

Impact of Jets

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29.1 Introduction

We see that whenever a jet of liquid impinges (*i.e.*, strikes) on a fixed plate, it experiences some force. As per Newton's Second Law of Motion, this force is equal to the rate of change of momentum of the jet. It has been observed that if the plate is not fixed, then the plate starts moving in the direction of the jet, because of the force.

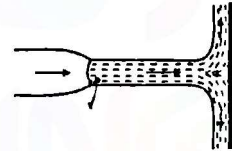
29.2 Force of Jet Impinging Normally on a Fixed Plate

Consider a jet of water impinging normally on a fixed plate as shown in Fig. 29.1.

Let V = Velocity of the jet in m/s, and

a = Cross-sectional area of the jet in m^2 .

$$\therefore \text{Mass of water flowing/s} = \frac{waV}{g} \text{ kg}$$



We know that the velocity of jet, in its original direction, is reduced to zero after the impact (as the plate is fixed). Therefore force exerted by the jet on the plate.

Fig. 29.1. Jet impinging normally on a fixed plate.

$$\begin{aligned} F &= \text{Mass of water flowing/s} \times \text{Change of velocity} \\ &= \frac{waV}{g} \times (V - 0) = \frac{waV^2}{g} \text{ kN} \end{aligned}$$

where w is the specific weight of water in kN/m^3 .

Example 29.1. A jet of water of 100 mm diameter impinges normally on a fixed plate with a velocity of 30 m/s. Find the force exerted on the plate.

Solution. Given : $d = 100 \text{ mm} = 0.1 \text{ m}$ and $V = 30 \text{ m/s}$.

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.1)^2 = 7.854 \times 10^{-3} \text{ m}^2$$

and force exerted on the plate,

$$\begin{aligned} F &= \frac{waV^2}{g} = \frac{9.81 \times (7.854 \times 10^{-3}) \times (30)^2}{9.81} \text{ kN} \\ &= 7.07 \text{ kN Ans.} \end{aligned}$$

Example 29.2. A jet of water 50 mm diameter is discharging under a constant head of 70 metres. Find the force exerted by the jet on a fixed plate. Take coefficient velocity as 0.9.

Solution. Given : $d = 50 \text{ mm} = 0.05 \text{ m}$; $H = 70 \text{ m}$ and $C_v = 0.9$.

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.05)^2 = 1.964 \times 10^{-3} \text{ m}^2$$

and velocity of the jet, $V = C_v \sqrt{2gH} = 0.9 \times \sqrt{2 \times 9.81 \times 70} = 33.4 \text{ m/s}$

\therefore Force exerted by the jet on the fixed plate,

$$F = \frac{waV^2}{g} = \frac{9.81 \times (1.964 \times 10^{-3}) \times (33.4)^2}{9.81} \text{ kN} \\ = 2.19 \text{ kN Ans.}$$

Example 29.3. A horizontal jet of water is issuing under an effective head of 25 m. Calculate the diameter of the jet, if the force exerted by the jet on a vertical fixed plate is 2.22 kN. Take $C_v = 1$.

Solution. Given : $H = 25 \text{ m}$; $F = 2.22 \text{ kN}$ and $C_v = 1$.

Let d = Diameter of the jet in metres.

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = 0.7854 d^2$$

and velocity of the jet, $V = C_v \times \sqrt{2gH} = 1 \times \sqrt{2 \times 9.81 \times 25} = 21 \text{ m/s}$

We also know that force exerted by the jet (F),

$$2.22 = \frac{waV^2}{g} = \frac{9.81 \times (0.7854 d^2) \times (21)^2}{9.81} = 346.4 d^2$$

$\therefore d^2 = 2.22/346.4 = 0.0064$ or $d = 0.08 \text{ m} = 80 \text{ mm Ans.}$

29.3 Force of Jet Impinging on an Inclined Fixed Plate

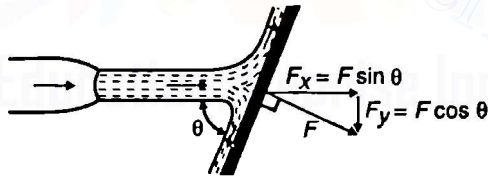


Fig. 29-2. Jet impinging on an inclined fixed plate.

