Chapter 05

DESIGN of SPRINGS

MARKS - 12

Introduction

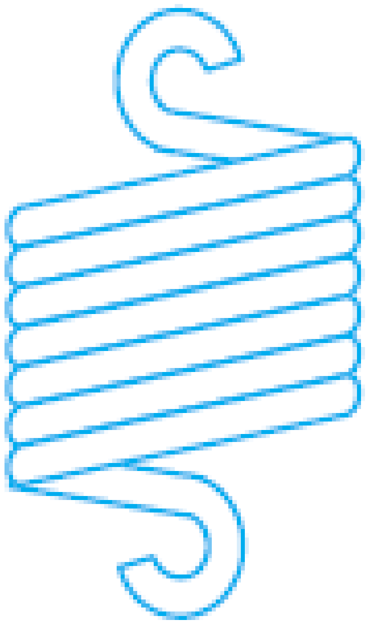
- A spring is defined as an elastic body whose function is to deflect or deform by storing the energy when loaded and recover its original shape when load is removed
- **Applications** –

1) It is used in spring balance and engine indicator to measure force.
2) To absorb shocks and vibrations in automobile, railway shock absorber for suspension.
3) To apply a force in clutch, brake and spring loaded valve.
4) To stored energy in clocks, toys, etc.

Classification of Spring

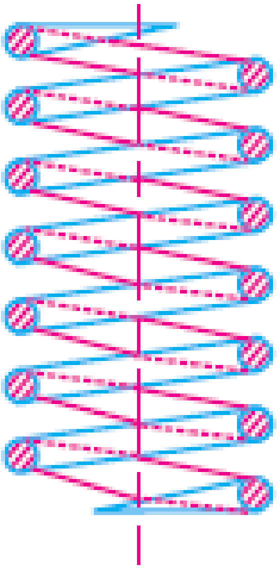
1) Helical spring
    a) Helical compression spring
    b) Helical tension spring
2) Conical or Volute spring
3) Torsional spring
    a) Helical torsional
    b) Spiral torsional
4) Leaf or Laminated spring
5) Disc or Belleville Spring.

- **Tensile Helical Spring** – The spring is designed to operate with a tension load, so the spring stretches as the load is applied to it.



(*b*) Tension helical spring.

- **Compression Helical Spring** – It is designed to operate with a compression load, so the spring gets shorter as the load is applied to it.



(*a*) Compression helical spring.

- **Closed coil helical springs** –

  In this spring, the helix angle is very small usually 10^0. the coil are so close that there is very small gap consecutive coils.

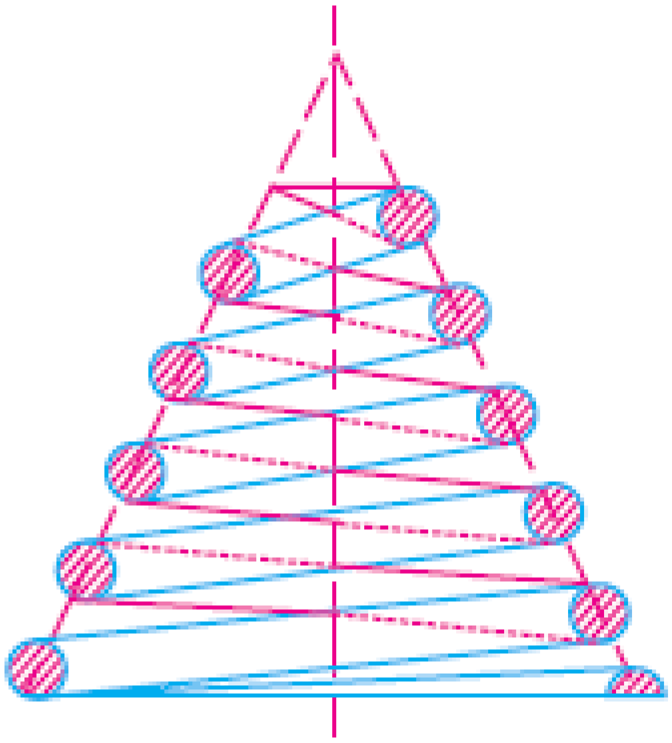
- **Open coil Helical spring** –

  In open coil helical spring, the helix angle is large usually more than 10^0. the wire is coiled is so that there is large gap between two consecutive coils.

- **Conical Spring** –

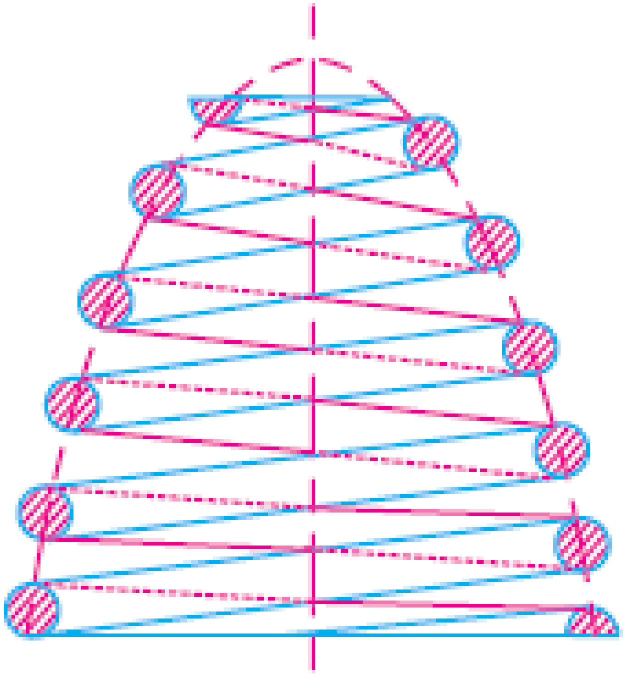
➢ The conical spring are make of wire coil are arranged in a shape of frustum of cone.

➢ These springs are work in compression and are Used where space limitation prohibit.



(*a*) Conical spring.

