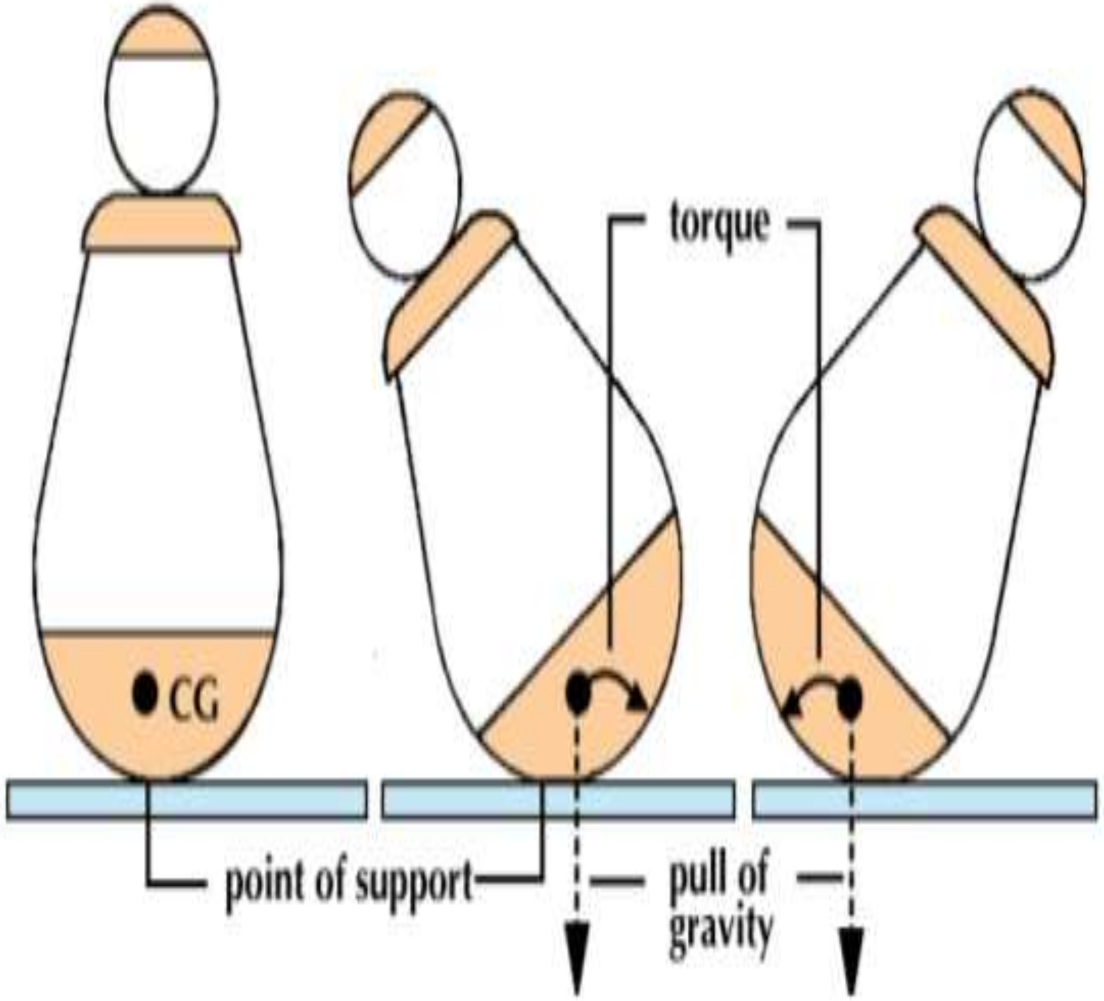
CENTRE OF GRAVITY

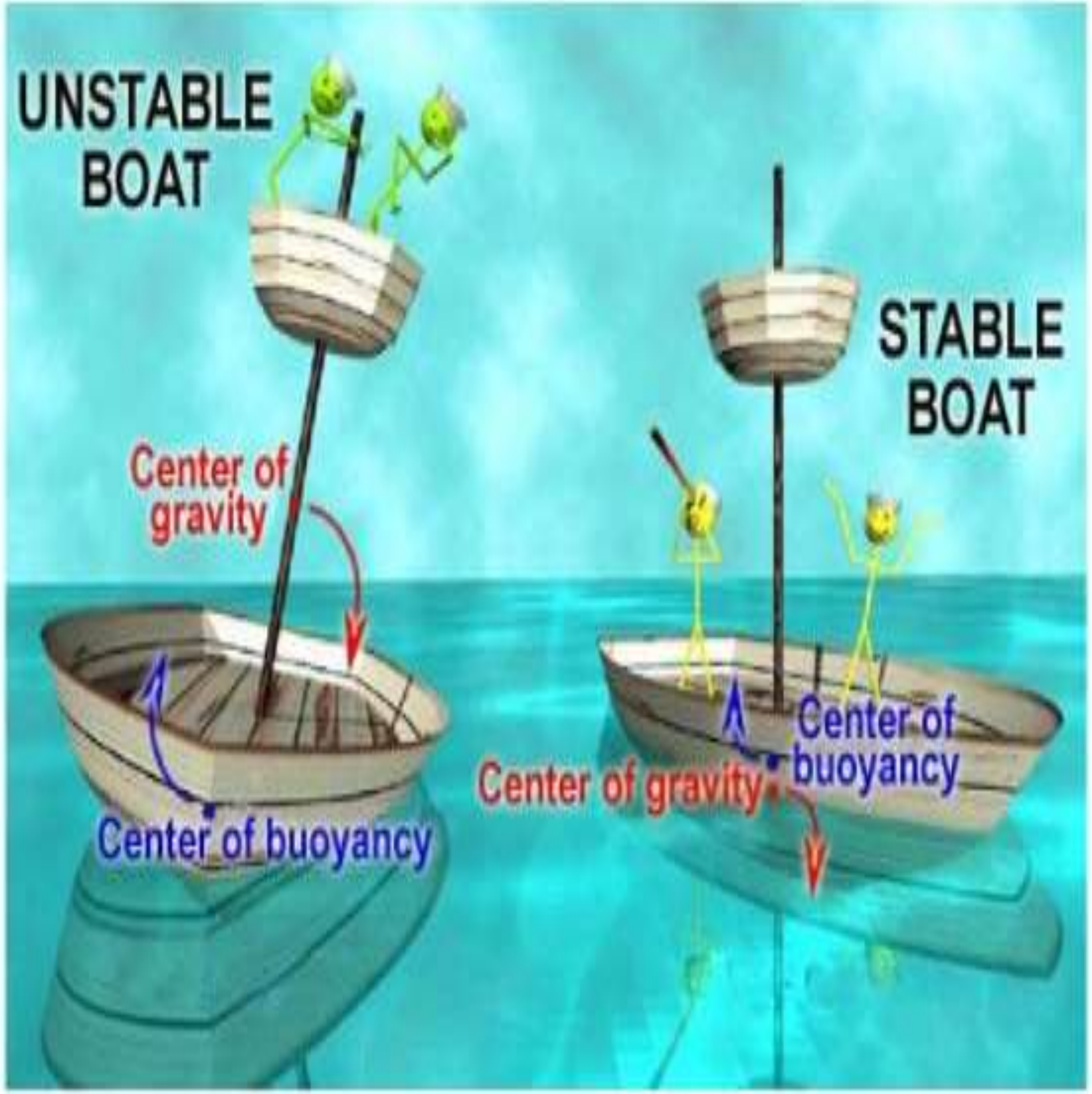
Centroid and Centre of Gravity

Centroid	Center of Gravity
• It is defined as a point about which the entire line, area or volume is assumed to be concentrated. • It is related to distribution of length, area and volume.	• It is defined as a point about which the entire weight of the body is assumed to be concentrated. • Center of mass. • It is related to distribution of mass.