Consider a jet impinging on an inclined fixed plate as shown in Fig. 29-2.

Let V = Velocity of jet in m/s, .

a = Cross-sectional area of the jet in m^2 , and

θ = Angle at which the plate is inclined with the jet.

We know that the force exerted by the jet in its original direction

= Mass of water flowing/s \times Change of velocity

$$= \frac{waV}{g} \times (V - 0) = \frac{waV^2}{g} \text{ kN}$$

\therefore Force exerted by the jet in a direction normal (i.e., perpendicular) to the plate,

$$F = \frac{waV^2}{g} \times \sin \theta = \frac{waV^2 \sin \theta}{g}$$

and the force exerted by the jet in the direction of flow,

$$F_x = F \sin \theta = \frac{waV^2 \sin \theta}{g} \times \sin \theta = \frac{waV^2 \sin^2 \theta}{g}$$

Similarly, force exerted by the jet in a direction normal to flow,

$$\begin{aligned} F_y &= F \cos \theta = \frac{waV^2 \sin \theta}{g} \times \cos \theta \\ &= \frac{2waV^2 \sin \theta \cos \theta}{2g} \quad \dots (\text{Multiplying and dividing by 2}) \\ &= \frac{waV^2 \sin 2\theta}{2g} \quad \dots (\because \sin 2\theta = 2 \sin \theta \cos \theta) \end{aligned}$$

Example 29.4. A jet of water of 100 mm diameter, moving with a velocity of 20 m/s strikes a stationary plate. Find the force on the plate in the direction of the jet, when

(a) the plate is normal to the jet, and

(b) the angle between the jet and plate is 45° .

Solution. Given : $d = 100 \text{ mm} = 0.1 \text{ m}$; $V = 20 \text{ m/s}$ and $\theta = 45^\circ$.

(a) Force on the plate when it is normal to the jet

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.1)^2 = 7.854 \times 10^{-3} \text{ m}^2$$

and force on the plate when it is normal to the jet,

$$F = \frac{waV^2}{g} = \frac{9.81 \times (7.854 \times 10^{-3}) \times (20)^2}{9.81} = 3.142 \text{ kN Ans.}$$

(b) Force on the plate when the angle between the jet and plate is 45°

We also know that force on the plate when the angle between the jet and plate is 45° ,

$$\begin{aligned} F &= \frac{waV^2}{g} \times \sin^2 \theta = 3.142 \sin^2 45^\circ \text{ kN} \\ &= 3.142 \times (0.707)^2 = 1.57 \text{ kN Ans.} \end{aligned}$$

Example 29.5. A 25 mm diameter jet exerts a force of 1 kN in the direction of flow against a flat plate, which is held inclined at an angle of 30° with the axis of the stream. Find the rate of flow.

Solution. Given : $d = 25 \text{ mm} = 0.025 \text{ m}$; $F_x = 1 \text{ kN}$ and $\theta = 30^\circ$.

Let Q = Rate of flow of water.