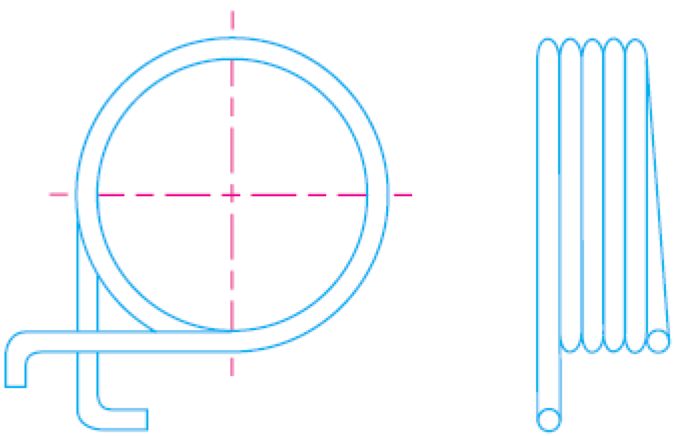
- **Volute Spring** –
- ➢ A *volute spring* is a compression spring in the form of a cone, designed so that under compression the coils are not forced against each other, thus permitting longer travel.



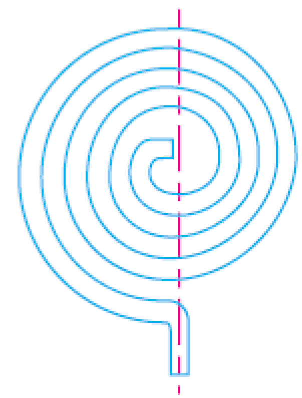
(*b*) Volute spring.

- **Torsional Spring** –

in which the load is an axial force, the load applied to a torsion spring is a torque or twisting force, and the end of the spring rotates through an angle as the load is applied.



(*a*) Helical torsion spring.



(*b*) Spiral torsion spring.

***Belleville spring** –

A disc shaped spring commonly used to apply tension to a bolt (and also in the initiation mechanism of pressure-activated landmines).

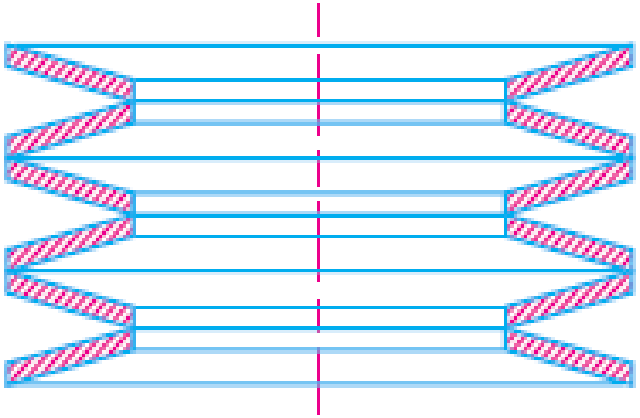


Fig. 23.5. Disc or bellevile springs.

- **Leaf spring** – A flat spring used in vehicle suspensions, electrical switches, and bows.

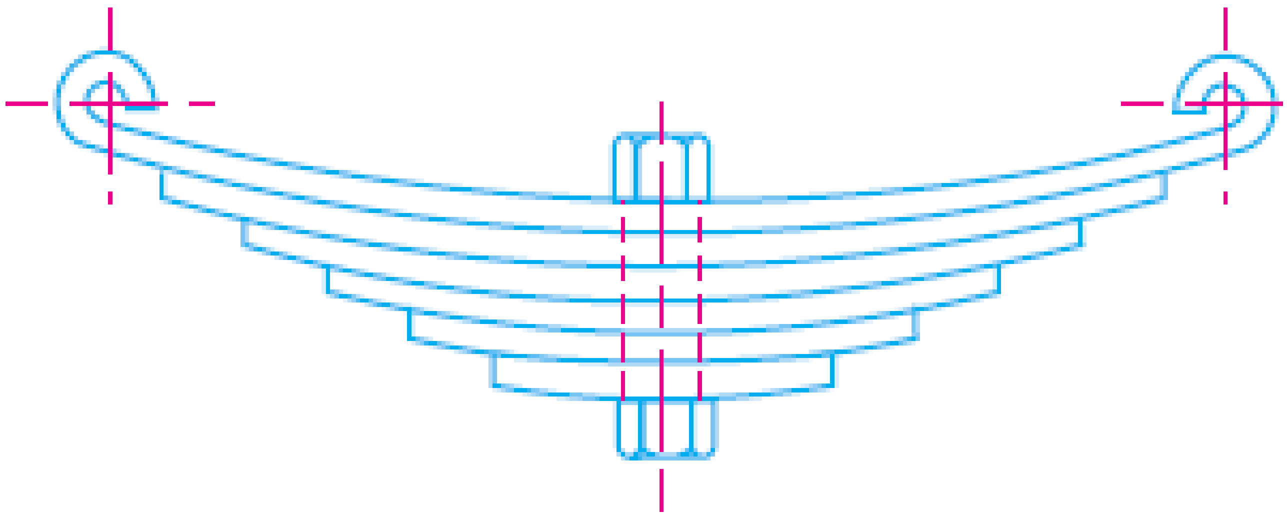
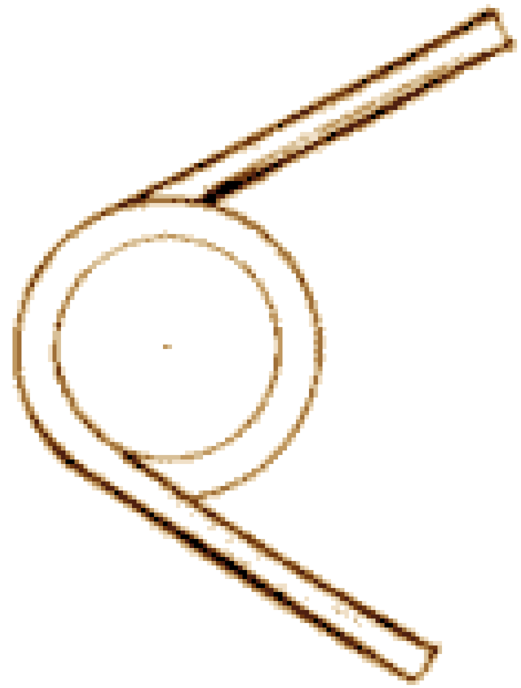


Fig. 23.4. Laminated or leaf springs.