Examples

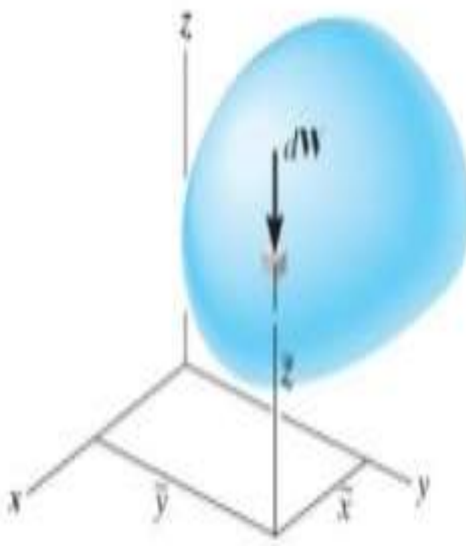


Examples

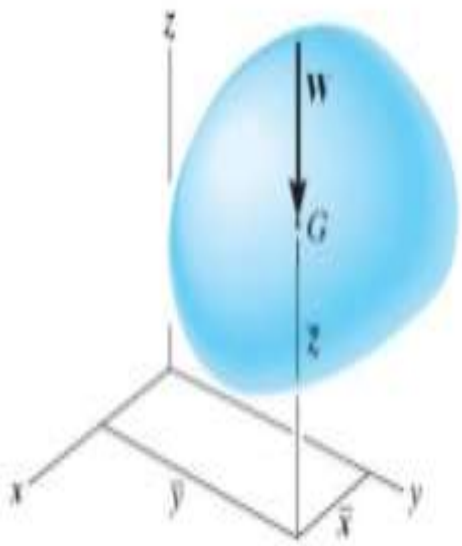


Center of Gravity

- Consider system of n particles fixed within a region of space.
- The weights of the particles can be replaced by a single (equivalent) resultant weight having defined point G of application



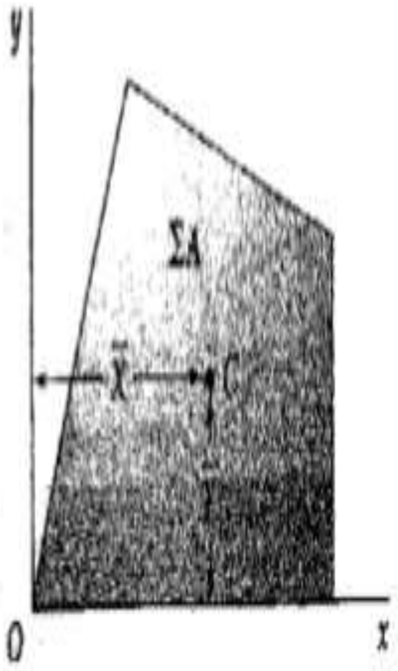
(a)

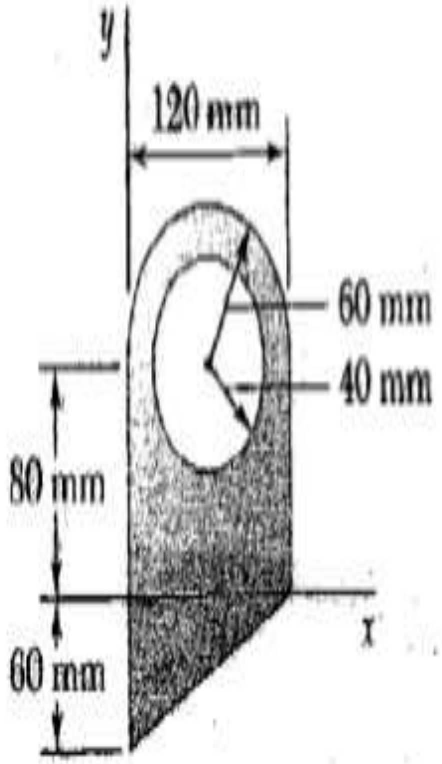


(b)

Axis of Reference

The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference (or sometimes with reference to some point of reference). The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating $\bar{y}$ and the left line of the figure for calculating $\bar{x}$.