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.025)^2 = 0.491 \times 10^{-3} \text{ m}^2$$

and velocity of the jet,

$$V = \frac{Q}{a} = \frac{Q}{0.491 \times 10^{-3}} = 2.04 \times 10^3 Q \text{ m/s}$$

We also know that force exerted by the jet (F_x),

$$\begin{aligned} 1 &= \frac{waV^2}{g} \times \sin^2 \theta \\ &= \frac{9.81 \times (0.491 \times 10^{-3}) \times (2.04 \times 10^3 Q)^2}{9.81} \times \sin^2 30^\circ \\ &= 2.04 \times 10^3 Q^2 \times (0.5)^2 = 0.51 \times 10^3 Q^2 \end{aligned}$$

$$\therefore Q^2 = 1/(0.51 \times 10^3) = 1.96 \times 10^{-3}$$

$$\text{or } Q = 0.0443 \text{ m}^3/\text{s} = 44.3 \text{ litres/s Ans.}$$

29.4 Force of Jet Impinging on a Hinged Plate

In the previous articles, we have studied that whenever a jet is impinging on a plate, it exerts some force on it. Now we shall discuss the effect of this force on a hinged plate. Now consider a plate hinged at O as shown in Fig. 29.3 (a).

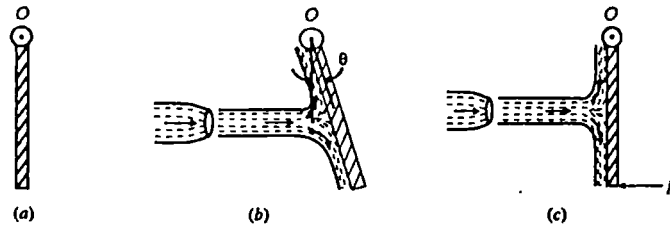


Fig. 29.3. Jet impinging on a hinged plate.

Let a jet be allowed to strike on a hinged plate. As a result of this jet strike, the plate will swing about the hinge, through some angle as shown in Fig. 29.3 (b). The angle, through which the plate will swing, may be found out by taking moments of the force of jet and the weight of plate about the hinge.

If it is required to keep the plate vertical, then some force (P) has to be applied on the plate as shown in Fig. 29.3 (c). The value of this force (P) may also be found out by taking moments about the hinge, as usual.

Note : If the jet of water strikes the plate at its centre, then the inclination of the plate, with the vertical, may be found out as discussed below.

$$\text{Force exerted by the jet, } F = \frac{waV^2}{g}$$

Moment of this force about the hinge

$$= F \times \frac{d}{2} \quad \dots (\text{where } d = \text{Depth of plate})$$

and restoring moment of the plate about the hinge

$$= W \times \frac{d}{2} \sin \theta \quad \dots (\text{where } W = \text{Weight of the plate})$$

Equating these two moments,

$$W \times \frac{d}{2} \sin \theta = F \times \frac{d}{2}$$

$$\therefore \sin \theta = \frac{F}{W} \quad \dots (i)$$

Now moment of the force P about the hinge

$$= P \cdot d$$

Again equating the moments of P and the force of jet,

$$P \cdot d = F \times \frac{d}{2}$$

$$\therefore P = \frac{F}{2} \quad \dots (ii)$$

Example 29.6. A jet of water 25 mm diameter strikes a flat plate normally at 30 metres/s, at a point 150 mm below the top of the plate. What force should be applied, 100 mm below the axis of the jet, in order to keep the plate vertical?

Solution. Given : $d = 25 \text{ mm} = 0.025 \text{ m}$; $V = 30 \text{ m/s}$.

Let

$P =$ Force required to keep the plate vertical.

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.025)^2 \text{ m}^2$$

$$= 0.491 \times 10^{-3} \text{ m}^2$$

and force exerted by the jet,

$$F = \frac{\rho a V^2}{g}$$

$$= \frac{9.81 \times (0.491 \times 10^{-3}) \times (30)^2}{9.81} \text{ kN}$$

$$= 0.442 \text{ kN} = 442 \text{ N}$$

Now taking moments about the hinge of the plate,

$$P \times 250 = 442 \times 150 = 66\,300$$

\therefore

$$P = 66\,300/250 = 265.2 \text{ N Ans.}$$

Example 29.7. A square plate weighing 150 N and of uniform thickness and 300 mm side is hung, so that it can swing freely about its upper horizontal edge. A horizontal jet of 20 mm diameter and having velocity of 20 m/s impinges on the plate. When the plate is vertical, the jet strikes the plate normally at its centre. Find the force, which must be applied at the lower edge of the plate, in order to keep the plate vertical.