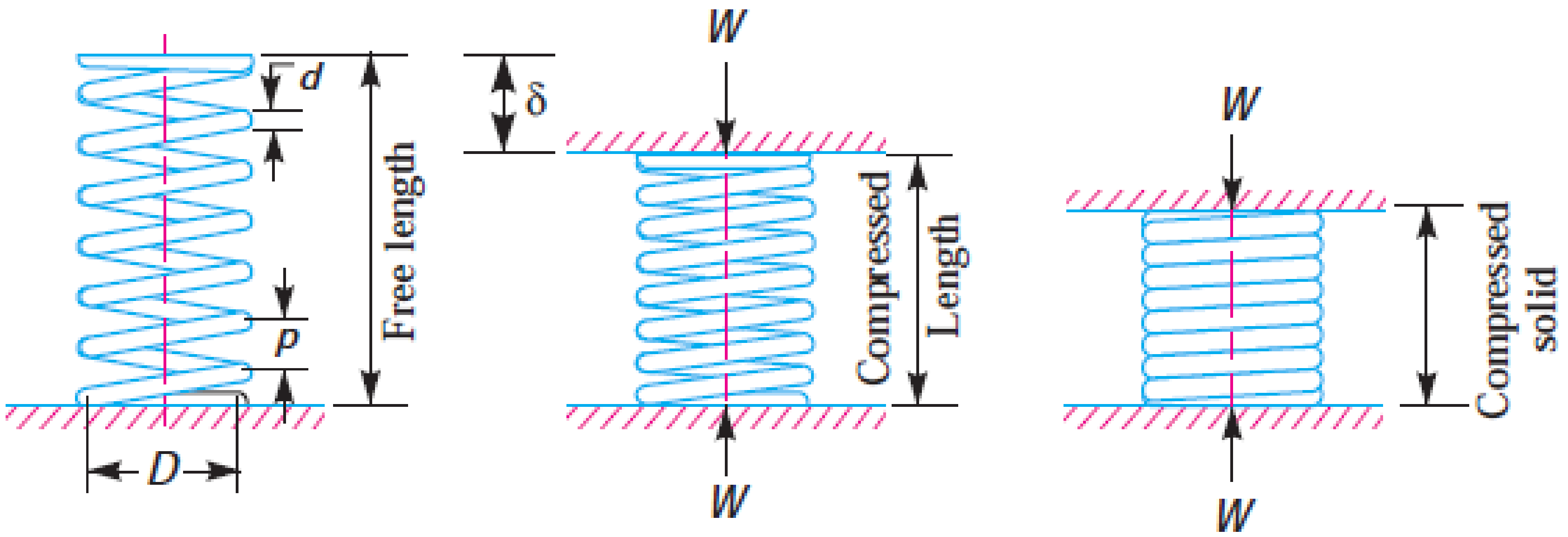
Compression

Tension

Torsion



Terminology for Helical Compression Spring



Terminology for Helical Compression Spring

1) Solid Length (L_s) –

When the compression spring is compressed until the coils comes in contact with each other then the spring is said to be solid. This is known as solid length.

$$Solid\ Length = L_s = n'd$$

$$Where\ n' = total\ no.of\ coils\ or turns$$

$$d = wire\ diameter\ in\ mm$$

2) Free length (L_F) –

It is the length of the spring in the free or unloaded condition.

Free length = solid length + max. compression + clearance between coils

$$L_F = n'd + \delta_{max} + 0.15\delta_{max}$$

$$L_F = n'd + \delta_{max} + (n'-1)\times 1mm$$

3) Spring index (C) –

It is defined as the ratio of mean coil diameter to the wire diameter.

Spring index = C = D_m/d

Where D_m = mean coil diameter

D = diameter of wire

4) Spring constant or spring stiffness or spring rate (K) –

It is defined as the load required per unit deflection of the spring.

$$K = \frac{F}{\delta}$$

5) Pitch (p) –

It is defined as the axial distance between adjacent coil when the spring is in uncompressed condition.

$$p = \frac{Free\,length}{n'-1}$$

$$p = \frac{L_F - L_s}{n'} + d$$

$$L_F = Free\,length$$

$$L_s = solid\ length$$

$$n' = total\ number\,of\ coils\ or\,turns$$

$$d = diameter\ of\ wire$$

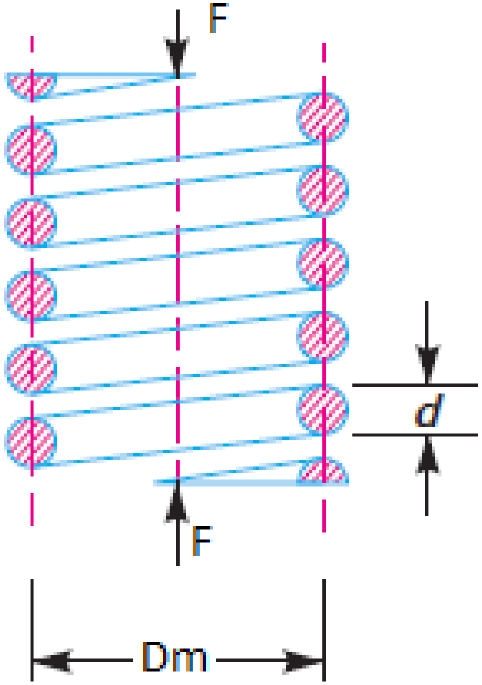
Desirable Properties of Spring Material

- It should have high resilience.
- It should have high static strength.
- It should have high fatigue strength.
- It should be ductile.
- It should be creep resistance.
- It should be non corrosive.

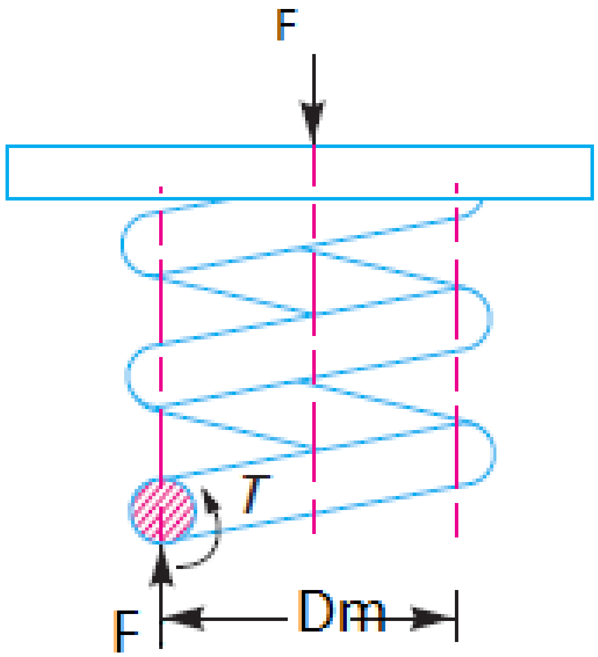
Material for Spring

- The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 per
- cent carbon and 0.60 to 1.0 per cent manganese.
- Music wire is used for small springs.
- Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

Stresses in Spring



(*a*) Axially loaded helical spring.