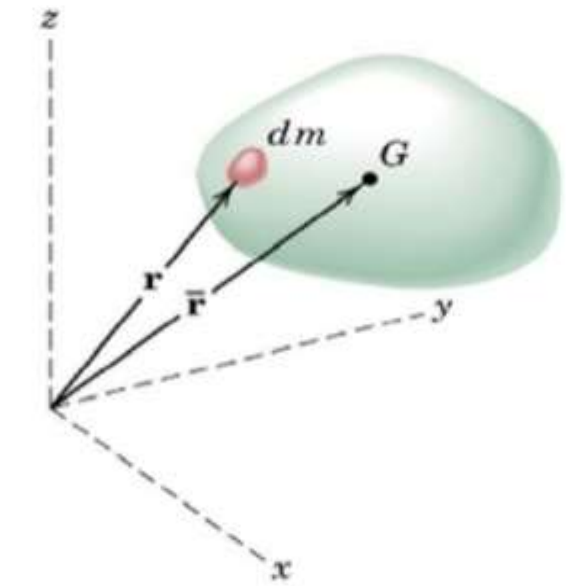


Center of Gravity

- Resultant weight = total weight of n particles , $W_R = \int dW$
- Sum of moments of weights of all the particles about x, y, z axes = moment of resultant weight about these axes
- moments about the x axis, $\bar{x}W_R = \tilde{x}_1 W_1 + \tilde{x}_2 W_2 + \ldots + \tilde{x}_n W_n$
- moments about y axis, $\bar{y}W_R = \tilde{y}_1 W_1 + \tilde{y}_2 W_2 + \ldots + \tilde{y}_n W_n$
- Generally,

$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int dW}; \qquad \bar{y} = \frac{\int \tilde{y}\,dW}{\int dW}; \qquad \bar{z} = \frac{\int \tilde{z}\,dW}{\int dW}$$

