Thomas Young:

Modulus of Elasticity

- Young's modulus, also known as the tensile modulus or elastic modulus, is a measure of the stiffness of an elastic material.
- Named after a British Scientist THOMAS YOUNG

Modulus of Elasticity

- $Stress \propto Strain$

Or $\frac{Stress}{Strain} = Constant = Modulus\ of\ elasticity$

Its unit is "pa" or N/m^2

NB: *If the **modulus of elasticity of a material** is large, it means a larger stress will produce only a small strain.*

Types of Modulus of Elasticity

- Corresponding to three types of strain, there are three moduli of elasticity;

i. Young's Modulus of elasticity, Y

ii. Bulk modulus of elasticity, $K = \frac{Normal\ stress}{Volumetric\ strain}$

iii. Modulus of rigidity, $\eta = \frac{Shearing\ stress}{Shearing\ strain}$

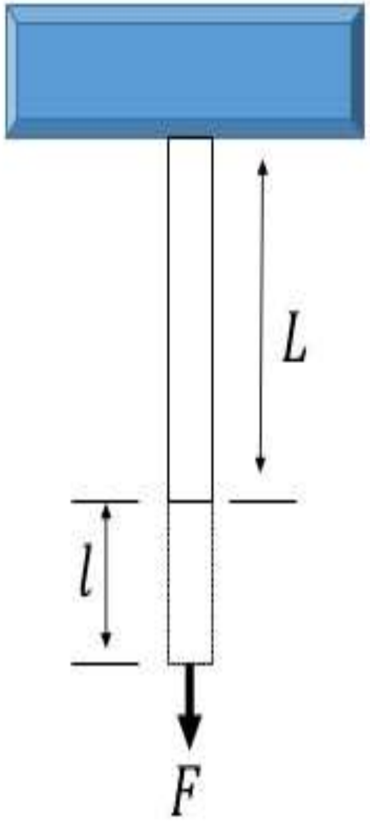
Young's Modulus (Y)

- It is defined as the ratio of normal stress to the longitudinal strain.

$$Y = \frac{Normal\ stress}{Longitudinal\ strain}$$

Y...

- Consider a wire of length L and area of cross section A fixed at one end to the rigid, then;
- Normal stress, $\sigma = \frac{F}{A}$
- Longitudinal Strain = $\frac{l}{L}$



Y…

$$Y = \frac{F/A}{l/L} = \frac{FL}{Al}$$

- If r is the radius of the wire, then $A = \pi r^2$

$$\therefore Y = \frac{FL}{\pi r^2 l}$$

Young's moduli of structural materials

	Young's modulus (N/m²)
Engineering materials	
Steel	2.1×10^{11}
Concrete	1.7×10^{10}
Rubber	7×10^{6}
Biological materials	
Bone	1.7×10^{10}
Cartilage	1.3×10^{7}
Tendon	1.9×10^{8}
Locust cuticle	9.4×10^{9}

E is the stress required to produce 100% strain

HOOK’S LAW:

It states “when a material is loaded within its elastic limit, the stress is proportional to the strain”. Mathematically, $= E$, a Constant. Hook’s Law holds good equally for tension as well as in compression.

MODULUS OF ELASTICITY:

Whenever a material is loaded within its elastic limit, the stress is directly proportional to strain.

Bulk Modulus (K)

- This refers to situations in which the volume (i.e. bulk) of a substance is changed by the application of external normal stress.

Bulk modulus, $K = \frac{Normal\ Strain}{Volumetric\ strain}$

Bulk Modulus…

- Normal stress, $\sigma = {}^{F}/_{A}$
- Volumetric strain, $\varepsilon_{vol} = -{}^{\Delta V}/_{V}$
- *The negative sign is included to indicate that the volume decreases with an increase in pressure.*
- $\therefore K = -\frac{F/A}{\Delta V/V} = -\frac{FV}{A\Delta V}$

i.e. Pressure, $p = F/A$

$$\therefore Y = -V\frac{p}{\Delta V}$$

Shear Modulus or Rigidity Modulus (η)

- This refers to situations in which the shape of a substance is changed by the application of tangential stress.