(*b*) Free body diagram

- Figure shows a free body diagram of a portion of the spring subjected to an axial force (F). The axial force induced the stresses at each and every section of the spring wire having diameter (d).
- While resisting the axial load the spring wire is subjected to

1. Twisting moment (T) about its own axis induced torsional shear stress.
2. Direct shear stress due to force (F)

- Let D_m=Mean coil diameter in mm
- F = Axial force on the spring in N
- d = wire diameter in mm
- n = number of active coils
- C = Spring index
- K = Stiffness of spring in N/mm
- G = Modulus of rigidity N/mm^2

$$\tau = Shear\ stress\ induced\ in\ wire\ N/mm^2$$

If the effect of curvature of the wire is neglectedd the resultant shear stress induced in the spring will be

$$\tau_R = \tau_t + \tau_d \quad ----(1)$$

1. *Torsional shear stress induced*

$$T = F \times \frac{D_m}{2} \quad ---(2)$$

$$And\ T = \frac{\pi}{16} \times \tau_t \times d^3$$

$$\therefore \tau_t = \frac{16T}{\pi d^3} = \frac{16 \times F \times D_m/2}{\pi d^3}$$

$$\tau_t = \frac{8FD_m}{\pi d^3}$$

2. *Direct shear stress induced*

$$\tau_d = \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2}$$

3. *Resultant shear stress will be*

$$\tau_R = \tau_t \pm \tau_d = \frac{8FD_m}{\pi d^3} \pm \frac{4F}{\pi d^2}$$

$$\tau_R = \frac{8FD_m}{\pi d^3}(1 + \frac{d}{2D_m})$$

$$\tau_R = \frac{8FD_m}{\pi d^3}\left(1 + \frac{0.5}{C}\right) where\, C = Spring\ Index = \frac{D_m}{d}$$

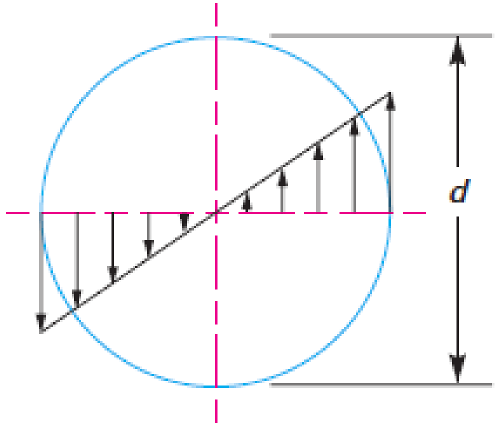
$Put\ K_s = Shear\ stress\, correction\ \ factor.$

$It\, is\, used\, to\, account\ \ for\ direct\ shear\ stress\, and\ effect$

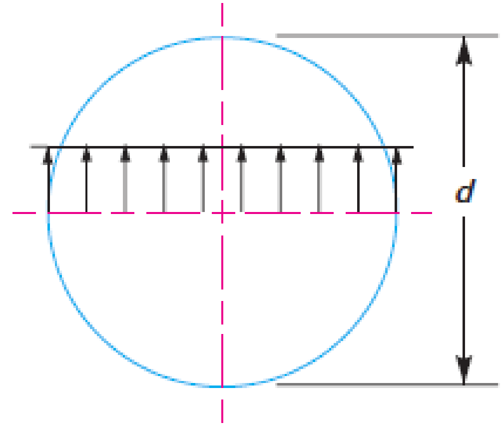
$due\, to\, torsion$

$$K_s = (1 + \frac{0.5}{C})$$

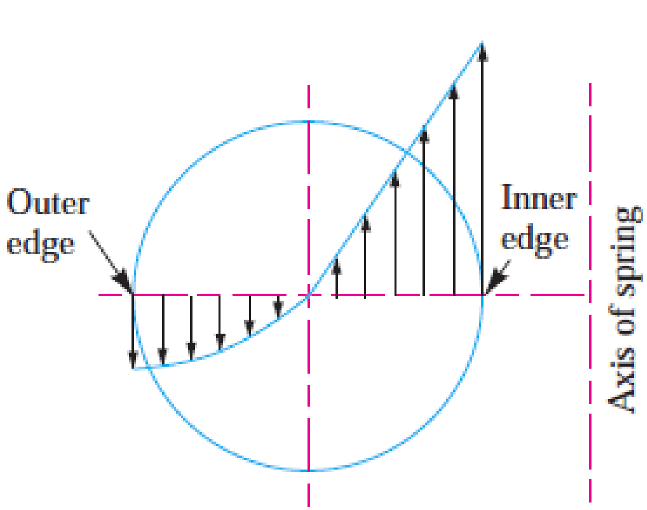
$$\therefore \tau_R = K_S\left(\frac{8FD_m}{\pi d^3}\right) = K_s\left(\frac{8FC}{\pi d^2}\right)$$



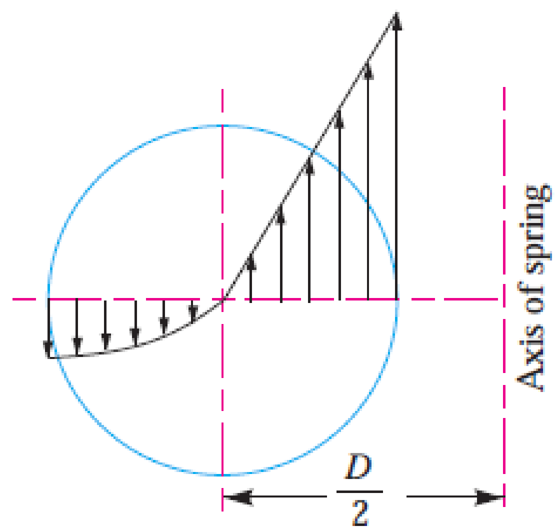
(*a*) Torsional shear stress diagram.



(*b*) Direct shear stress diagram.