If the plate is allowed to swing freely, find the inclination to the vertical, which the plate will assume under the action of jet.

Solution. Given : $W = 150 \text{ N}$; Side of square plate = 300 mm; $d = 20 \text{ mm} = 0.02 \text{ m}$; $V = 20 \text{ m/s}$

Force required to keep the plate vertical

Let

$P =$ Force required to keep the plate vertical.

We know that cross-sectional area of the jet,

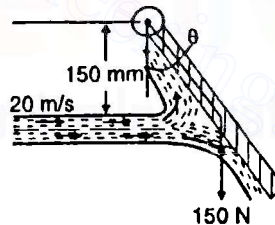
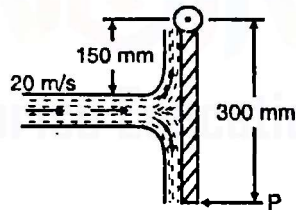


Fig. 29.5

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.02)^2 = 0.314 \times 10^{-3} \text{ m}^2$$

and force exerted by the jet,
$$F = \frac{\rho a V^2}{g} = \frac{9.81 \times (0.314 \times 10^{-3}) \times (20)^2}{9.81} = 125.6 \text{ N}$$

Now taking moments about the hinge of the plate,

$$P \times 300 = 125.6 \times 150 = 18\,840$$

\therefore

$$P = 18\,840/300 = 62.8 \text{ N Ans.}$$

Angle through which the plate will be inclined

Let

$\theta =$ Angle with the vertical through which the plate will be inclined.

From the geometry of the figure, we find that

$$\sin \theta = \frac{F}{W} = \frac{125.6}{150} = 0.8373 \quad \text{or } \theta = 56.9^\circ \quad \text{Ans.}$$

Example 29.8. A jet of water of 30 mm diameter strikes a hinged square plate weighing 100 N at its centre. Find the velocity of the jet in order to deflect the plate through an angle of 30° .

Solution. Given : $d = 30 \text{ mm} = 0.03 \text{ m}$; $W = 100 \text{ N}$ and $\theta = 30^\circ$.

Let $V =$ Velocity of the jet.

We know that cross-sectional area of the jet,

$$\begin{aligned} a &= \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.03)^2 \\ &= 0.707 \times 10^{-3} \text{ m}^2 \end{aligned}$$

and force exerted by the jet,

$$\begin{aligned} F &= \frac{waV^2}{g} = \frac{9.81 \times (0.707 \times 10^{-3}) V^2}{9.81} \text{ kN} \\ &= 0.707 \times 10^{-3} V^2 \text{ kN} = 0.707 V^2 \text{ N} \end{aligned}$$

From the geometry of the figure we find that

$$\sin 30^\circ = \frac{F}{W} \quad \text{or } 0.5 = \frac{0.707 V^2}{100} = 0.707 \times 10^{-2} V^2$$

$$\therefore V^2 = 0.5 / (0.707 \times 10^{-2}) = 70.7$$

or

$$V = \sqrt{70.7} = 8.41 \text{ m/s} \quad \text{Ans.}$$

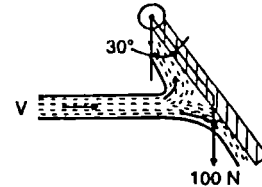


Fig. 29-6.

