3. Keys and Couplings

Keys –

- A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.
- It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses.
- A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

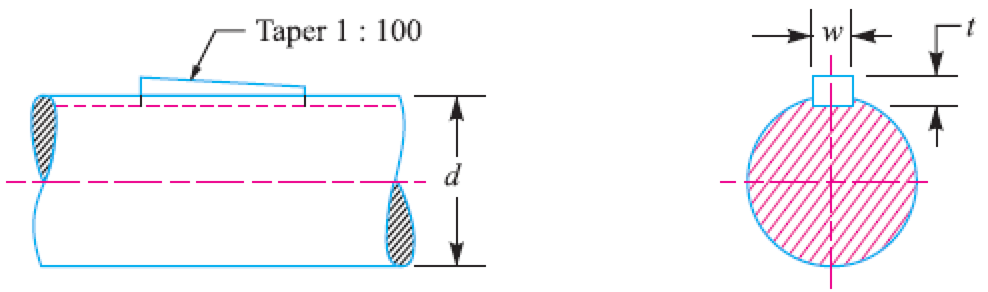
- The following types of keys are important from the subject point of view :

1. **Sunk keys,**
2. **Saddle keys,**
3. **Tangent keys,**
4. **Round keys,**
5. **Splines.**

Sunk Keys

- The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1. ***Rectangular sunk key*. A rectangular sunk key is shown in Fig.**



The usual proportions

- **Width of key, *w = d / 4***
- ***thickness of key, t = 2w / 3 = d / 6***
- where *d = Diameter of the shaft or diameter of the hole in the hub.*
- The key has taper 1 in 100 on the top side only.

2. *Square sunk key.*

- The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.*
- $w = t = d / 4$

3. *Parallel sunk key.*

- The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. Gib-head key.

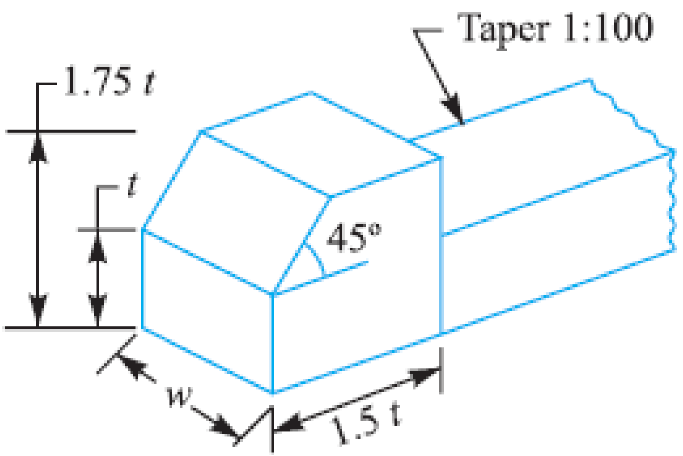
It is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key.

The usual proportions of the gib head key are :

Width, *w* = *d* / 4 ;

and thickness at large end,

***t* = 2*w* / 3 = *d* / 6**



Effect of Keyways

- A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft.
- This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft.
- It other words, the torsional strength of the shaft is reduced.
- The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

$where, e = Shaft\ strength\ factor$

$$e = \frac{strength\ of\ shaft\ with\ keyway}{strength\ of\ shaft\ without\ keyway\ for\ same\ shaft}$$

$w = width\ of\ keyway$

$d = diameter\ of\ shaft$

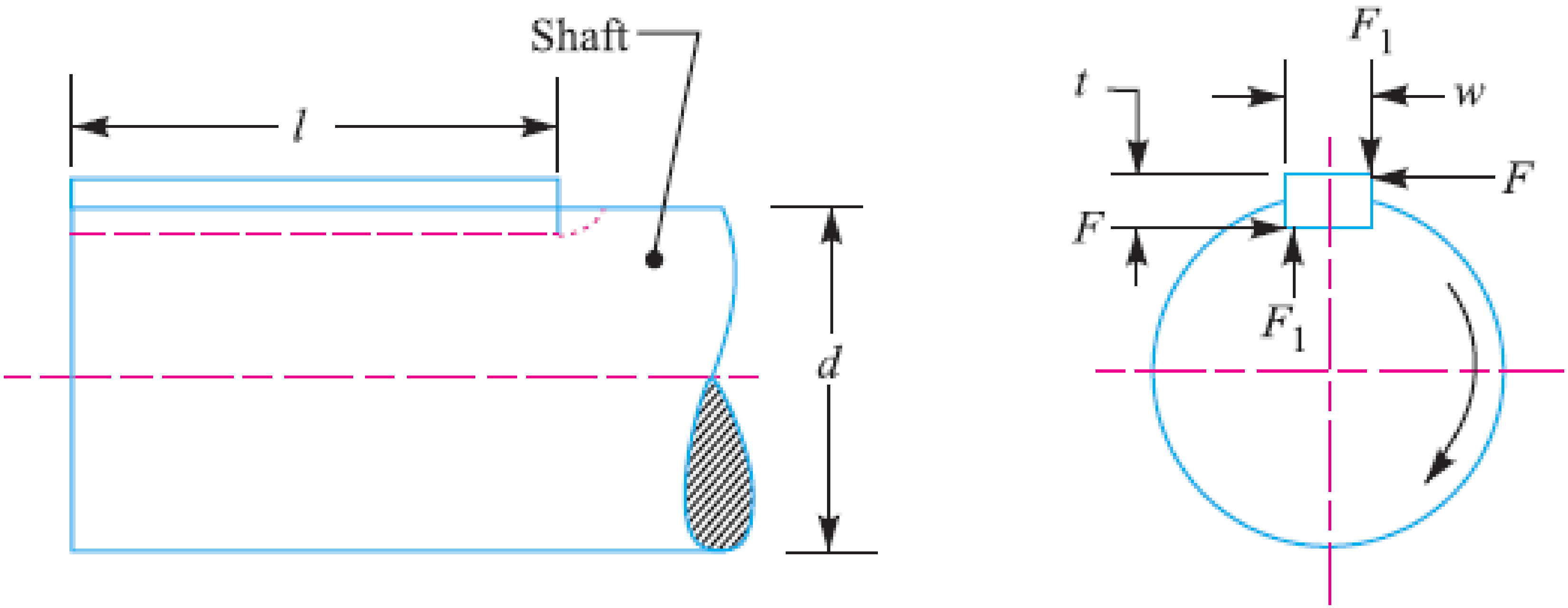
$h = depth\ of\ keyway = \left(\frac{thickness\ of\ keyway(t)}{2}\right)$

$If\ shaft\ is\ too\ long\ and\ sliding\ keyway\ then$

$K_o = \mathrm{Re}duction\ factor\ for\ angular\ twist$

$$K_o = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

Design of Key



Let T = *Torque transmitted by the shaft,*

F = *Tangential force acting at the circumference of the shaft,*

d = *Diameter of shaft,*

l_k = *Length of key,*

w_k = *Width of key.*

t_k = *Thickness of key, and*

τ_k and σ_{crk} = *Shear and crushing stresses for the material of key.*

1) Direct Shear stress in key –

$$\tau_d = \frac{Shear\ \ force}{Area} = \frac{F}{w \times l}$$

$$\therefore F = w \times l \times \tau_d$$

$$Also\,Torque = T = Force \times \mathrm{Re}\,dial\ dis\,\mathrm{tan}\,ce$$

$$T = F \times \frac{d}{2}$$

$$\therefore T = w \times l \times \tau_d \times \frac{d}{2}$$

2) Crushing stress in key –

$$\sigma_{cr} = \frac{Compressive\ force}{Area} = \frac{F}{A}$$

$$\sigma_{cr} = \frac{F}{l \times \frac{t}{2}}$$

$$\therefore F = l \times \frac{t}{2} \times \sigma_{cr}$$

$$also, Torque = T = F \times \frac{d}{2}$$

$$T = l \times \frac{t}{2} \times \sigma_{cr} \times \frac{d}{2}$$

Couplings

- Shafts are available in varying length from 6 to 10 meters for easy handling and transportation.
- Larger length shafts can not be manufactured in correct for the use of power transmission.
- But in actual practice, larger length shafts are required for transmission of torque and power.
- This requirement will be fulfilled by the use of coupling which joined two or more shafts so coupling is a device used to join two or more shafts.