(*c*) Resultant torsional shear and direct shear stress diagram.



(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

Wahl's Correction Factor (K_w)

- Wahl's correction factor or stress factor is the modification of shear stress factor (K_s).
- Prof. A. M. Wahl's first determined more accurate way of finding out resultant shear stress induced in the spring wire analytically.
- The curvature of the wire increases the shear stress on the inner surface of the spring and decreases it slightly on the outer surface.
- This curvature effect stress is localized and is significant only when fatigue load is present.

- For static loading, this stress can be neglected.
- In order to consider the effect of both direct shear stress and curvature effect, the shear stress correction factor (K_s) is replaced by another factor 'K_w' known as Wahl's correction factor.

It is given by

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Where, $(\frac{4C-1}{4C-4})$ *gives the correction for curvature effect and*

$(\frac{0.615}{C})$ *gives the correction for direct shear stress effect*

- Hence, the maximum shear stress induced in a spring which is at inner surface is given by –

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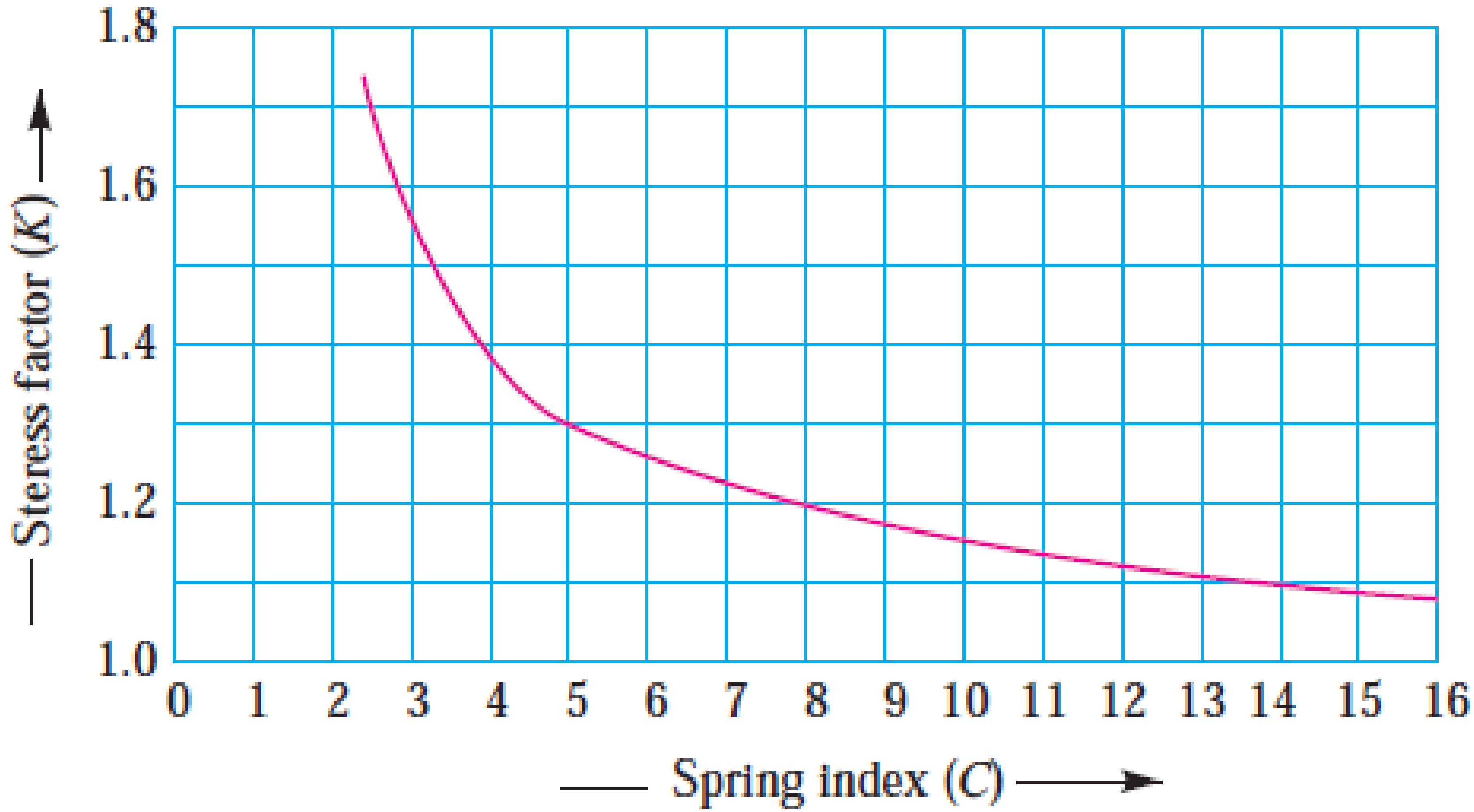
$$\tau = K_w \times \frac{8FD_m}{\pi d^3}$$

Fig. shows the effect of spring index ‘C’ on the Wahl’s factor.

The Wahl’s factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.



1.8
1.6
1.4
1.2
1.0
Steress factor (K)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Spring index (C)

Stresses in springs

$$1)\, Torque = F \times \frac{D_m}{2}$$

$$2)\, Shear\ stress = \tau = \frac{8FD_m}{\pi d^3} = \frac{8FC}{\pi d^2}$$

Neglecting the curvature effect.

$$3)\, shear\ stress = \tau = K_w \times \frac{8FD_m}{\pi d^3} = K_w \times \frac{8FC}{\pi d^2}$$