EXERCISE 29-1

1. A jet of water of 50 mm diameter moving with a velocity of 10 m/s strikes a flat fixed plate. Calculate the force exerted by the jet, if the plate is held normal to the direction of the jet. [Ans. 196 N]
2. A horizontal jet of water of 30 mm diameter is impinging on a vertical fixed plate. If the force exerted by the jet is 282.5 N, find the velocity of the jet. [Ans. 20 m/s]
3. A jet of water of 100 mm diameter is striking a plate with a velocity of 35 m/s. Find the force exerted by the jet in a direction normal to the plate, if the plate is inclined at 30° with the jet. [Ans. 4.81 kN]
4. A jet of water of 75 mm diameter moving with a velocity of 15 m/s impinges on a fixed plate. Find the force exerted on the plate in the direction of the jet, when the angle between the plate and the jet is 25° . [Ans. 177.5 N]
5. A horizontal jet of water of 25 mm diameter is striking a vertical plate, hinged at the top, with a velocity of 12 m/s. If the plate is allowed to swing freely, find that inclination, the plate will assume due to impact of the jet. Take weight of the plate as 180 N. [Ans. 23.13°]
6. A jet of water of 50 mm diameter strikes a square plate at its centre. The plate weighing 80 N is hinged at its top. Find the velocity of the jet, which will keep the plate in equilibrium at an angle of 30° to the vertical. [Ans. 4.5 m/s]

29.5 Force of Jet Impinging on a Moving Plate

Consider a jet of water impinging normally on a plate. As a result of the impact of the jet, let the plate move in the direction of the jet as shown in Fig. 29-7.

Let

$V =$ Velocity of the jet in m/s,

$a =$ Cross-sectional area of the jet in m^2 , and

$v =$ Velocity of the plate, as a result of the impact of jet in m/s.

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A little consideration will show that the relative velocity of the jet with respect to the plate is equal to $(V-v)$ m/s.

For analysis purposes, it will be assumed that the plate is fixed and the jet is moving with a velocity of $(V-v)$ m/s. Therefore force exerted by the jet,

$$\begin{aligned}
 F &= \text{Mass of water flowing per second} \\
 &\quad \times \text{Change of velocity} \\
 &= \frac{wa(V-v)}{g} \times [(V-v) - 0] \\
 &= \frac{wa(V-v)^2}{g} \text{ kN} \\
 \text{and work done by the jet} &= \text{Force} \times \text{Distance} \\
 &= \frac{wa(V-v)v}{g} \text{ kN-m}
 \end{aligned}$$

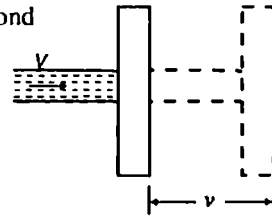


Fig. 29-7. Jet impinging on a moving plate.

Example 29-9. A jet of water 50 mm diameter and moving with a velocity of 26 m/s is impinging normally on a plate. Determine the pressure on the plate, when (a) it is fixed and (b) it is moving with a velocity of 10 m/s in the direction of the jet.

Also determine the work done per second by the jet.

Solution. Given : $d = 50 \text{ mm} = 0.05 \text{ m}$; $V = 26 \text{ m/s}$ and $v = 10 \text{ m/s}$.

Pressure on the plate when it is fixed

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.05)^2 = 1.964 \times 10^{-3} \text{ m}^2$$

and pressure on the plate,
$$P_1 = \frac{waV^2}{g} = \frac{9.81 \times (1.964 \times 10^{-3}) \times (26)^2}{9.81} = 1.33 \text{ kN Ans.}$$

Pressure on the plate when it is moving

We know that pressure on the plate when it is moving,

$$\begin{aligned}
 P_2 &= \frac{wa(V-v)^2}{g} = \frac{9.81 \times (1.964 \times 10^{-3}) \times (26 - 10)^2}{9.81} \\
 &= 0.503 \text{ kN Ans.}
 \end{aligned}$$

Work done by the jet

We also know that work done by the jet

$$\begin{aligned}
 &= \text{Force} \times \text{Distance} = 0.503 \times 10 = 5.03 \text{ kN-m} \\
 &= 5.03 \text{ kJ Ans.}
 \end{aligned}$$

29-6 Force of Jet Impinging on a Series of Vanes

A little consideration will show, that the case of a jet impinging on a moving plate is merely a theoretical one, which seldom arises in practice; because it requires continuously a jet following the moving plate. But in actual practice, a case similar to this arises, in which a jet of water impinges on a series of vanes, mounted on the circumference of a large wheel as shown in Fig. 29-8.