Center of mass

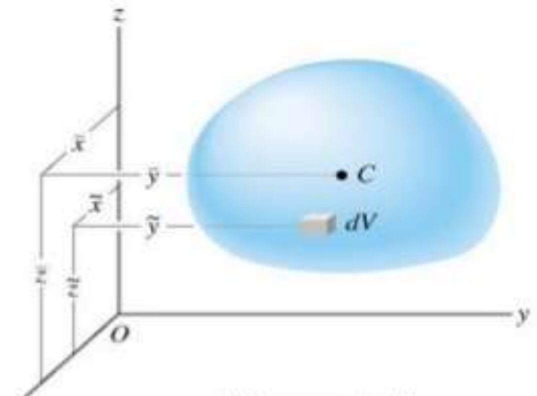


$$\bar{x} = \frac{\int \tilde{x}\,dm}{\int dm}$$

$$\bar{y} = \frac{\int \tilde{y}\,dm}{\int dm}$$

$$\bar{z} = \frac{\int \tilde{z}\,dm}{\int dm}$$

Center of Volume / Volume centroid



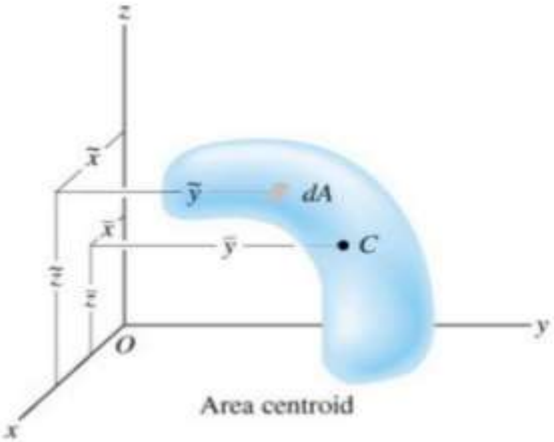
Volume centroid

$$\bar{x} = \frac{\int_V \tilde{x}\,dV}{\int_V dV}$$

$$\bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV}$$

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV}$$

Center of Area / Area centroid



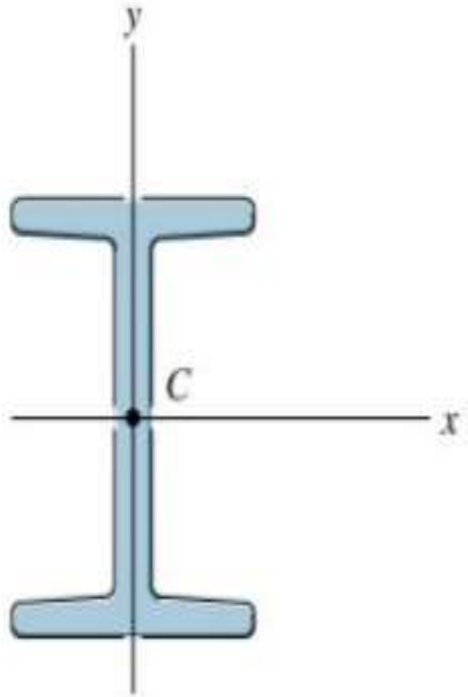
Area centroid

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA}$$

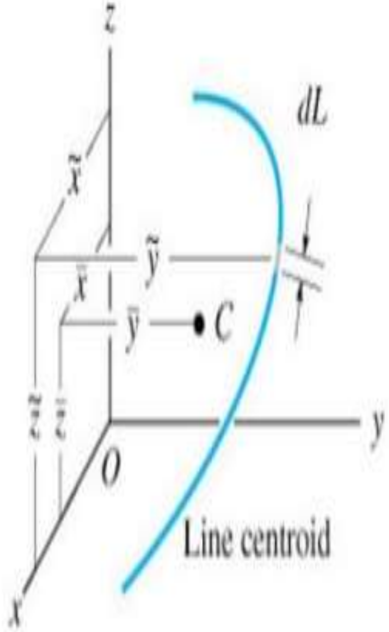
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA}$$