Considering the curvature effect

$$K_w = Wahl's\ correction\ factor = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$4)\ Deflection = \delta = \frac{8FC^3n}{Gd}$$

$$5)\ Stiffness = \frac{F}{\delta} = \frac{Gd}{8C^3n}$$

$$6)\ Energy\ stored\ in\ spring = U = \frac{1}{2}F \times \delta$$

$$7)\ Energy\ stored\ per\ unit\ volume = u = \frac{U}{V}$$

$$8)\ Volume\ of\ spring = V = \frac{\pi}{4}d^2 \times \pi D_m n$$

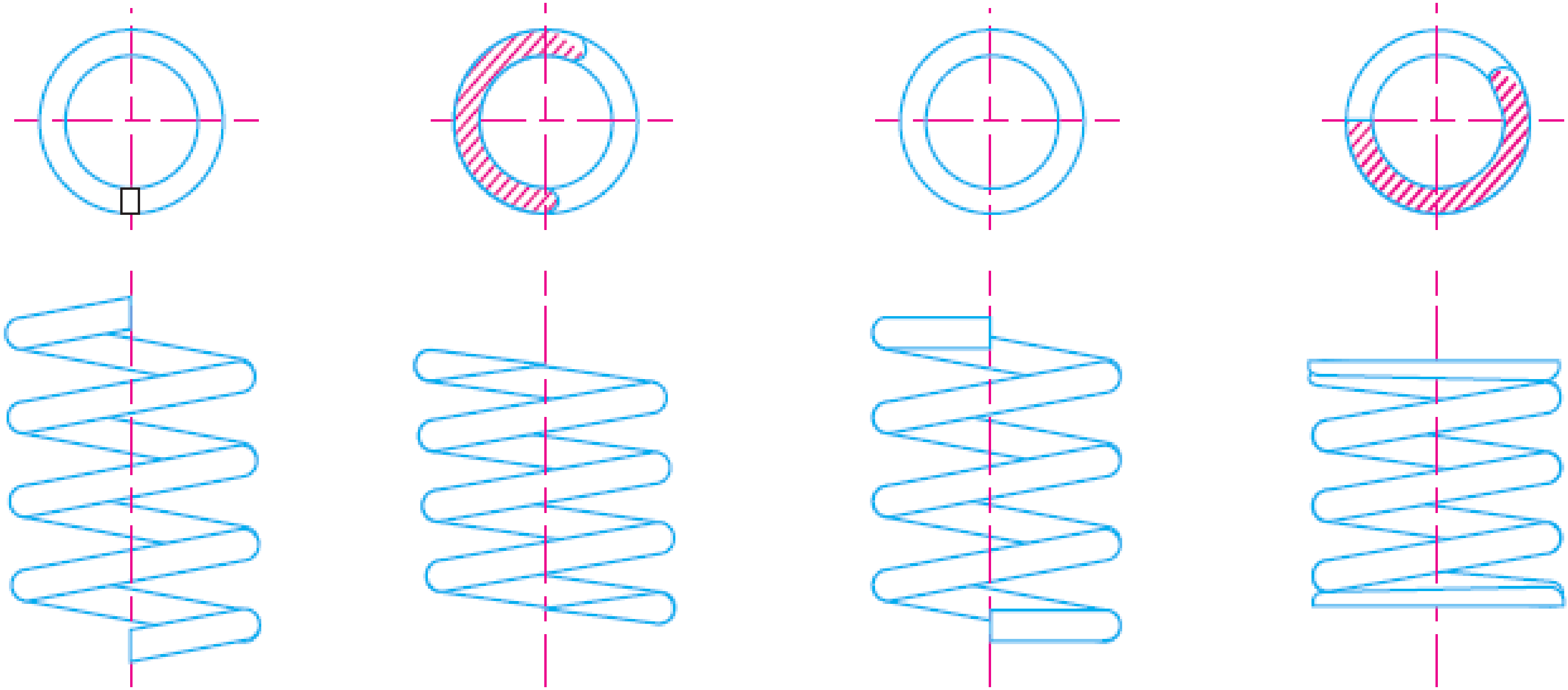
- Where G = Modulus of rigidity in N/mm^2
- n = number of active coils or turns
- n’ = total number of coils or turns
- C = spring index
- D_m = mean coil diameter in mm
- d = wire diameter in mm

$$D_o = D_m + d$$

$$D_i = D_m - d$$

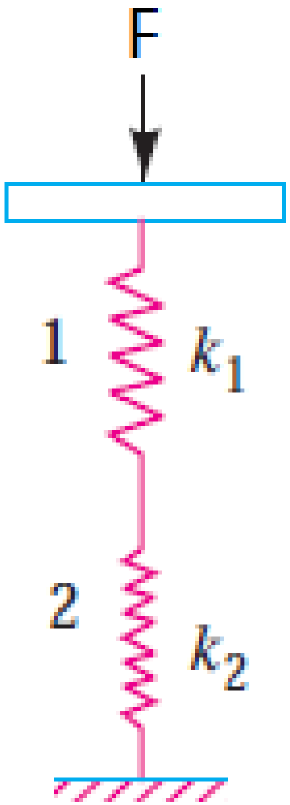
Types of ends for Helical Compression Spring

Types of end	Total no. of coils (n’)	Free length (L_F)	Solid length (L_s)
Plain end	n	P n + d	(n + 1) d
Plain and Ground end	n	P n	n d
Square end	n + 2	P n + 3d	(n + 3) d
Square and Ground end	n + 2	P n + 2d	(n + 2) d



(*a*) Plain ends. (*b*) Ground ends. (*c*) Squared ends. (*d*) Squared and ground ends.

Springs in Series



Springs in series.

Consider two springs connected in series as shown in Figure

Let F = Load carried by the springs,

$\delta 1$ = Deflection of spring 1,

$\delta 2$ = Deflection of spring 2,

K_1 = Stiffness of spring 1 = $W / \delta 1$, and

K_2 = Stiffness of spring 2 = $W / \delta 2$

A little consideration will show that when the springs are connected in series,

then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.

$\therefore$ *Total deflection of the spring*

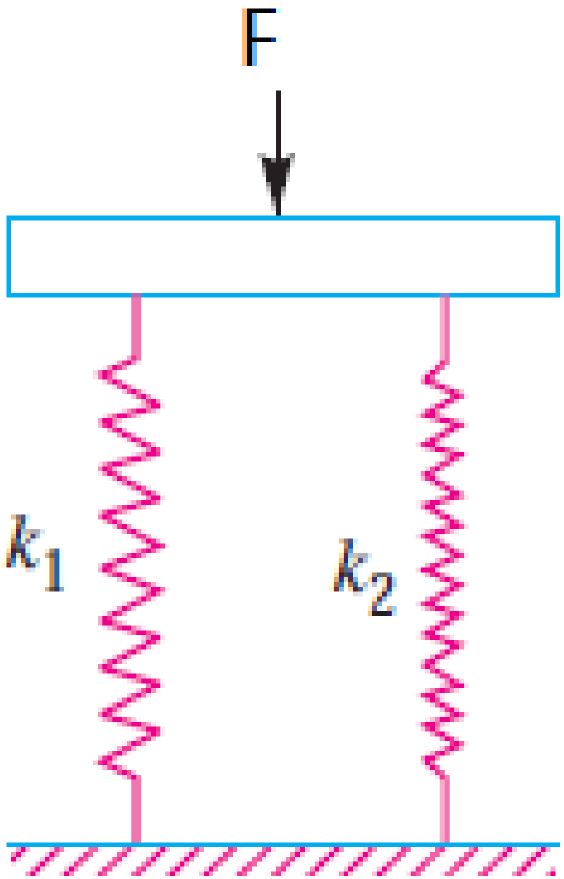
$$\delta = \delta_1 + \delta_2$$

$$or\ \frac{F}{K} = \frac{F}{K_1} + \frac{F}{K_2}$$

$$\therefore \frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

Where $K =$ *combined stiffness of the spring*

Springs in Parallel



Springs in parallel.