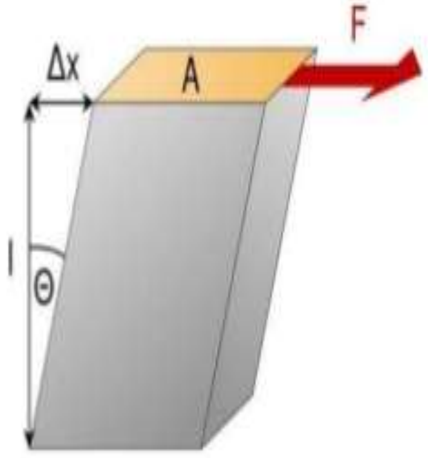
$$\eta = \frac{Tangental\ stress}{Shear\ strain}$$

Shear Modulus…

- $tangental\ stress = \frac{F}{A}$
- Shear strain = θ = tanθ

$= \frac{\Delta x}{l}$

$$\therefore\ \eta = \frac{Fl}{A\Delta x}$$



Strain Energy (U)

What is Strain Energy ?

- When a body is subjected to gradual, sudden or impact load, the body deforms and work is done upon it. If the elastic limit is not exceed, this work is stored in the body. This work done or energy stored in the body is called strain energy.

- Energy is stored in the body during deformation process and this energy is called "Strain Energy".

Strain energy = Work done

Strain Energy...

- Work done (ΔW) is given by;

$$\Delta W = F\Delta x$$

- Acceding to Hooke's law;

$$F = kx$$

The total work done in increasing the extension from *0* to *x* is given by;

$$W = \int_0^x kxdx$$

$$= {}^1/_2\,(kx^2)$$

$$\therefore \boldsymbol{W = U = {}^1/_2 Fx}$$

Strain Energy...

- The ***U*** equation can be expressed in another useful form;

$$U = \frac{1}{2} x \left(\frac{F}{A}\right) x \left(\frac{x}{L}\right) x\ LA$$

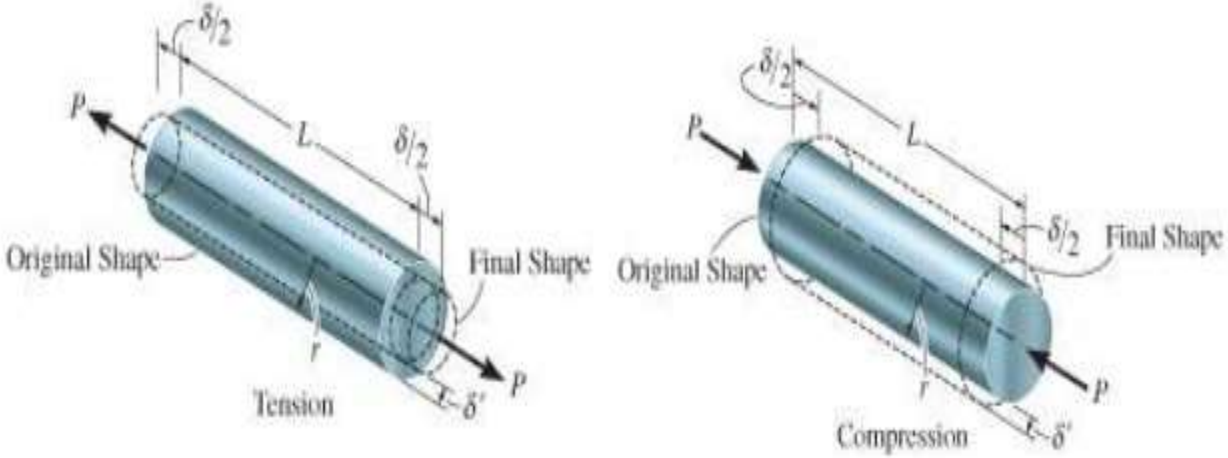
$$U= \frac{1}{2}\ x\ stress\ x\ strain\ x\ volume$$

$$\therefore Strain\ energy\ per\ unit\ volume\ is;$$

$$U= \frac{1}{2}\ x\ stress\ x\ strain$$

POISSON'S RATIO

- When body subjected to axial tensile force, it elongates and contracts laterally
- Similarly, it will contract and its sides expand laterally when subjected to an axial compressive force



POISSON'S RATIO…

- Strains of the bar are:

$$\varepsilon_{long} = \frac{\delta}{L} \qquad \varepsilon_{lat} = \frac{\delta'}{r}$$

- Early 1800s, S.D. Poisson realized that within elastic range, ration of the two strains is a constant value, since both are proportional.

$$Poisson's\ ratio,\ \nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$