$$\bar{z} = \frac{\int_A \tilde{z}\,dA}{\int_A dA}$$

Axis of symmetry



Center of Line / Line centroid

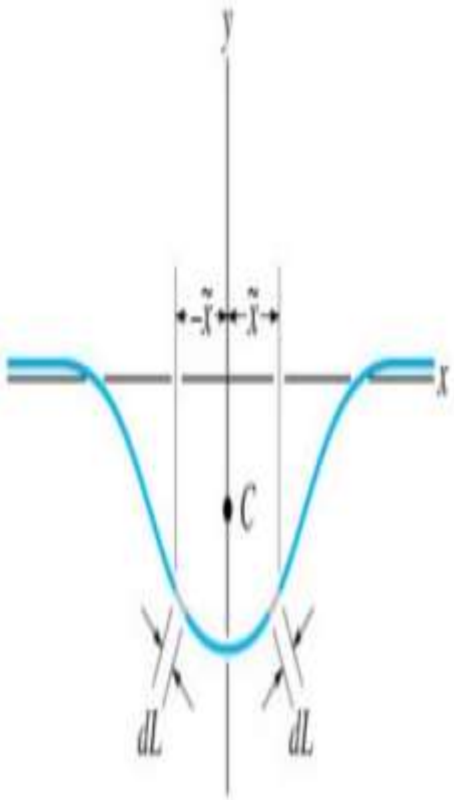


$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL}$$

$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL}$$

$$\bar{z} = \frac{\int_L \tilde{z}\, dL}{\int_L dL}$$

Axis of symmetry

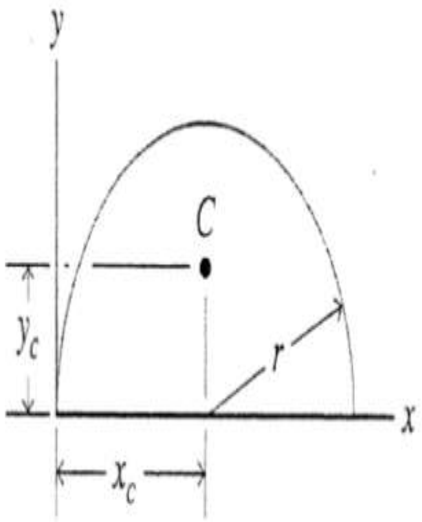


Semicircular arc

$L = \pi r$

$x_C = r$

$y_C = \dfrac{2r}{\pi}$

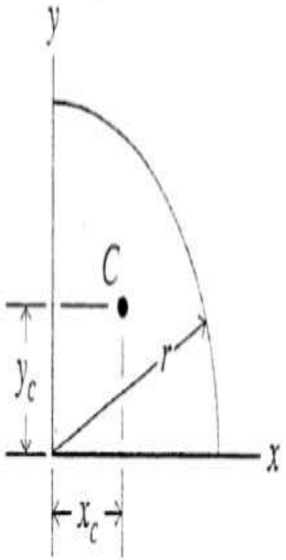


Quarter circular arc

$L = \dfrac{\pi r}{2}$

$x_C = \dfrac{2r}{\pi}$

$y_C = \dfrac{2r}{\pi}$

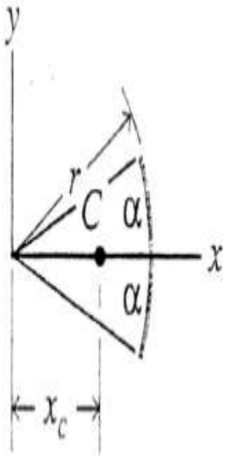


Circular arc

$L = 2r\alpha$

$x_C = \dfrac{r \sin \alpha}{\alpha}$

$y_C = 0$

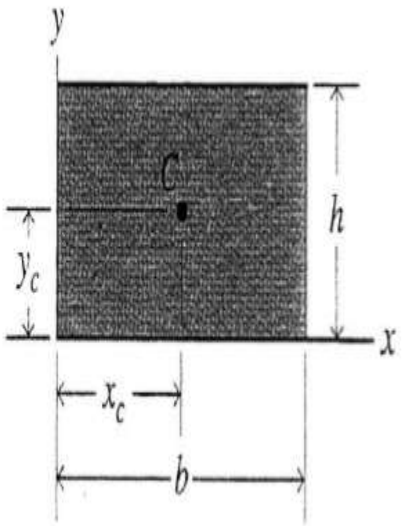


Rectangular area

$A = bh$

$x_C = \dfrac{b}{2}$

$y_C = \dfrac{h}{2}$

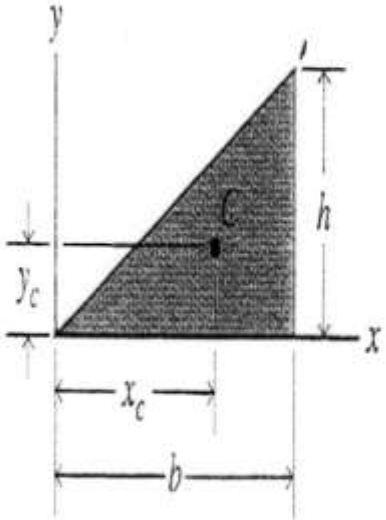


Triangular area

$A = \dfrac{bh}{2}$

$x_C = \dfrac{2b}{3}$

$y_C = \dfrac{h}{3}$

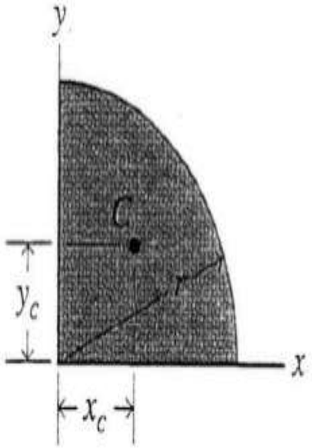


Quadrant of a circle

$A = \dfrac{\pi r^2}{4}$

$x_C = \dfrac{4r}{3\pi}$

$y_C = \dfrac{4r}{3\pi}$

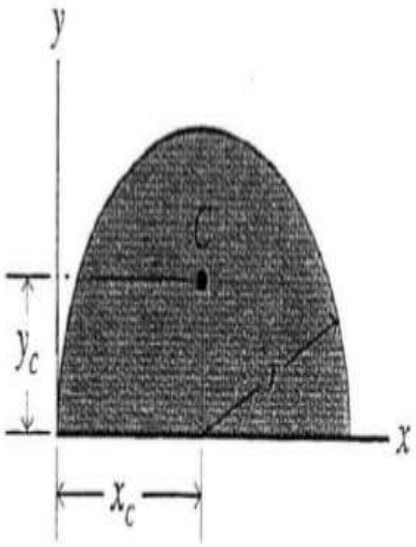


Semicircular area

$$A = \frac{\pi r^2}{2}$$

$$x_C = r$$

$$y_C = \frac{4r}{3\pi}$$



Quadrant of a circle

$$A = \frac{\pi r^2}{4}$$

$$x_C = \frac{4r}{3\pi}$$

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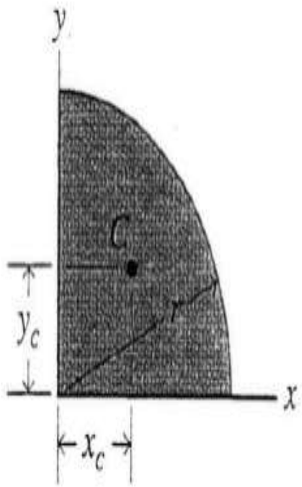