Consider two springs connected in parallel as shown in Figure

Let F = Load carried by the springs,

F_1 = Load shared by spring 1,

F_2 = Load shared by spring 2,

K_1 = Stiffness of spring 1, and

K_2 = Stiffness of spring 2.

A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.

$$We\,know\,that$$

$$F = F_1 + F_2$$

$$K \times \delta = K_1 \times \delta + K_2 \times \delta$$

$$K \times \delta = \delta(K_1 + K_2)$$

$$\therefore K = K_1 + K_2$$

$$where\, K = Combined\ stiffnes\ of\ the\, spring$$

$$\delta = Deflection\ \ produced$$

Surge in Springs

- When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire.
- A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils.
- In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely.

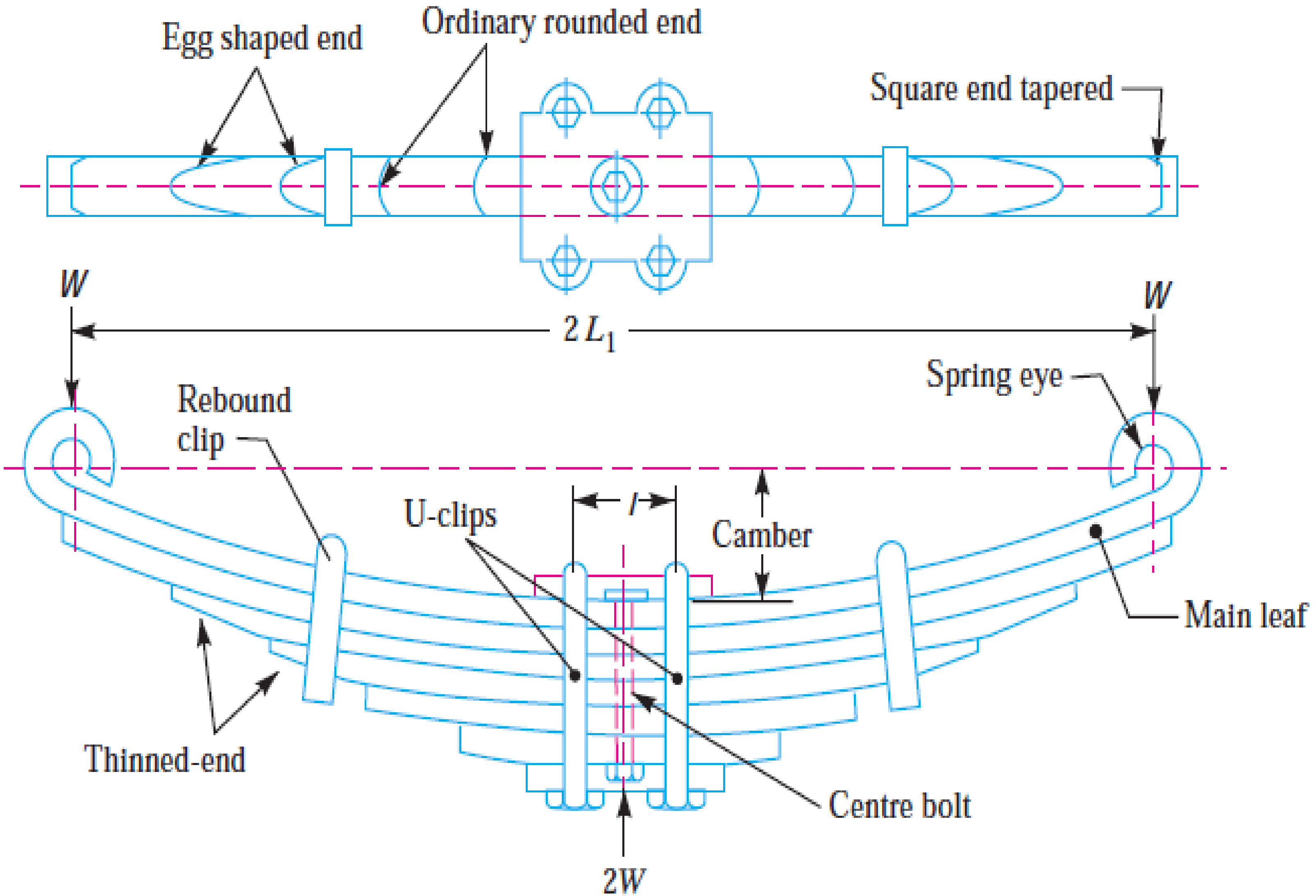
- If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur.
- This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge.***

Methods to avoid Surge in Springs

- The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Semi elliptical leaf spring



- The Fig shows a laminated semi- elliptic spring.
- The longest leaf is called as master leaf and has its ends formed in the shape of an eye through which bolts are passed to secure the spring to its supports.
- Usually, the eyes through which spring is attached to shackle are provided with bushing of anti-friction material.
- The other leaves are called as graduated leaves which are arranged in the order of decreasing length and then clamped to the master leaf with the help of strip.

- The camber shown in the figure is known as positive camber.
- The central clamp is required to hold the leaves of the spring.
- However, the bolt holes required to engage the bolts to clamp the leaves weaken the spring to some extent.
- Rebound clips help to share the load from the master leaf to the graduated leaf.

Cont.

- Since master leaf has to withstand vertical bending loads as well as the loads due to sideways of vehicle, therefore due to presence of stresses caused by these loads, it is usual to provide two full length leaves and rest as graduated leaves.

