1 - h_2 = 6{\cdot}5 - 4{\cdot}25 = 2{\cdot}25 \text{ m}$$

and the discharge through the metre,

$$Q = \frac{C.\, a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$= \frac{0{\cdot}99 \times (70{\cdot}69 \times 10^{-3}) \times (7{\cdot}854 \times 10^{-3})}{\sqrt{(70{\cdot}69 \times 10^{-3})^2 - (7{\cdot}854 \times 10^{-3})^2}} \times \sqrt{2 \times 9{\cdot}81 \times 2{\cdot}25} \text{ m}^3\text{/s}$$

$$= \frac{549{\cdot}6 \times 10^{-6}}{70{\cdot}25 \times 10^{-3}} \times 6{\cdot}644 = 52 \times 10^{-3} \text{ m}^3\text{/s} = 52 \text{ litres/s}$$ **Ans.**

Example 7·10. *A horizontal venturimeter 160 mm × 80 mm is used to measure the flow of an oil of specific gravity 0·8. Determine the deflection of the oil-mercury gauge, if the discharge of the oil is 50 litres/s. Take coefficient of venturimeter as 1.*

Solution. Given : $d_1 = 160$ mm $= 0{\cdot}16$ m; $d_2 = 80$ mm $= 0{\cdot}08$ m; Specific gravity of oil = 0·8; $Q = 50$ litres/s $= 50 \times 10^{-3}$ m^3/s and $C = 1$.

Let h = Deflection of oil-mercury gauge in m.

We know that the area at inlet,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0{\cdot}16)^2 = 20{\cdot}11 \times 10^{-3} \text{ m}^2$$

and area at throat, $$a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0{\cdot}08)^2 = 5{\cdot}027 \times 10^{-3} \text{ m}^2$$

We also know that the deflection of oil mercury gauge,

$$h = \left(\frac{13{\cdot}6 - 0{\cdot}8}{0{\cdot}8} \times h\right) = 16\,h \text{ m of oil}$$

and discharge of the oil through venturimetre, (Q),

$$50 \times 10^{-3} = \frac{C \,.\, a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$= \frac{1 \times (20{\cdot}11 \times 10^{-3}) \times (5{\cdot}027 \times 10^{-3})}{\sqrt{(20{\cdot}11 \times 10^{-3})^2 - (5{\cdot}027 \times 10^{-3})^2}} \times \sqrt{2 \times 9{\cdot}81 \times 16h}$$

$$= \frac{101{\cdot}1 \times 10^{-6}}{19{\cdot}46 \times 10^{-3}} \times 17{\cdot}7\sqrt{h} = 91{\cdot}96 \times 10^{-3}\sqrt{h}$$

$$\therefore \quad \sqrt{h} = \frac{50 \times 10^{-3}}{91{\cdot}96 \times 10^{-3}} = 0{\cdot}544$$

or $$h = (0{\cdot}544)^2 = 0{\cdot}296 \text{ m} = 296 \text{ mm} \quad \textbf{Ans.}$$

Example 7·11. *A venturimeter is to be fitted to a 250 mm diameter pipe, in which the maximum flow is 7200 litres per minute and the pressure head is 6 metres of water. What is the minimum diameter of throat, so that there is no negative head in it ?*

Solution. Given : d_1 = 250 mm; Q = 7200 litres/min = 120 litres/s = 120×10^{-3} m^3/s and pressure head (h) = 6 m.*

Let a_2 = Area of the throat.