Let

V = Velocity of the jet in m/s,

a = Cross-sectional area of the jet in m^2 , and

v = Velocity of the vanes, as a result of the jet, in m/s.

From the geometry of the figure, we find that

$$\sin \theta = \frac{F}{W} = \frac{125.6}{150} = 0.8373 \quad \text{or } \theta = 56.9^\circ \quad \text{Ans.}$$

Example 29-8. A jet of water of 30 mm diameter strikes a hinged square plate weighing 100 N at its centre. Find the velocity of the jet in order to deflect the plate through an angle of 30° .

Solution. Given : $d = 30 \text{ mm} = 0.03 \text{ m}$; $W = 100 \text{ N}$ and $\theta = 30^\circ$.

Let $V =$ Velocity of the jet.

We know that cross-sectional area of the jet,

$$\begin{aligned} a &= \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.03)^2 \\ &= 0.707 \times 10^{-3} \text{ m}^2 \end{aligned}$$

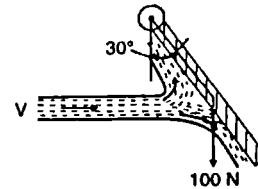


Fig. 29-6.

$$\begin{aligned} \text{and force exerted by the jet, } F &= \frac{waV^2}{g} = \frac{9.81 \times (0.707 \times 10^{-3}) V^2}{9.81} \text{ kN} \\ &= 0.707 \times 10^{-3} V^2 \text{ kN} = 0.707 V^2 \text{ N} \end{aligned}$$

From the geometry of the figure we find that

$$\sin 30^\circ = \frac{F}{W} \quad \text{or } 0.5 = \frac{0.707 V^2}{100} = 0.707 \times 10^{-2} V^2$$

$$\begin{aligned} \therefore V^2 &= 0.5 / (0.707 \times 10^{-2}) = 70.7 \\ \text{or } V &= \sqrt{70.7} = 8.41 \text{ m/s} \quad \text{Ans.} \end{aligned}$$

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 F &= \text{Mass of water flowing per second} \\
 &\quad \times \text{Change of velocity} \\
 &= \frac{wa(V-v)}{g} \times [(V-v) - 0] \\
 &= \frac{wa(V-v)^2}{g} \text{ kN} \\
 \text{and work done by the jet} &= \text{Force} \times \text{Distance} \\
 &= \frac{wa(V-v)v}{g} \text{ kN-m}
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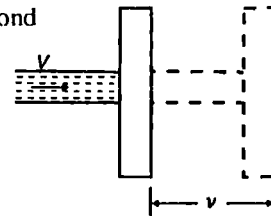


Fig. 29-7. Jet impinging on a moving plate.

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Also determine the work done per second by the jet.

Solution. Given : $d = 50 \text{ mm} = 0.05 \text{ m}$; $V = 26 \text{ m/s}$ and $v = 10 \text{ m/s}$.

Pressure on the plate when it is fixed

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.05)^2 = 1.964 \times 10^{-3} \text{ m}^2$$

and pressure on the plate,
$$P_1 = \frac{waV^2}{g} = \frac{9.81 \times (1.964 \times 10^{-3}) \times (26)^2}{9.81} = 1.33 \text{ kN Ans.}$$

Pressure on the plate when it is moving

We know that pressure on the plate when it is moving,

$$\begin{aligned}
 P_2 &= \frac{wa(V-v)^2}{g} = \frac{9.81 \times (1.964 \times 10^{-3}) \times (26 - 10)^2}{9.81} \\
 &= 0.503 \text{ kN Ans.}
 \end{aligned}$$