Requirements of a Good Shaft Coupling

- **A good shaft coupling should have the following requirements :**

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. It should have no projecting parts.

Factors Consider in Selection of Coupling

1) Torque requirement.
2) Speed involved.
3) Shaft misalignment.
4) Operating condition.
5) Cyclic operation.
6) Direction of rotation.
7) Life of coupling.
8) Duty or work involved.

- **Why a coupling should be placed as close to a bearing as possible?**

Answer – Coupling should be placed as close to a bearing because of following reasons

1. It gives minimum vibrations.
2. Bending load on the shaft can be minimized.
3. It increases power transmission stability.
4. To avoid deflections of shaft.

Types of Couplings

1) **Rigid coupling –**

   it is used to connect two shafts which are parallel and in alignment.

a) **Sleeve or muff coupling.**

b) **Clamp or split muff coupling.**

c) **Flange coupling.**

2) Flexible coupling –

It is used to connect two shafts which are parallel and not in alignment.

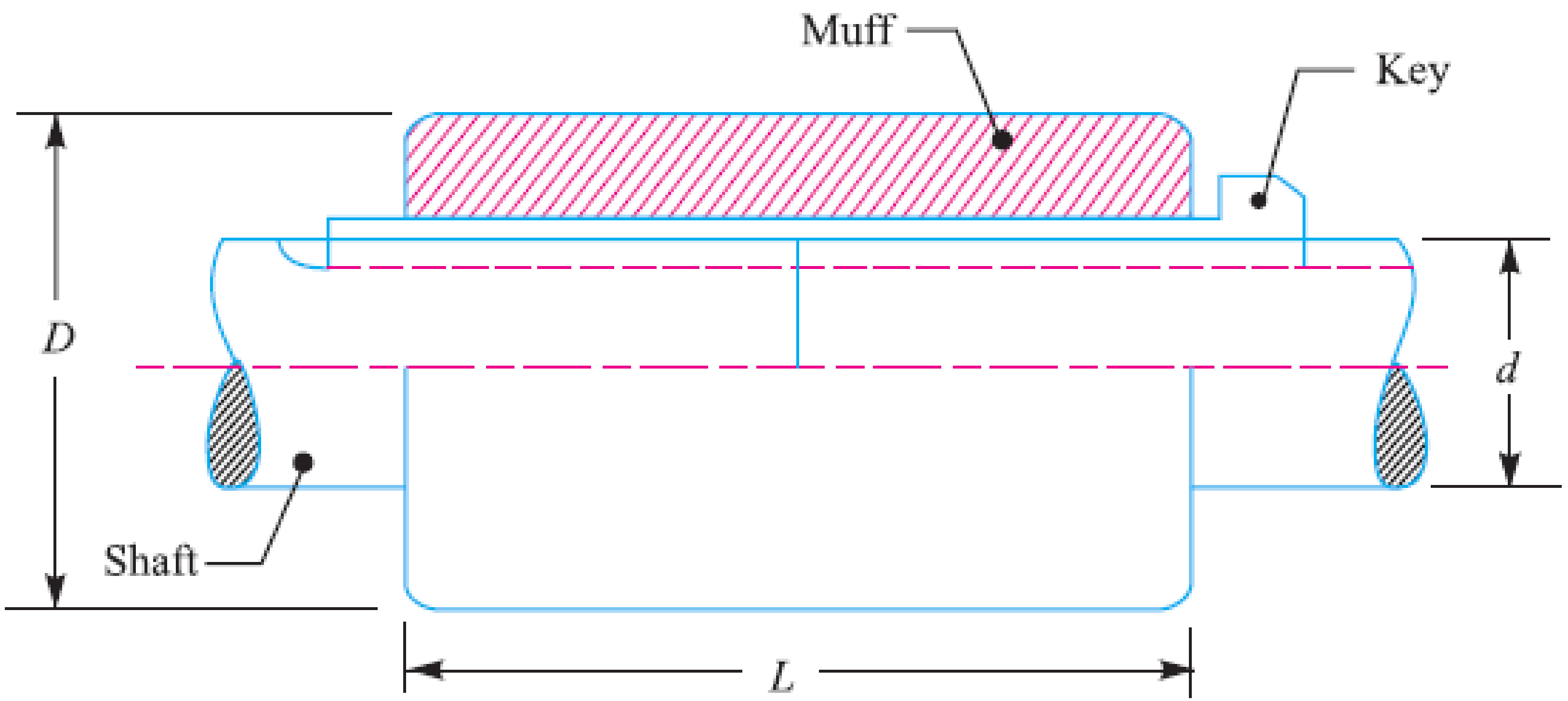
a) **Bushed pin type flexible coupling.**