We know that the area at inlet,

$$a_1 = \frac{\pi}{4}(d_1)^2 = \frac{\pi}{4} \times (0{\cdot}25)^2 = 49{\cdot}09 \times 10^{-3} \text{ m}^2$$

and flow through the venturimeter (Q),

$$120 \times 10^{-3} = \frac{C \,.\, a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh} = \frac{C a_1}{\sqrt{\dfrac{a_1^2 - a_1^2}{a_2^2}}} \times \sqrt{2gh} = \frac{C a_1}{\sqrt{\left(\dfrac{a_1}{a_2}\right)^2 - 1}} \times \sqrt{2gh}$$

$$= \frac{1 \times (49{\cdot}09 \times 10^{-3})}{\sqrt{\left(\dfrac{a_1}{a_2}\right)^2 - 1}} \times \sqrt{2 \times 9{\cdot}81 \times 6} = \frac{532{\cdot}6 \times 10^{-3}}{\sqrt{\left(\dfrac{a_1}{a_2}\right)^2 - 1}}$$

---

*Pressure head, $h = \frac{p_1}{w} - \frac{p_2}{w}$

Since the pressure head at the throat is not to be negative, or maximum it can be zero. Therefore venturihead

$$h = 6 - 0 = 6 \text{ m of water.} \quad \ldots\left(\because \frac{p_1}{w} = 6 \text{ m given}\right)$$

$$\sqrt{\left(\frac{a_1}{a_2}\right)} - 1 = \frac{532 \cdot 6 \times 10^{-3}}{120 \times 10^{-3}} = 4 \cdot 44$$

Squaring both sides of the above equation,

$$\left(\frac{a_1}{a_2}\right)^2 - 1 = 19 \cdot 7 \quad \text{or} \quad \left(\frac{a_1}{a_2}\right)^2 = 19 \cdot 7 + 1 = 20 \cdot 7$$

Taking square root of both sides,

$$\frac{a_1}{a_2} = \sqrt{20 \cdot 7} = 4 \cdot 55$$

or
$$a_2 = \frac{a_1}{4 \cdot 55} = \frac{49 \cdot 09 \times 10^{-3}}{4 \cdot 55} = 10 \cdot 79 \times 10^{-3} \text{ m}^2$$

∴ Diameter of throat
$$d_2 = \sqrt{\frac{4a_2}{\pi}} = \sqrt{\frac{4 \times 10 \cdot 79 \times 10^{-3}}{\pi}} \text{ m} \qquad \ldots\left[\because \text{Area} = \frac{\pi}{4} \times (d_2)^2\right]$$

$$= 0 \cdot 117 \text{ m} = 117 \text{ mm} \quad \textbf{Ans.}$$

7·14 Inclined Venturimeter

Sometimes, a venturimeter is fitted to an inclined (or even a vertical) pipe as shown in Fig. 7·10 (*a*) and (*b*).

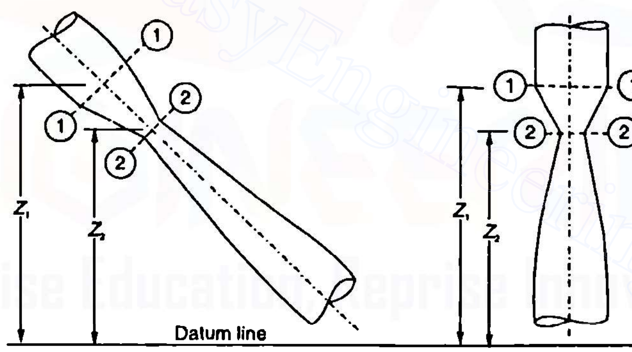

Fig. 7·10.

The same formula for discharge through the venturimeter, (as we derived in Art. 7·13) holds good. The discharge through an inclined venturimeter may also be found out, first by finding out the velocity at either section (by using Bernoulli's equation) and then by multiplying the velocity with the respective area of flow.

Example 7·12. *200 mm × 100 mm venturimeter is mounted in a vertical pipe carrying water, the flow being upwards. The throat section is 200 mm above the entrance section of the venturimeter. For a certain flow through the meter, the differential gauge between the throat and entrance indicates a gauge of deflection of 250 mm. Assuming the venturi coefficient as 0·98, find the discharge.*

Solution. Given : d_1 = 200 mm = 0·2 m; d_2 = 100 mm = 0·1 m; h = 250 mm = 0·25 m of mercury and C = 0·98.