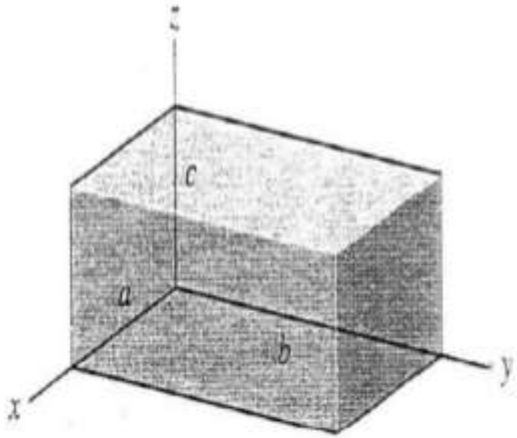
Rectangular parallelepiped

$$V = abc$$

$$x_C = \frac{a}{2}$$

$$y_C = \frac{b}{2}$$

$$z_C = \frac{c}{2}$$

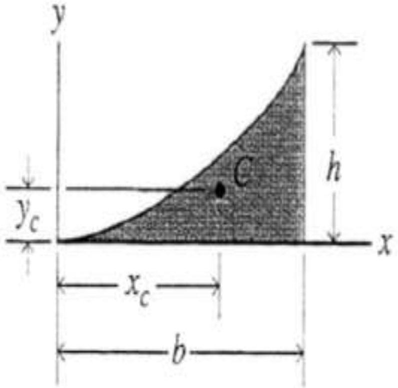


Parabolic spandrel

$$A = \frac{bh}{3}$$

$$x_C = \frac{3b}{4}$$

$$y_C = \frac{3h}{10}$$

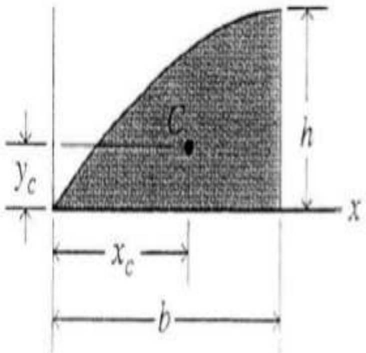


Quadrant of a parabola

$$A = \frac{2bh}{3}$$

$$x_C = \frac{5b}{8}$$

$$y_C = \frac{2h}{5}$$



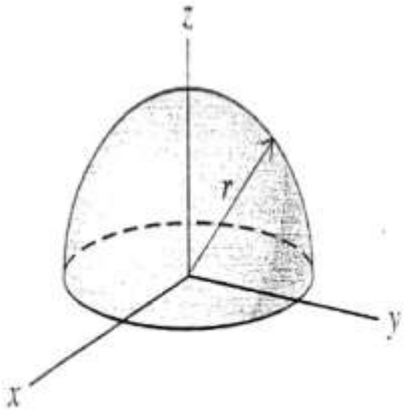
Hemisphere

$$V = \frac{2\pi r^3}{3}$$

$$x_C = 0$$

$$y_C = 0$$

$$z_C = \frac{3r}{8}$$



CENTRE OF GRAVITY (CG)

- Centre of Gravity (CG) of a uniform rod is at middle point $=\frac{L}{2}$

- Centre of Gravity (CG) of a rectangle/ parallelogram is at that point where its two diagonal meet.

- Centre of Gravity (CG) of a triangle is that point where three medians meet (median is the line which develops by joining its vertex and middle point of the opposite face).

- Centre of Gravity (CG) of a semicircle is at a distance of $\frac{4r}{3\pi}$ from its base measure along vertical axis.