b) **Universal coupling.**

c) **Oldham coupling.**

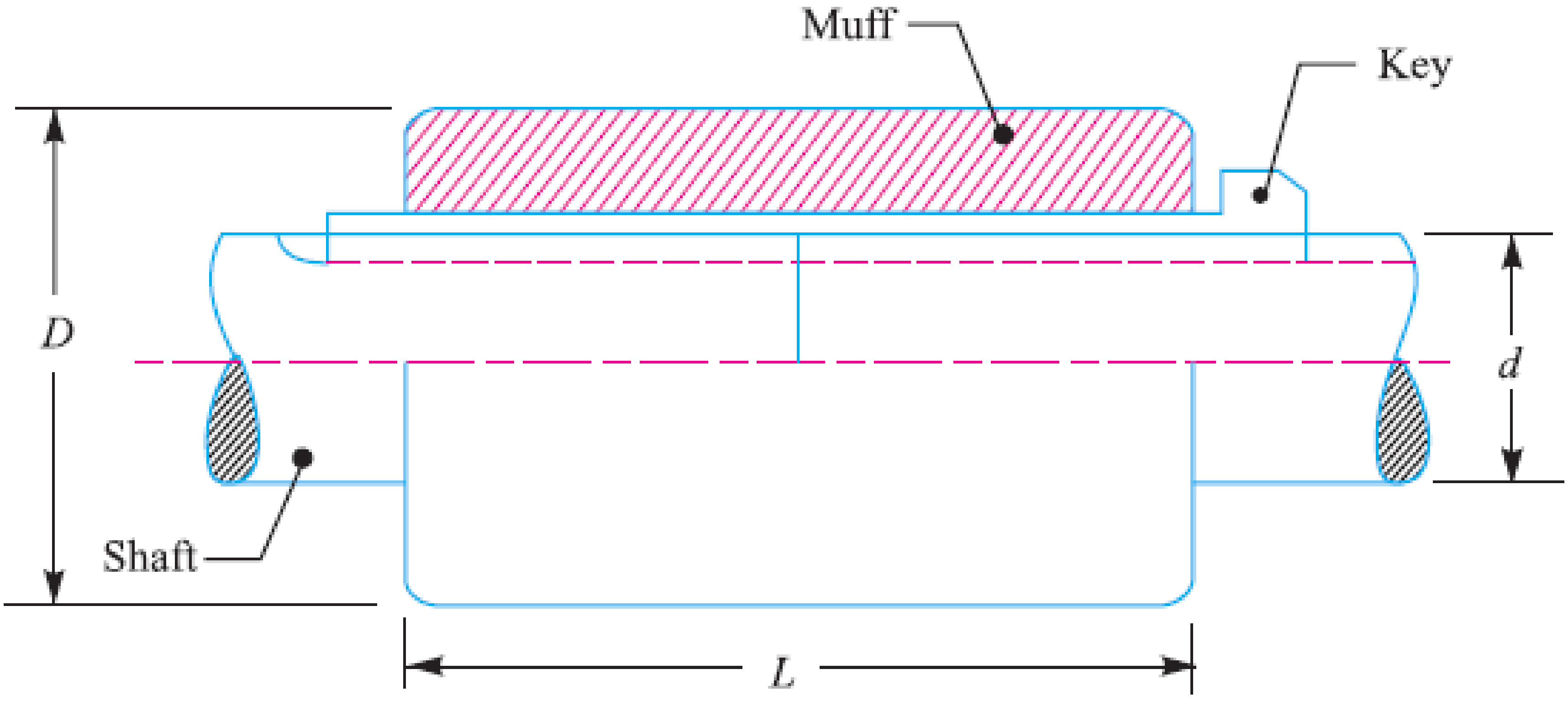
Parameters	Rigid Coupling	Flexible Coupling
Purpose	Rigid couplings are used to connect two shafts which are perfectly aligned.	Flexible couplings are used to connect two shafts having small misalignment.
Alignment	Rigid couplings can not tolerate any misalignment between two shafts.	Flexible couplings can tolerate small amount of misalignment between two shafts.
Shock & Vibration	Rigid couplings can not absorb shock and vibration.	Flexible couplings can absorb shock and vibration.
Deflection	In rigid couplings, shaft deflection is less.	In flexible couplings, shaft deflection is more.
Cost	These are less expensive	These are more expensive.

1. Muff or Sleeve Coupling



It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque.

Design of Muff or Sleeve Coupling



Advantages – 1. It is simple in construction

2. It has no projection part.

- Muff or Sleeve Coupling

1) Design of Shaft –

The diameter of shaft should be calculated as discussed in earlier section of shaft.

2) Design of Sleeve –

The sleeve is design by considering a hollow shaft.

According to standard proportion –

Outside diameter of sleeve = D = 2d + 13

Length of sleeve = L = 3.5d

3) *Considering the torsional shear failure of sleeve*

$$T = \frac{\pi}{16} \times \tau \times D^3 (1 - k^4)$$

$$Where\ k = \frac{d}{D}$$

This equation is used to check the shear stress in sleeve

4) *Design of key*

For rectangular key

$$w_k = \frac{d}{4} \quad \& \quad t_k = \frac{d}{6}$$

For square key

$$w_k = t_k = \frac{d}{4}$$

And length of key $= l_k = \frac{L}{2} = \frac{3.5d}{2}$

After that key is to checked for

Shearing and crushing

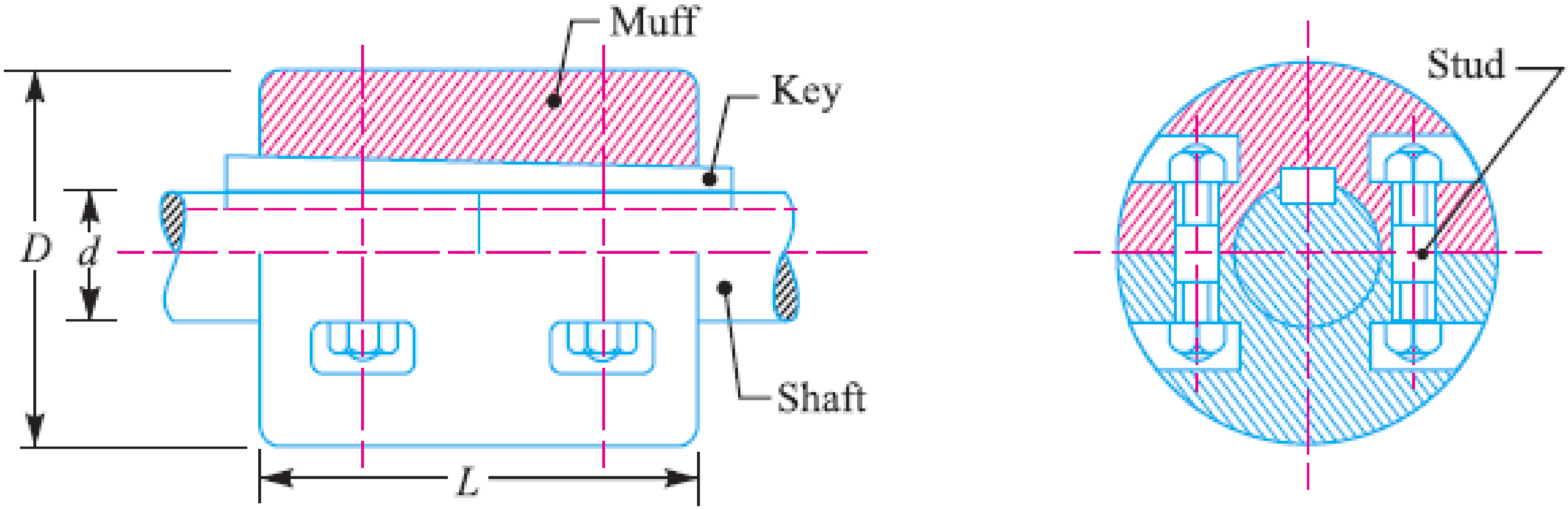
$$i)\, T = l_k \times w_k \times \tau_k \times \frac{d}{2} \quad --- shearing$$

$$ii)\, T = l_k \times \frac{t_k}{2} \times \sigma_{cr\,k} \times \frac{d}{2} \quad --- crushing$$

Split Muff Coupling

- It is also known as ***split muff coupling.***
- ***In*** this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig.
- The halves of the muff are made of cast iron. The shaft ends are made to about each other and a single key is fitted directly in the keyways of both the shafts.
- One-half of the muff is fixed from below and the other half is placed from above.

- Both the halves are held together by means of mild steel studs or bolts and nuts.
- The number of bolts may be two, four or six.
- The nuts are recessed into the bodies of the muff castings.
- This coupling may be used for heavy duty and moderate speeds.
- The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the couplings.