We know that the area at inlet,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0 \cdot 2)^2 = 31 \cdot 42 \times 10^{-3} \text{ m}^2$$

and the area at throat, $a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0{\cdot}1)^2 = 7{\cdot}854 \times 10^{-3}$ m²

We also know that manometer reading,

$$h = 0{\cdot}25\,(13{\cdot}6 - 1) = 3{\cdot}15 \text{ m of water}$$

and discharge through the venturimeter,

$$Q = \frac{C\,a_1 a_2}{\sqrt{(a_1^2 - a_2^2)}}\sqrt{2gh}$$

$$= \frac{0{\cdot}98 \times (31{\cdot}42 \times 10^{-3}) \times (7{\cdot}854 \times 10^{-3})}{\sqrt{(31{\cdot}42 \times 10^{-3})^2 - (7{\cdot}854 \times 10^{-3})^2}} \times \sqrt{2 \times 9{\cdot}81 \times 3{\cdot}15} \text{ m}^3/\text{s}$$

$$= \frac{241{\cdot}8 \times 10^{-6}}{30{\cdot}42 \times 10^{-3}} \times 7{\cdot}861 = 62{\cdot}5 \times 10^{-3} \text{ m}^3/\text{s} = 62{\cdot}5 \text{ litres/s} \quad \textbf{Ans}$$

Example 7·13. *Find the throat diameter of a venturimeter, when fitted to a horizontal main 100 mm diameter having a discharge of 20 litres/s. The differential U-tube mercury manometer shows a deflection giving a reading of 0·6 m. Venture coefficient is 0·95.*

In case, this venturimeter is introduced in a vertical pipe with the water flowing upwards, find the difference in the readings of mercury gauge. The dimensions of pipe and venturimeter remain unaltered as well as the discharge through the pipe.

Solution. Given : d_1 = 100 mm = 0·1 m; Q = 20 litres/s = 20×10^{-3} m³/s; h = 0·6 m of mercury and C = 0·95.

Throat diameter of the venturimeter

Let a_2 = Area of the throat in m²

We know that the area of the main,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0{\cdot}1)^2 = 7{\cdot}854 \times 10^{-3} \text{ m}^2$$

and the difference of pressure heads,

$$h = 0{\cdot}6\,(13{\cdot}6 - 1) = 7{\cdot}56 \text{ m}$$

∴ Discharge through the venturimetre (Q),

$$20 \times 10^{-3} = \frac{Ca_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh} = \frac{Ca_1}{\sqrt{\dfrac{a_1^2 - a_2^2}{a_2^2}}}\sqrt{2gh} = \frac{Ca_1}{\sqrt{\left(\dfrac{a_1}{a_2}\right)^2 - 1}}\sqrt{2gh}$$

$$= \frac{0{\cdot}95 \times (7{\cdot}854 \times 10^{-3})}{\sqrt{\left(\dfrac{a_1}{a_2}\right)^2 - 1}} \times \sqrt{2 \times 9{\cdot}81 \times 7{\cdot}56} = \frac{90{\cdot}87 \times 10^{-3}}{\sqrt{\left(\dfrac{a_1}{a_2}\right)^2 - 1}}$$

$$\sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1} = \frac{90{\cdot}87 \times 10^{-3}}{20 \times 10^{-3}} = 4{\cdot}544$$

Squaring both sides of the above equation,

$$\left(\frac{a_1}{a_2}\right)^2 - 1 = 20{\cdot}64 \quad \text{or} \quad \left(\frac{a_1}{a_2}\right)^2 = 20{\cdot}64 + 1 = 21{\cdot}64$$

and now taking square root of both sides,

$$\frac{a_1}{a_2} = \sqrt{21 \cdot 64} = 4 \cdot 652$$

$$a_2 = \frac{a_1}{4 \cdot 652} = \frac{7 \cdot 854 \times 10^{-3}}{4 \cdot 652} = 1 \cdot 688 \times 10^{-3} \text{ m}^2$$

or dia. of throat,

$$d = \sqrt{\frac{4a_2}{\pi}} = \sqrt{\frac{4 \times (1 \cdot 688 \times 10^{-3})}{\pi}} \text{ m} \quad \ldots\left[\because \text{Area} = \frac{\pi}{4} \times (d_2)^2\right]$$

$$= 0 \cdot 046 \text{ m} = 46 \text{ mm} \quad \textbf{Ans.}$$

Difference in the readings of mercury gauge, when the venturimeter is introduced in a vertical pipe