Utility

- **Centre bolt –** The leaf spring is made up of number of leaves. These leaves are held together by a bolt at the centre known as centre bolt.
- **U-clamp –** The leaf is clamped to the axle by means of U-clamp.
- **Rebound clip –** They are located at intermediate position in the length of spring, so that, the graduated leaves can also share the stresses induced in the full length leaves, when the spring rebounds.

- **Camber –** the leaves are initially given curvature, so that they will tend to straighten under the action of load. This is called as camber.
- **Material for Leaf Spring –**
- The automobile leaf springs are made up of oil hardened and tempered alloy steels such as 50Cr1, 50Cr1V23, 55Si2Mn90.

Advantages of Leaf Spring

- In addition to an energy absorbing device, leaf spring acts as a structural member.
- Leaf springs are used in automobile suspensions due to capability to take lateral loads, brake torque and driving torque in addition to shocks.

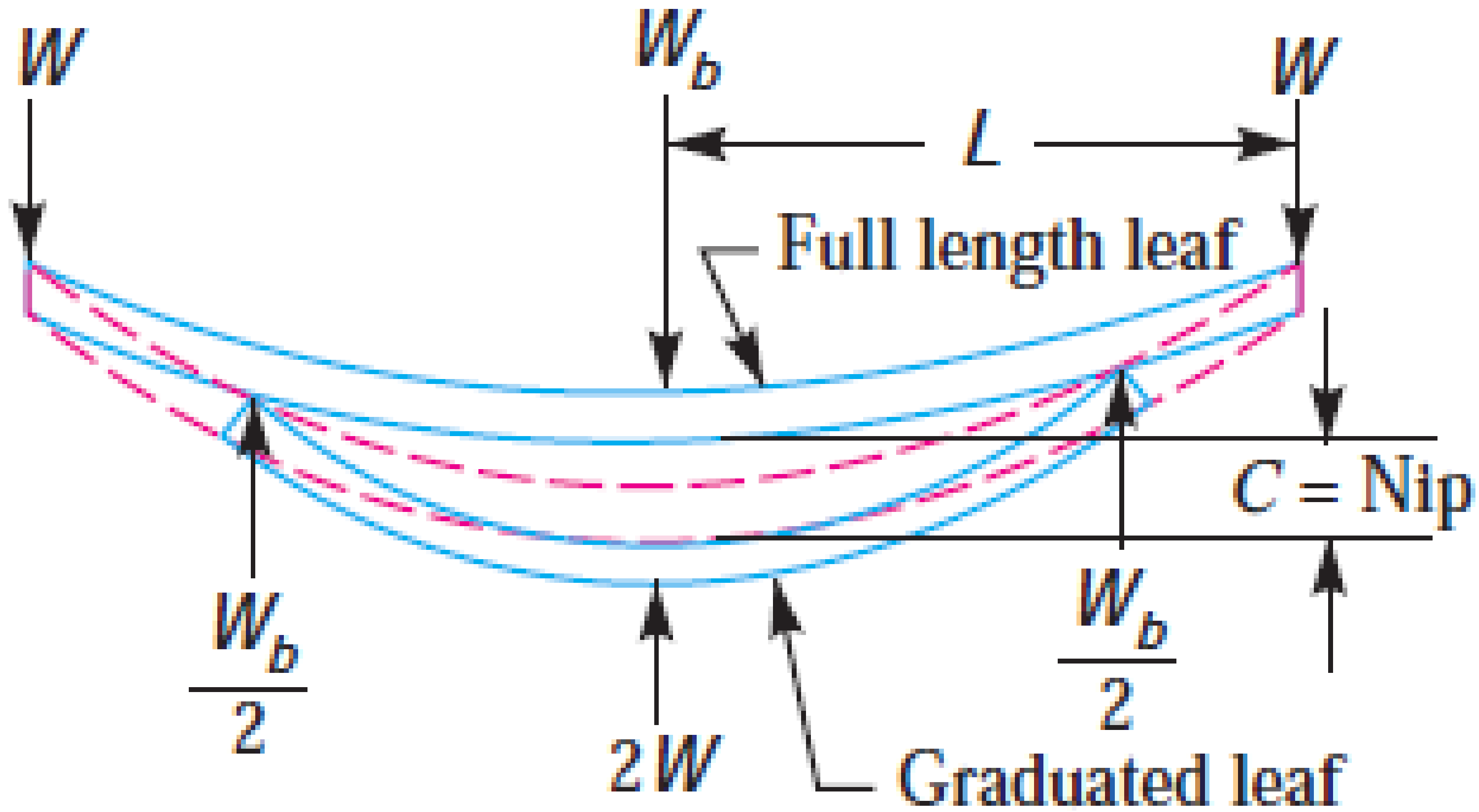
Nipping in Leaf Springs

$We\,know\,that$

$$Stress\,in\;full\;length\;leaves = \sigma_F = \frac{18WL}{bt^2(2n_g + 3n_f)}\;and$$

$$Stree\,in\;graduated\;leaves = \sigma_G = \frac{12WL}{bt^2(2n_g + 3n_f)}$$

We have already discussed that the stress in the full length leaves is 50% greater than the stress in the graduated leaves. In order to utilize the material to the best advantage, all the leaves should be equally stressed.



W
W
b
W
L
Full length leaf
C = Nip
W
b
2
2W
W
b
2
Graduated leaf

- This may be achieved by pre-stressing the leaves.
- The pre-stressing of the spring can be done by giving a greater radius of curvature to the full length leaves than graduated leaves, as shown in Fig. before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the leaves. This initial gap, as shown by *C in Fig. is called* ***nip.***
- ***When the central bolt,*** holding the various leaves together, is tightened, the full length leaf will bend back as shown dotted in Fig. and have an initial stress in a direction opposite to that of the normal load.

- The graduated leaves will have an initial stress in the same direction as that of the normal load.
- When the load is gradually applied to the spring, the full length leaf is first relieved of this initial stress and then stressed in opposite direction. Consequently, the full length leaves will be stressed less than the graduated leaf.
- This process of pre-stressing the spring by giving different radii of curvature before assembly is known as **nipping.**
- The initial gap between the leaves may be adjusted so that under maximum load condition the stress in all the
- leaves is equal.

- Normally the nip is adjusted to give stress in full length leaves slightly less than the graduated leaves. This is desirable in automobile, because full length leaves are expected to take transverse forces in addition to bending load.