Split Muff Couplings

Design procedure

- *T = Torque transmitted by the shaft,*
- *d = Diameter of shaft,*
- d_b *= Root or effective diameter of bolt,*
- *n = Number of bolts,*
- σ*t = Permissible tensile stress for bolt material,*
- μ = Coefficient of friction between the muff and shaft, and
- *L = Length of muff.*

1) Design of Shaft –

The diameter of shaft should be calculated as discussed in earlier section of shaft.

2) Design of Sleeve –

The sleeve is design by considering a hollow shaft.

According to standard proportion –

Outside diameter of sleeve = D = 2d + 13

Length of sleeve = L = 3.5d

3) *Considering the torsional shear failure of sleeve*

$$T = \frac{\pi}{16} \times \tau \times D^3 (1 - k^4)$$

$$Where\, k = \frac{d}{D}$$

This equation is used to check the shear stress in sleeve

4) *Design of key*

For rectangular key

$$w_k = \frac{d}{4} \quad \& \quad t_k = \frac{d}{6}$$

For square key

$$w_k = t_k = \frac{d}{4}$$

And length of key $= l_k = \frac{L}{2} = \frac{3.5d}{2}$

After that key is to checked for

Shearing and crushing

$$i)\ T = l_k \times w_k \times \tau_k \times \frac{d}{2} \quad ---shearing$$

$$ii)\ T = l_k \times \frac{t_k}{2} \times \sigma_{cr\,k} \times \frac{d}{2} \quad ---crushing$$

The bolts are subjected to tensile load

$$\sigma_t = \frac{force(f)}{\frac{\pi}{4} \times d_b^2}$$

$$\therefore \text{Force exerted by each bolt} = f = \frac{\pi}{4} \times d_b^2 \times \sigma_t$$

The force exerted by the bolts on each side of the shaft is

$$f = \frac{\pi}{4} \times d_b^2 \times \sigma_t \times \frac{n}{2}$$

$\therefore$ *For uniform pressure distribution over the surface due to pressure 'p' on the shaft & muff*

$$P = \frac{force}{projected\ area}$$

$$P = \frac{\frac{\pi}{4} \times d_b^2 \times \sigma_t \times \frac{n}{2}}{\frac{1}{2} \times L \times d}$$

$$Frictional\ \ force\ between\ each\ shaft\ \&\ muff$$

$$F = \mu \times \text{Pr}essure \times area$$

$$F = \mu \times P \times \frac{1}{2}\pi dL$$

$$F = \mu \times \frac{\frac{\pi}{4} \times d_b^2 \times \sigma_t \times \frac{n}{2}}{\frac{1}{2} \times L \times d} \times \frac{1}{2} \times \pi dL$$

$$F = \mu \times \frac{\pi^2}{8} \times d_b^2 \times \sigma_t \times n$$

$$\therefore Torque\, transmitted\, by\, coupling\; is$$

$$T = F \times \frac{d}{2}$$

$$T = \mu \times \frac{\pi^2}{8} \times d_b^2 \times \sigma_t \times n \times \frac{d}{2}$$

$$T = \frac{\pi^2}{16} \times \mu \times d_b^2 \times \sigma_t \times n \times d$$

$$From\, this\, equation\; d_b\; can\, be\, calculated\,.$$

Flange Couplings

- A flange coupling usually applies to a coupling having two separate cast iron flanges.
- Each flange is mounted on the shaft end and keyed to it.
- The faces are turned up at right angle to the axis of the shaft.
- One of the flange has a projected portion and the other flange has a corresponding recess. This helps to bring the shafts into line and to maintain alignment.

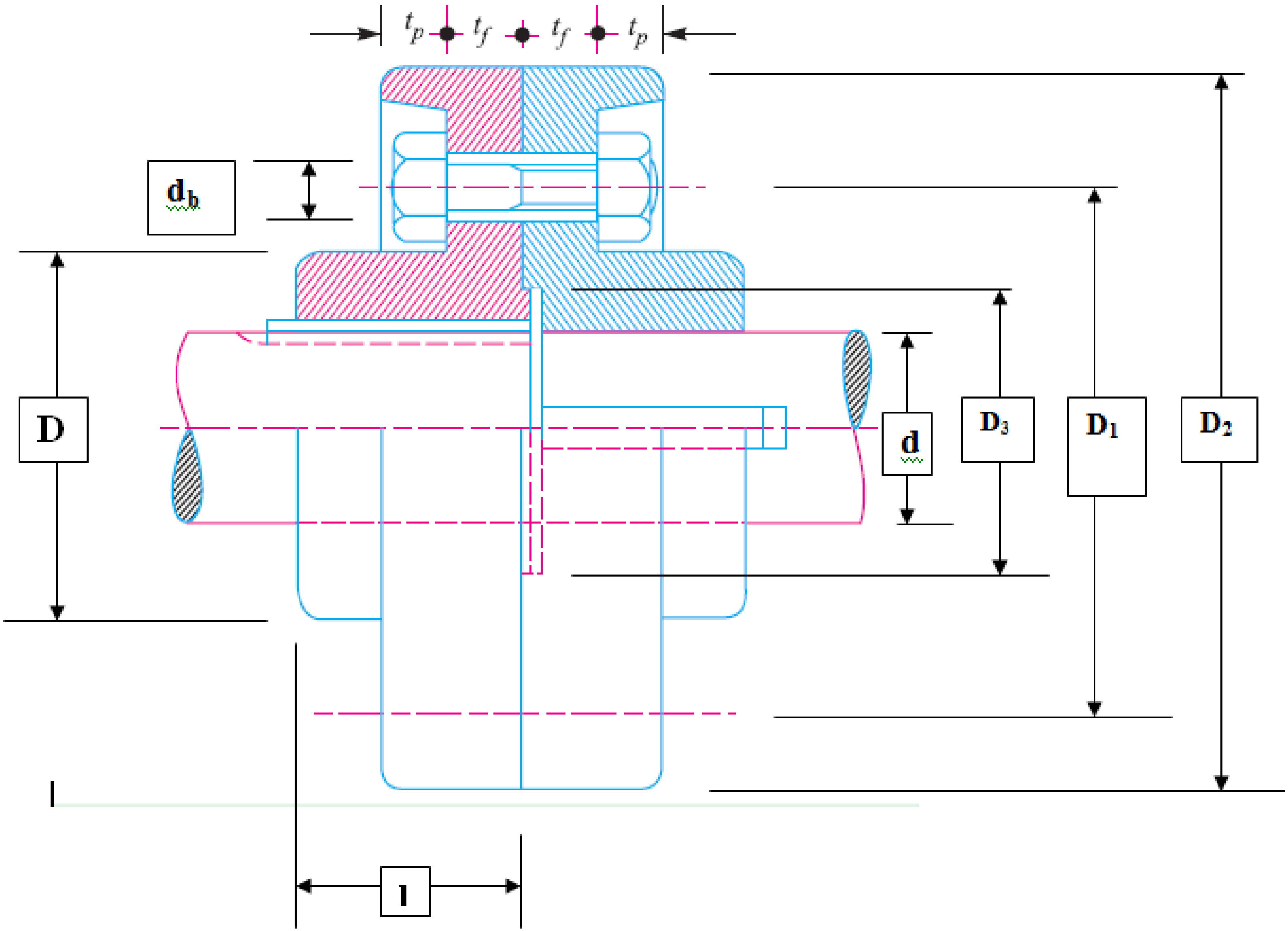
- The two flanges are coupled together by means of bolts and nuts. The flange coupling is adopted to heavy loads and hence it is used on large shafting.
- The flange couplings are of the following three types :