The difference in the readings of mercury gauge will be the same as that when the venturimeter is introduced in a horizontal pipe. *i.e.*, 600 mm of mercury. **Ans.**

Example 7·14. *A 300 mm × 150 mm venturimeter is provided in a vertical pipeline carrying oil of specific gravity 0·9, the flow being upwards. The difference in elevations of the throat section and entrance section of the venturimeter is 300 mm. The differential U-tube mercury manometer shows a gauge deflection of 250 mm. Calculate*

(i) discharge of the oil, and

(ii) pressure difference between the entrance and throat section.

Take the coefficient of meter as 0·98 and the specific gravity of the mercury as 13·6.

Solution. Given : $d_1 = 300$ mm $= 0 \cdot 3$ m ; $d_2 = 150$ mm $= 0 \cdot 15$ m; Specific gravity of oil $= 0 \cdot 9$; Difference of elevations of throat section and entrance section $= 300$ mm $= 0 \cdot 3$ m; $h = 250$ mm $= 0 \cdot 25$ m of mercury and $C = 0 \cdot 98$.

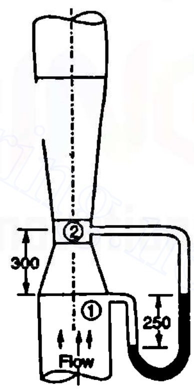

Fig. 7·11

(i) Discharge of the oil

We know that the area of inlet,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0 \cdot 3)^2 \text{ m}^2$$

$$= 70 \cdot 69 \times 10^{-3} \text{ m}^2$$

and the area at throat,

$$a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0 \cdot 15)^2 \text{ m}^2$$

$$= 17 \cdot 67 \times 10^{-3} \text{ m}^2$$

We also know that the difference of pressure head,

$$h = 0 \cdot 25 \left(\frac{13 \cdot 6 - 0 \cdot 9}{0 \cdot 9}\right) = 3 \cdot 53 \text{ m.}$$

and discharge of the oil,

$$Q = \frac{C a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$= \frac{0 \cdot 98 \times (70 \cdot 69 \times 10^{-3}) \times (17 \cdot 67 \times 10^{-3})}{\sqrt{(70 \cdot 69 \times 10^{-3})^2 - (17 \cdot 67 \times 10^{-3})^2}} \times \sqrt{2 \times 9 \cdot 81 \times 3 \cdot 53} \text{ m}^3/\text{s}$$

$$= \frac{1224 \times 10^{-6}}{68 \cdot 45 \times 10^{-3}} \times 8 \cdot 322 = 149 \times 10^{-3} \text{ m}^3/\text{s} = 149 \text{ litres/s} \quad \textbf{Ans.}$$

(ii) Pressure difference between the entrance and throat section

Let p_1 = Pressure at the entrance, and

p_2 = Pressure at the throat section.

We know that the velocity of oil at entrance,

$$v_1 = \frac{Q}{a_1} = \frac{149 \times 10^{-3}}{70 \cdot 69 \times 10^{-3}} = 2 \cdot 11 \text{ m/s}$$

and the velocity of oil at throat, $v_2 = \dfrac{Q}{a_2} = \dfrac{149 \times 10^{-3}}{17 \cdot 67 \times 10^{-3}} = 8 \cdot 43$ m/s

Applying Bernoulli's equation for the entrance and throat section,

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}$$

$$0 + \frac{(2 \cdot 11)^2}{2 \times 9 \cdot 81} + \frac{p_1}{w} = 0 \cdot 3 + \frac{(8 \cdot 43)^2}{2 \times 9 \cdot 81} + \frac{p_2}{w}$$

$$0 + 0 \cdot 227 + \frac{p_1}{w} = 0 \cdot 3 + 3 \cdot 622 + \frac{p_2}{w} = 3 \cdot 922 + \frac{p_2}{w}$$

$$\therefore \quad \frac{p_1}{w} - \frac{p_2}{w} = 3 \cdot 922 - 0 \cdot 227 = 3 \cdot 695 \text{ m of oil} \quad \textbf{Ans.}$$

7·15 Orifice Metre

An orifice metre is used to measure the discharge in a pipe. An orifice metre, in its simplest form, consists of a plate having a sharp edged circular hole known as an orifice. This plate is fixed inside a pipe as shown in Fig. 7·12.

A mercury manometer is inserted to know the difference of pressures between the pipe and the throat (*i.e.*, orifice).

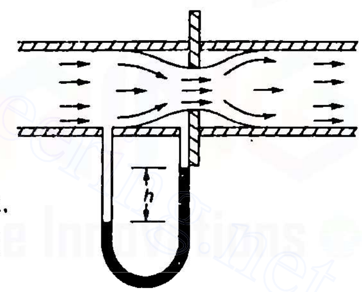

Fig. 7·12. Orifice metre.

Let h = Reading of the mercury manometer,

p_1 = Pressure at inlet,

v_1 = Velocity of liquid at inlet,

a_1 = Area of pipe at inlet, and

p_2, v_2, a_2 = Corresponding values at the throat.

Now applying Bernoulli's equation for inlet of the pipe and the throat,

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w} \quad \ldots(i)$$

$$\frac{p_1}{w} - \frac{p_2}{w} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \ldots(\because Z_1 = Z_2)$$

or

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \frac{1}{2g}(v_2^2 - v_1^2) \quad \ldots(ii)$$

Since the discharge is continuous, therefore $a_1 \cdot v_1 = a_2 \cdot v_2$

$$v_1 = \frac{a_2}{a_1} \times v_2 \quad \text{or} \quad v_1^2 = \frac{a_2^2}{a_1^2} \times v_2^2$$

Substituting the above value of v_1^2 in equation (*ii*),

$$h = \frac{1}{2g}\left(v_2^2 - \frac{a_2^2}{a_1^2} \times v_2^2\right) = \frac{v_2^2}{2g}\left(1 - \frac{a_2^2}{a_1^2}\right) = \frac{v_2^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_1^2}\right)$$

$$\therefore \qquad v_2^2 = 2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right) \quad \text{or} \quad v_2 = \sqrt{2gh}\left(\frac{a_1}{\sqrt{a_1^2 - a_2^2}}\right)$$

We know that the discharge,

$$Q = \text{Coefficient of orifice metre} \times a_2 \, . \, v_2$$

$$= \frac{C\, a_1\, a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh} \qquad \text{...(Same as in Art. 8·14)}$$

Note : The value of the coefficient of orifice metre will be such, which will also include the factor for coefficient of contraction for the orifice (for details of coefficient of contraction, please refer to Art. 8·6).

Example 7·15. *An orifice metre consisting of 100 mm diameter orifice in a 250 mm diameter pipe has coefficient equal to 0·65. The pipe delivers oil (sp. gr. 0·8). The pressure difference on the two sides of the orifice plate is measured by a mercury oil differential manometer. If the differential gauge reads 80 mm of mercury, calculate the rate of flow in litres/s.*

Solution. Given : $d_2 = 100$ mm = 0·1 m; $d_1 = 250$ mm = 0·25 m; $C = 0·65$; Specific gravity of oil = 0·8 and $h = 0·8$ m of mercury.