1) Unprotected type flange coupling
2) Protected type flange coupling
3) Marine type flange coupling

Protected Type Flange Coupling

- **Protected Type Flange Coupling**

Protected Type Flange Coupling



$$Let,\ d = diameter\ of\ shaft\ in\ mm$$

$$D = outer\ diamter\ of\ hub$$

$$D_1 = diameter\ of\ bolt\ circle$$

$$D_2 = outer\ diameter\ of\ flange$$

$$D_3 = diameter\ of\ flange\ recess$$

$$l = length\ of\ hub$$

$$t_f = thickness\ of\ flange$$

$$t_p = thickness\ of\ protective\ flange$$

$$d_b = nominal\ diameter\ of\ bolt$$

$$n = number\ of\ bolts$$

- The design of rigid flange coupling can be done in two different ways, depending upon the fit of the bolts in flange holes.
- If bolts are fitted in reamed holes and are finger-tight, in such case, the torque is transmitted by the shear resistance and crushing resistance of the bolts.
- If the bolts are fitted in large clearance holes and are tightened sufficiently with pre-load, in such case, the torque is transmitted from one flange to the other not through the bolts but due to friction between the two flanges.

1. **Design of Shaft & Key –**

   The shaft and keys are designed as discussed in earlier sections.

2. Dimensions of coupling as standard proportions

$$Outer\ diameter\ of\ hub = D = 2d$$

$$Diameter\ of\ bolt\ circle = D_1 = 3d$$

$$Outer\ diameter\ of\ flange = D_2 = 4d$$

$$Diameter\ of\ flange\ recess = D_3 = 1.1D = 2.2d$$

$$Length\ of\ hub = l = 1.5d$$

$$Thickness\ of\ flange = t_f = 0.5d$$

$$Thickness\ of\ protective\ flange = t_p = 0.25d$$

$$Number\ of\ bolts = n = 3\ for\ d\ upto\ 35\ mm$$

$$= 4\ for\ d\ upto\ 55\ mm$$

$$= 6\ for\ d\ upto\ 150\ mm$$

$$= 8\ for\ d\ upto\ 230\ mm$$

3. Design of Hub –

The hub is subjected to a torsional shear stress.

Considering it as a hollow shaft.

$$T = \frac{\pi}{16} \times \tau_h \times D^3 (1 - k^4)$$

$$where\, k = \frac{d}{D}$$

For safety

$$\tau_h \leq \tau_{Given}$$

4. Design of Flange –

The flange is subjected to a direct shear at the junction with the hub.

$$T = shear\ area \times direct\ shear\ stress \times outside\ radius\ of\ hub$$

$$T = \pi D t_f \times \tau_f \times \frac{D}{2}$$

For the safety of flange against shear failure

$$\tau_f \le \tau_{Given}$$

5. Design of Bolts

a) If bolts are fitted in reamed holes –

In such case, the bolts are subjected to a direct shear stress and crushing stress.

i) Considering the Shearing of bolts –

$$T = no.of\ bolts \times shear\ area\ of\ each\ bolts \times$$

$$shear\ stress \times radius\ of\ bolt\ circle$$

$$T = n \times \frac{\pi d_b^2}{4} \times \tau_b \times \frac{D_1}{2}$$

$$From\ this\ equation\ d_b\ may\ be\ obtained$$

ii) Considering crushing failure of bolts –

The bolts as well as the contact area of flange are subjected to crushing stress.

$$T = no.of\ bolts \times projected\ area\ of\ each\ bolt \times$$
$$in\ contact\ with\ flange \times crushing\ stress \times radius\ of\ bolt\ circle$$

$$T = n \times d_b t_f \times \sigma_{crb} \times \frac{D_1}{2}$$

From this equation crushing stress in bolt is to be checked

For safety of bolts

$$\sigma_{crb} \leq \sigma_{cr(Given)}$$

b) If bolts are fitted in large clearance holes –

In such case, the torque is transmitted from one flange to the other due to friction between them.

Hence, according to uniform intensity of pressure theory, the torque transmitting capacity of flange coupling is given by –

$$T = \frac{2}{3} \times \mu \times n \times W \left[\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \right]$$

$$where\ n = number\ of\ bolts$$

$$\mu = coefficient\ of\ friction\ between\ two\ flanges$$

$$W = preload\ in\ each\ bolt$$

$$r_o = outer\ radius\ of\ flange = \frac{D_2}{2}$$

$$r_i = radius\ of\ flange\ recess = \frac{D_3}{2}$$

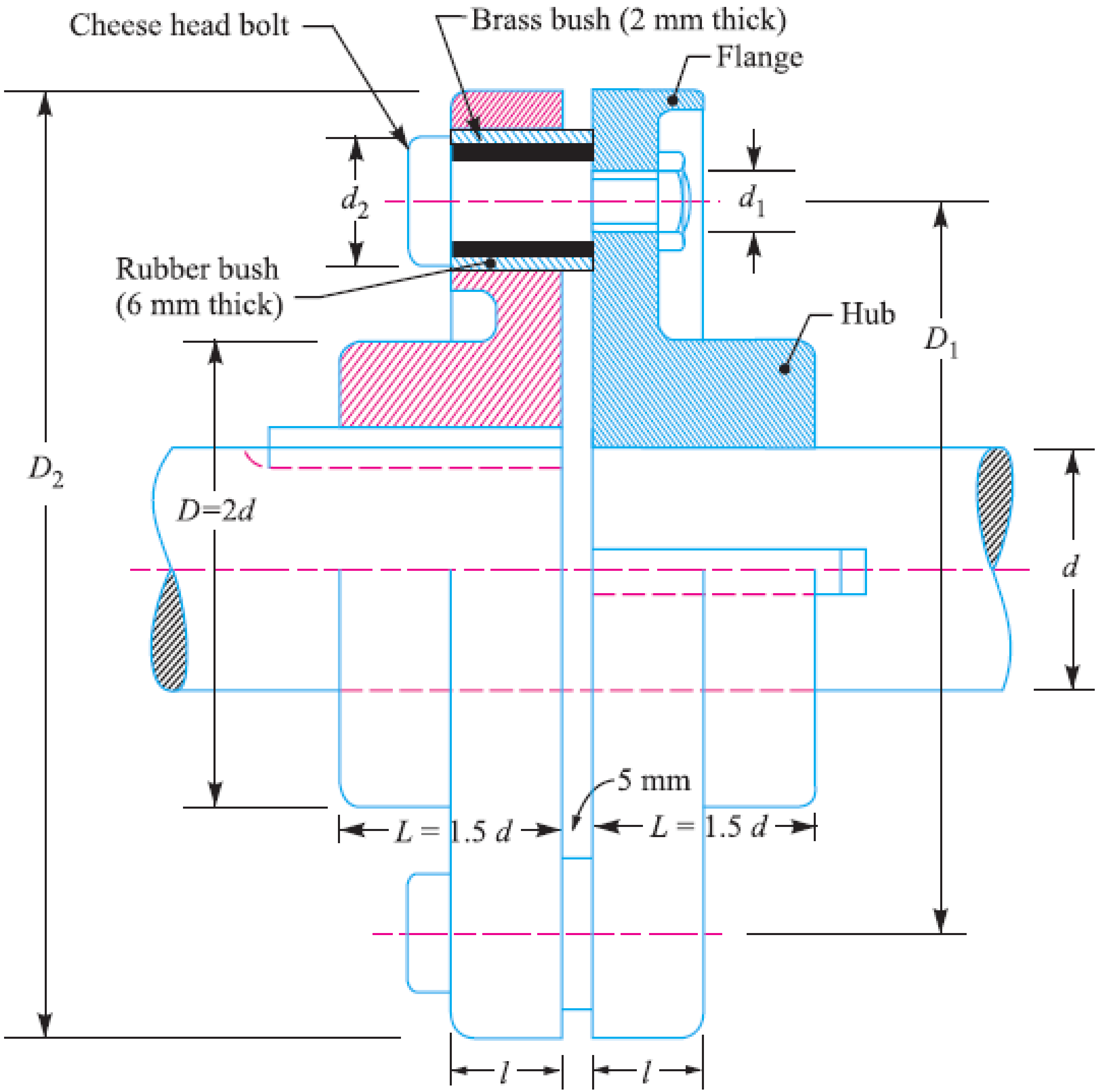
The industrial practice is to select the coupling dimensions for the given shaft from standard tables and check the stresses induced in various parts.