We know that the area of pipe,

$$a_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (0·25)^2 = 49·09 \times 10^{-3}\ \text{m}^2$$

and area of throat,
$$a_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (0·1)^2 = 7·854 \times 10^{-3}\ \text{m}^2$$

We also know that the pressure difference,

$$h = 0·8\left(\frac{13·6 - 0·8}{0·8}\right) = 12·8 \text{ m of oil}$$

and rate of flow,
$$Q = \frac{C\, . \, a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

$$= \frac{0·65 \times (49·09 \times 10^{-3}) \times (7·854 \times 10^{-3})}{\sqrt{(49·09 \times 10^{-3})^2 - (7·854 \times 10^{-3})^2}} \times \sqrt{2 \times 9·81 \times 12·8}\ \text{m}^3/\text{s}$$

$$= \frac{250·6 \times 10^{-6}}{48·45 \times 10^{-3}} \times 15·85 = 82 \times 10^{-3}\ \text{m}^3/\text{s} = 82 \text{ litres/s} \quad \textbf{Ans.}$$

7·16 Pitot Tube

A Pitot* tube is an instrument to determine the velocity of flow at the required point in a pipe or a stream. In its simplest form, a pitot tube consists of a glass tube bent a through 90° as shown in Fig. 7·13.

The lower end of the tube faces the direction of the flow as shown in Fig. 7·13. The liquid rises up in the tube due to the pressure exerted by the flowing liquid. By measuring the rise of liquid in the tube, we can find out the velocity of the liquid flow.

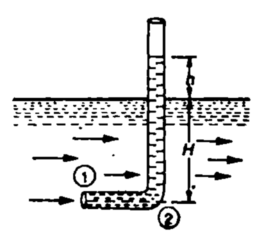

Fig. 7·13. Pitot tube.

Let h = Height of the liquid in the pitot tube above the surface,

* Pitot, Henri (1695—1771) was a French scientist and an engineer. He conducted a series of experiments on the flow of water in channels and pipes.

H = Depth of tube in the liquid, and

v = Velocity of the liquid.

Applying Bernoulli's equation for the sections 1 and 2,

$$H+\frac{v^2}{2g} = H+h \qquad ...(\because Z_1 = Z_2)$$

or
$$h = \frac{v^2}{2g}$$

$\therefore$
$$v = \sqrt{2gh}$$

Note : It has been experimentally found that if the pitot tube is placed, with its nose facing side way, in the flow, there will be no rise of the liquid in the tube. But if a pitot tube is placed, with its nose facing down stream, the liquid level in the tube will be depressed.

Example 7·16. *A pitot tube was inserted in a pipe to measure the velocity of water in it. If the water rises the tube is 200 mm, find the velocity of water.*

Solution. Given : h = 200 mm = 0·2 m.

We know that the velocity of water in the pipe,

$$v = \sqrt{2gh} = \sqrt{2\times 9{\cdot}81\times 0{\cdot}2} = 1{\cdot}98 \text{ m/s} \quad \textbf{Ans.}$$

EXERCISE 7·2

1. A 200 mm × 120 mm venturimeter is installed in a pipe carrying water. If the mercury differential manometer shows a reading of 200 mm, find the discharge through the pipe. Take coefficient for the venturimeter as 0·98. [**Ans.** 83·6 litres/s]

2. A venturimeter has 400 mm diameter at the main and 150 mm at the throat. If the difference of pressures is 250 mm of mercury and the metre coefficient is 0·97, calculate the discharge of oil through the venturimeter. Take specific gravity of oil as 0·75. [**Ans.** 158·6 litres/s]

3. A venturimeter is inserted in a 150 mm diameter pipe carrying an oil of sp. gr. 0·85. If the differential head indicated by the mercury manometer is 0·25 m, and the rate of flow is 80 litres/s, find the diameter of the venturimeter at its throat. Take coefficient for the venturimeter as 0·97. [**Ans.** 92 mm]

4. In a laboratory, a 100 mm × 50 mm venturimeter was used, which recorded a discharge of 18 litres of water per second, when the mercury reading was 300 mm. What is the value of venturimeter coefficient ? [**Ans.** 0·972]

5. A 150 mm × 75 mm venturimeter is connected in a pipe discharging water, which is inclined at an angle of 45° with the horizontal. Find the discharge through the venturimeter, if the mercury gauge shows a deflection of 175 mm and coefficient for the venturimeter as 0·95. [**Ans.** 28·5 litres/s]