Bushed Pin Type Flexible Coupling

- A bushed-pin flexible coupling, as shown in Fig. is a modification of the rigid type of flange coupling.
- The coupling bolts are known as pins.
- The rubber or leather bushes are used over the pins.
- The two halves of the coupling are dissimilar in construction.
- A clearance of 5 mm is left between the face of the two halves of the coupling.

- There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.
- In designing the bushed-pin flexible coupling, the proportions of the rigid type flange coupling are modified.
- The main modification is to reduce the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm2.
- In order to keep the low bearing pressure, the pitch circle diameter and the pin size is increased.

- **Bushed Pin type Flexible Coupling**



Cheese head bolt
Brass bush (2 mm thick)
Flange
Rubber bush
(6 mm thick)
Hub
d_2
d_1
D_1
D_2
$D=2d$
d
5 mm
$L = 1.5\ d$
$L = 1.5\ d$
l
l

$$Let, d = diameter\ of\ shaft\ in\ mm$$

$$D = outer\ diameter\ of\ hub$$

$$D_1 = diameter\ of\ bolt\ circle$$

$$D_2 = outer\ diameter\ of\ flange$$

$$l = length\ of\ hub$$

$$t_f = thickness\ of\ flange$$

$$t_p = thickness\ of\ protective\ flange$$

$$l_b = length\ of\ bush\ in\ flange$$

$$d_b = nominal\ diameter\ of\ pin$$

$$d_1 = diameter\ of\ enlarged\ portion\ of\ pin$$

$$d_2 = outer\ diameter\ of\ rubber\ bush$$

$$n = number\ of\ pins$$

1. <u>Design of shaft and key –</u>

- The shaft and key are designed as discussed in earlier sections.

2. <u>Dimensions of coupling as standard proportions –</u>

$$outer\ diameter\ of\ hub = D = 2d$$

$$length\ of\ hub = l = 1.5d$$

$$Thickness\ of\ flange = t_f = 0.5d$$

$$Thickness\ of\ protective\ flange = t_p = 0.25d$$

$$Number\ of\ pins = n = 3\ for\ d\ upto\ 30\,mm$$

$$4\ for\ d\ upto\ 75\,mm$$

$$6\ for\ d\ upto\ 110\,mm$$

$$8\ for\ d\ upto\ 150\,mm$$

3. Design of hub –

- The hub is subjected to a torsional shear stress.
- Considering it as hollow shaft.

$$T = \frac{\pi}{16} \times \tau_h \times D^3 (1 - k^4)$$

$$Where, \tau_h = torsional\ shear\ stress\ in\ hub$$

$$For\ safety\ of\ hub$$

$$\tau_h < \tau_{(Given)}$$

$$Then\ the\ design\ of\ hub\ is\ safe$$

4. <u>Design of Flange</u> –

- The flange is subjected to a direct shear stress at the junction of the hub.

$$T = shear\ area \times direct\ shear\ stress \times outside\ radius\ of\ hub$$

$$T = n D t_f \times \tau_f \times \frac{D}{2}$$

For safety of flange against shear

$$\tau_f < \tau_{(Given)}$$

Then the design of flange is safe.

5. Design of pins –

A) Calculate the dimensions of pin

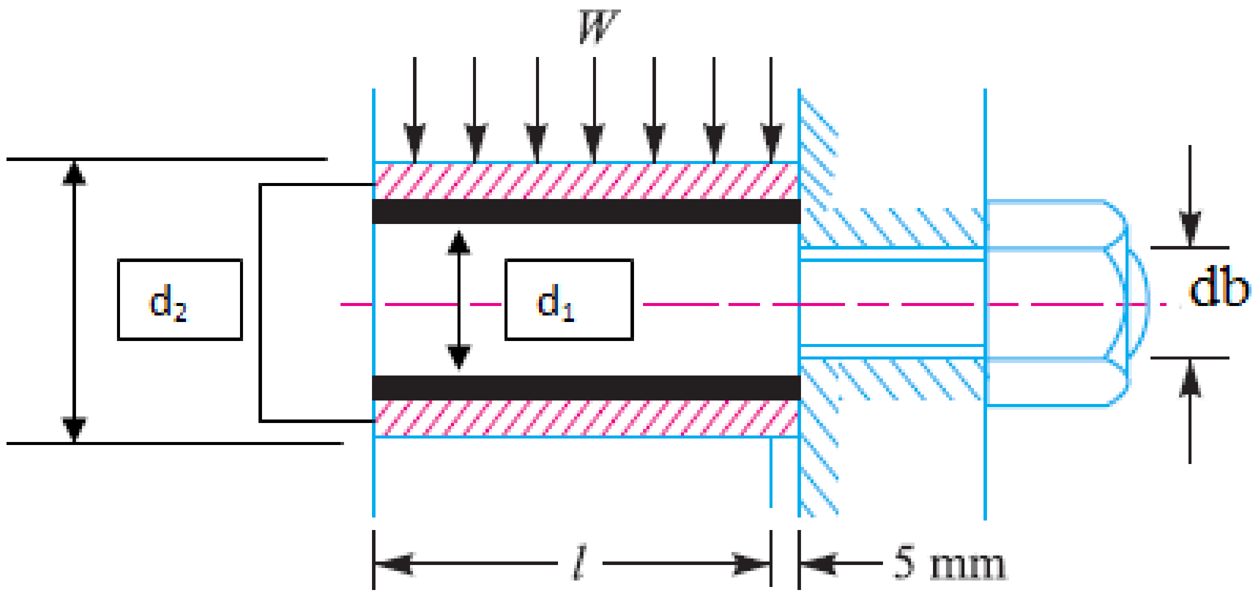
i) Select the number of pins (n)

ii) Nominal diameter of pin

$$d_b = \frac{0.5d}{\sqrt{n}}$$

The standard nominal diameter is selected.

iii) Diameter of enlarged portion of pin (d_1) –



The diameter of enlarged portion of the pin is taken as –

$$\mathbf{d_1 = d_b + 4\ mm}$$

iv) <u>Outer diameter of rubber bush (d_2)</u> –

- It is assumed that the brass bush of 2 mm thickness and rubber bush of 6 mm thickness are fitted on the enlarged portion of pin.
- Hence, the outer diameter of the rubber bush is –

$$d_2 = d_1 + (2 \times 2) + (2 \times 6)$$

v) <u>Diameter of bolt circle (D_1)</u> –

- Considering the distance of 8 mm between the outer surface of hub and the rubber bush.

$$D_1 = D + d_2 + (2 \times 8)$$

vi) <u>Length of bush in flange (l_b)</u> –

- Length of bush in flange is calculated by considering the bearing pressure on the pin.
- Bearing pressure acting on each pin is –

$$F = bearing\ pressure \times projected\ area\ of\ pin$$

$$F = P_b \times d_2 \times l_b \quad ---(1)$$

$$Torque\ transmitted\ is$$

$$T = n \times F \times \frac{D_1}{2}$$

$$Put\ the\ value\ of\ F\ from\ eq.(1)$$

$$T = n \times P_b \times d_2 \times l_b \times \frac{D_1}{2}$$

$$From\ this\ equation\ 'l_b'\ can\ be\ calculated$$

B) Stresses induced in pin –

- As each pin is rigidly fastened by nut to one of the flanges, it acts as a cantilever beam.
- The uniformly distributed load acting on the pin is equivalent to a point load 'F' acting at the centre of length 'lb'.
- Due to this force 'F' the pin is subjected to a **bending stress** and **direct shear stress.**

- **Bending stress in pin –**

$The \max imum\, B.M.on\ pin\, is$

$$M = F\left(\frac{l_b}{2} + 5\right)$$

$\therefore The \max imum\, bending\ stress\, in\ pin\, is -$

$$\sigma_b = \frac{M}{Z} = \frac{F\left(\frac{l_b}{2} + 5\right)}{\frac{\pi d_b^3}{32}}$$

$$\therefore \sigma_b = \frac{32F\left(\frac{l_b}{2} + 5\right)}{\pi d_b^3}$$

$$Where, F = \frac{T}{n \times \frac{D_1}{2}}$$

- **Direct shear stress in pin –**
- The direct shear stress induced in the pin is –

$$\tau_b = \frac{F}{\frac{\pi d_b^2}{4}}$$

$$\tau_b = \frac{4F}{\pi d_b^2}$$

- **Principal stress in pin –**
- As the pin is subjected to a bending stress and a direct stress, the maximum shear stress and the maximum principal stress induced in the pin are

$$\tau_{\max} = \sqrt{(\frac{\sigma_b^2}{2} + \tau_b^2} \ \ and$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + \sqrt{(\frac{\sigma_b^2}{2} + \tau_b^2}$$

for safety of pin

$$\tau_{\max} < \tau_{(Given)}$$

$$\sigma_{\max} < \sigma_{(Given)}$$

6) Outer diameter of flange (D_2) –

- The outer diameter of flange is taken such that the bolt circle is at equidistance from the outer surface of the flange and the outer surface of the hub.

$$D_1 = \frac{D + D_2}{2}$$

$$\therefore D_2 = 2D_1 - D$$

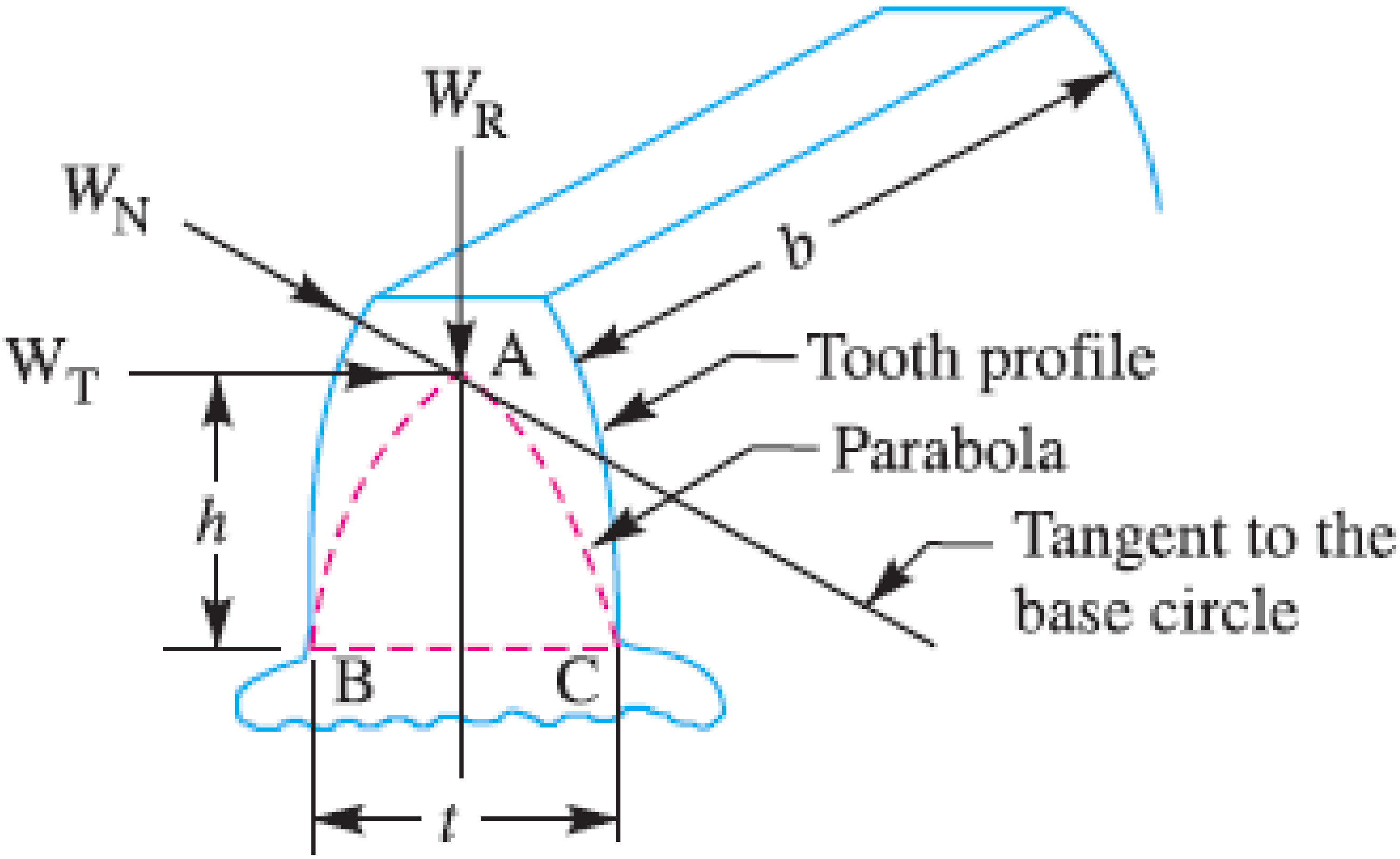
Design of Spur Gears

- Gears are defined as toothed wheels or multilobed cams which are used to transmit power and motion from one shaft to another shaft when the distance between the two shaft is small.
- It is called as positive drive and the velocity ratio remains constant.
- The gears are transmit large power and are compact in construction.
- Also they are able to transmit the motion at very low speed.