6. A 200 mm non-standard orifice is installed in a 250 mm pipe carrying water. When the flow is 165 litres/s, the mercury differential gauge reads 50 mm. Compute the value of coefficient for the orifice metre. [**Ans.** 0·872]

7. A pitot tube is installed in the centre of a pipe 80 mm diameter. Find the velocity of water in the centre of the pipe, if the water rises 300 mm in the tube. [**Ans.** 2·43 m/s]

QUESTIONS

1. What do you understand by the term total head of a moving fluid ? Explain clearly the difference between the total energy and total head of a moving fluid.

2. Derive Bernoulli's equation by any method.

3. State the limitations of the Bernoulli's theorem.

4. Name some practical applications of Bernoulli's theorem.

5. Derive an equation to measure the quantity of water flowing through a venturimeter.
6. Sketch a venturimeter and state why a certain angle of divergence is to be maintained.
7. What is an orifice metre ? Derive an expression for the discharge through an orifice metre.
8. Sketch a Pitot tube and explain how it is used to measure the velocity of a flowing liquid.

OBJECTIVE TYPE QUESTIONS

1. For a perfect incompressible liquid flowing in a continuous stream, the total energy of a particle remains the same, while the particle moves from one position to the other. This statement is called

   (*a*) continuity equation (*b*) Bernoulli's equation
   (*c*) Pascal's law (*d*) Archimedes' principle

2. According to Bernoulli's equation

   (*a*) $Z+p+v = C$ (*b*) $Z+\frac{p}{w}+\frac{v}{g} = C$
   (*c*) $Z+\frac{p}{w}+\frac{v^2}{g} = C$ (*d*) $Z+\frac{p}{w}+\frac{v^2}{2g} = C$

3. Bernoulli's equation is applied for

   (*a*) venturimeter (*b*) orifice metre
   (*c*) pitot tube (*d*) all of these

4. A venturimeter is used to measure

   (*a*) velocity of a flowing liquid (*b*) pressure of a flowing liquid
   (*c*) discharge of a flowing liquid (*d*) all of these

5. In order to avoid the tendency of separation at the throat in a venturimeter, the ratio of diameter at throat to that of the pipe should be

   (*a*) $\frac{1}{16}$ to $\frac{1}{8}$ (*b*) $\frac{1}{8}$ to $\frac{1}{4}$ (*c*) $\frac{1}{4}$ to $\frac{1}{3}$ (*d*) $\frac{1}{3}$ to $\frac{1}{2}$

ANSWERS

1. (*b*) 2. (*d*) 3. (*d*) 4. (*c*) 5. (*d*)

8

Flow Through Orifices

(Measurement of Discharge)

8·1 Introduction

An opening, in a vessel, through which the liquid flows out is known as an orifice. This hole or opening is called an orifice, so long as the level of the liquid on the upstream side is above the top of the orifice. The usual purpose of an orifice is the measurement of discharge.

An orifice may be provided in the vertical side of a vessel or in the base. But the former is more common.

8·2 Types of Orifices

There are many types of orifices, depending upon their size, shape and nature of discharge. But the following are important from the subject point of view :

1. According to size :
   (*a*) Small orifice, and
   (*b*) Large orifice.
2. According to shape :
   (*a*) Circular orifice,
   (*b*) Rectangular orifice, and
   (*c*) Triangular orifice.
3. According to shape of the edge :
   (*a*) Sharp-edged orifice, and
   (*b*) Bell mouthed orifice.
4. According to nature of discharge :
   (*a*) Fully submerged orifice, and
   (*b*) Partially submerged orifice.

All the above-mentioned orifices will be discussed in details at their appropriate places in the book. Before entering into details of the flow, through all types or orifices, following definitions should be clearly understood at this stage :

8·3 Jet of Water

The continuous stream of a liquid, that comes out or flows out of an orifice, is known as the jet of water.