Design Considerations for a Gear Drive

- The power to be transmitted.
- The velocity ratio or speed of the gear drive.
- The central distance between the two shaft.
- Input speed of the driving gear.
- The strength of gear teeth so that they will not fail under static loading or dynamic loading under normal running condition.
- Wear characteristics of the gear tooth for a long satisfactory life.
- The use of space and material and cost should be economical.

Lewis Equation for Static Beam Strength of Spur Gear Teeth



- Consider each tooth as a cantilever beam
- loaded by a normal load (W_N) *as shown in Fig.*

  It is resolved into two components *i.e.*

  tangential component (W_T) *and radial component* (W_R) *acting perpendicular and parallel to the centre* line of the tooth respectively.
- The tangential component (W_T) *induces a bending stress which tends to* break the tooth.
- The radial component (W_R) *induces a compressive stress of relatively small magnitude*, therefore its effect on the tooth may be neglected.

- Hence, the bending stress is used as the basis for design calculations.
- The critical section or the section of maximum bending stress may be obtained by drawing a parabola through *A and tangential to the tooth curves at B and C.*
- *This parabola, as* shown dotted in Fig., outlines a beam of uniform strength, *i.e. if the teeth are shaped like a* parabola, it will have the same stress at all the sections.
- But the tooth is larger than the parabola at every section except *BC.*

- *We therefore, conclude that the section BC is the section of maximum stress* or the critical section.
- The maximum value of the bending stress (or the permissible working stress), at the section *BC is given by*

$$\sigma_b = \frac{M \times y}{I} ----(1)$$

$$Where, M = Maximum\ B.M.of\ critical\ section\ BC$$

$$M = W_T \times h$$

$$W_T = Tangential\ load\ acting\ at\ the\ tooth$$

$$h = Length\ of\ the\ tooth$$

$$y = Half\ thickness\ of\ the\ tooth = \frac{t}{2}$$

$$I = M.I.\ about\ the\ center\ line\ of\ the\ tooth$$

$$I = \frac{bt^3}{12}$$

$$b = Face\ width\ of\ gear$$

Put the values of M, y & I *in equation* (1)

$$\sigma_b = \frac{(W_T \times h) \times \frac{t}{2}}{\frac{bt^3}{12}}$$

$$\sigma_b = \frac{(W_T \times h) \times 6}{bt^2}$$

$$\therefore W_T = \frac{\sigma_b \times bt^2}{6h} \quad --(2)$$

In this equation, t & h *are variables depending upon the size of the tooth (i.e. circular pitch) and its tooth profile.*

$$Let, t = x \times P_c$$

$$\& \, h = k \times P_c$$

$$where, x \,\&\, k \; are \; cons\tan t$$

$$\therefore W_T = \sigma_b \times b \times \frac{x^2 P_c^2}{6kP_c}$$

$$Put \; y = \frac{x^2}{6k}$$

$$\therefore W_T = \sigma_b \times b \times P_c \times y$$

$$\therefore W_T = \sigma_b \times b \times \pi M \times y$$

- The quantity of ‘y’ is known as Lewis form factor or tooth form factor and W_T is called the **beam strength of the tooth.**
- The value of ‘y’ in terms of number of teeth, may be given by

$$y = 0.124 - \frac{0.684}{T} \Rightarrow for\ 14\frac{1}{2}^0\ composite\ \&\ full\ depth\ involute\ system$$

$$y = 0.154 - \frac{0.912}{T} \Rightarrow for\ 20^0\ full\ depth\ involute\ system.$$

$$y = 0.175 - \frac{0.841}{T} \Rightarrow for\ 20^0\ stub\ system.$$

Permissible Working stress (Bending stress) for Gear Teeth in the Lewis equation

- The permissible working stress in the Lewis equation depends upon the material for which an allowable static stress may be determined.
- The allowable static stress is the stress at the elastic limit of the material.
- It is also called as basic stress.

- The permissible working (bending) stress, according to Barth formula is –

$$\sigma_b = \sigma_o \times C_V$$

$$where, C_V = Velocity \;\; factor$$

$$C_V = \frac{3}{3+v} \Rightarrow \; for\; ordinary\; gear\; operating\; at\; velocity$$

$$upto\, 12.5\, m/s$$

$$C_V = \frac{4.5}{4.5+v} \Rightarrow \; for\; carefully\; gear\; operating\; at\; velocity$$

$$upto\, 12.5\, m/s$$

$$C_V = \frac{6}{6+v} \Rightarrow \; for\; accurate\; cut\; and\; velocity$$

$$upto\, 20\, m/s$$

Power Transmission Capacity of Spur Gear in Bending

- The design tangential tooth (W_T) load and power transmitted (P) and the pitch line velocity is given by equation.

$$W_T = \frac{P}{V} \times C_S$$

$$Where, W_T = Permissible \tan gential\ tooth\ load\ in\ N$$

$$W_T = \sigma_b \times b \times \pi m \times y = (\sigma_o C_V) b \times \pi m \times y$$

$$P = Power\ transmitted\ in\ watt$$

$$V = Pitch\ line\ velocity\ in\ m/s = \frac{\pi DN}{60}$$

$$C_S = Service\ factor$$

$$D = P.C.D.\ of\ gear\ in\ meters$$

$$Circular\ pitch = P_C = \frac{\pi D}{T} = \pi m (\because D = mT)$$

$$where, m = \text{mod}\,ule\ in\ meter$$

$$T = number\ of\ teeth.$$

$$V = \frac{\pi D N}{60}$$

$$V = \frac{\pi m \times T \times N}{60}$$

$$V = \frac{P_C \times T \times N}{60}$$

$$where, N = speed\ in\ rpm.$$

Modes / Causes of Gear tooth Failure

1. Bending failure or tooth breakage –

- Gear tooth behave like a cantilever beam subjected to a repetitive bending stress. The tooth may break due to repetitive bending stress.
- The tooth breakage occurs when the repetitive bending stress induced in the gear tooth exceed the bending endurance strength of the gear tooth.
- In other words the tooth breakage occurs when the total load acting on the gear tooth exceed beam strength of the gear tooth.
- The tooth breakage can be avoided by adjusting the parameters such as module and face width in the gear design.

2. Wear Failure –

- The wear is the phenomenon which removes the complete layer of the surface or makes craters or scratches on the surface.
- The different types of wear failure in the gear tooth are discuss below.

a) Pitting –

- It is the surface fatigue failure due to repetitive contact stresses.
- The pitting starts when total load acting on the gear tooth exceeds the wear strength of the gear tooth.

b) Scoring –

- Scoring is essentially a lubrication failure.
- In adequate lubrication along with the high tooth load and poor surface finish result in breakdown of the oil film and cause the metal to metal contact.
- This causes rapid alteration welding and tearing at high spots which is known as stick-slip phenomenon.

c) Abrasive Wear –

- Abrasive wear is a surface damage caused by particles trapped in between the meeting teeth surfaces.
- These particles may be present in lubricant as impurities, may be the dirt entering the gearbox from outside or may be flakes of material detached from the tooth surface.
- Abrasive wear may be minimize by proper filtration of the lubricant, providing complete enclosure for gear, increasing the surface hardness and use of high viscosity oils.

d) Corrosive Wear –

- The corrosive wear is due to chemical action by the improper lubricant or sometime it may be due to surrounding atmosphere which may be corrosive in nature.
- The remedies against corrosive wear are using proper lubricant with proper additives and providing complete enclosure for gears.

THE END