Work done by the jet

We also know that work done by the jet

$$\begin{aligned}
 &= \text{Force} \times \text{Distance} = 0.503 \times 10 = 5.03 \text{ kN-m} \\
 &= 5.03 \text{ kJ Ans.}
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A little consideration will show, that the case of a jet impinging on a moving plate is merely a theoretical one, which seldom arises in practice; because it requires continuously a jet following the moving plate. But in actual practice, a case similar to this arises, in which a jet of water impinges on a series of vanes, mounted on the circumference of a large wheel as shown in Fig. 29-8.

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a = Cross-sectional area of the jet in m^2 , and

v = Velocity of the vanes, as a result of the jet, in m/s.

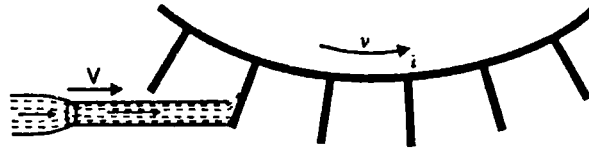


Fig. 29-8. Jet impinging on a series of vanes.

A little consideration will show, that the jet of water, after impinging on the vanes, will be moving with a velocity of v m/s.

We know that the force exerted by the jet

$$F = \text{Mass of water flowing per second} \times \text{Change of velocity} \\ = \frac{waV}{g} \times (V - v) = \frac{waV(V - v)}{g} \quad \dots(i)$$

$$\text{and work done by jet} = \text{Force} \times \text{Distance} = \frac{waV(V - v) \times v}{g} \quad \dots(ii)$$

$$\therefore \text{Work done per kN of water} = \frac{1}{g} (V - v) \times v \quad \dots(iii)$$

and energy of the jet water per kN of water

$$= \frac{v^2}{2g}$$

Efficiency,

$$\eta = \frac{\text{Work done per kN of water}}{\text{Energy per kN of water}} \\ = \frac{\frac{1}{g} (V - v) \times v}{\frac{v^2}{2g}} = \frac{2(V - v) \times v}{v^2}$$

Note : The efficiency is also equal to

$$\frac{\text{Work done by the jet}}{\text{Energy of the jet}}$$

Example 29-10. A jet of water 60 mm in diameter, moving with a velocity of 20 m/s, strikes a flat vane, which is normal to the axis of the stream. Find (i) the force exerted by the jet, when the vane moves with a velocity of 8 m/s. (ii) the force exerted by the jet, if instead of one flat vane, there is a series of vanes, so arranged that each vane appears successively before the jet in the same position and always moving with a velocity of 8 m/s.

Solution. Given : $d = 60 \text{ mm} = 0.06 \text{ m}$; $V = 20 \text{ m/s}$ and $v = 8 \text{ m/s}$.

(i) Force exerted by the jet when a single vane is moving

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.06)^2 = 2.827 \times 10^{-3} \text{ m}^2$$

and force exerted by the jet,

$$F_1 = \frac{wa(V - v)^2}{g} = \frac{9.81 \times (2.827 \times 10^{-3}) \times (20 - 8)^2}{9.81} \text{ kN} \\ = 0.407 \text{ kN Ans.}$$

(ii) Force exerted by the jet when a series of vanes are moving

We also know that force exerted by the jet,

$$F_2 = \frac{waV(V-v)}{g} = \frac{9.81 \times (2.827 \times 10^{-3}) \times 20(20-8)}{9.81} \text{ kN} \\ = 0.678 \text{ kN Ans.}$$

Example 29-11. A jet of water 50 mm in diameter, moving with velocity of 15 m/s, impinges on a series of vanes moving with a velocity of 6 m/s. Find (a) Force exerted by the jet. (b) Work done by the jet, and (c) Efficiency of the jet.

Solution. Given : $d = 50 \text{ mm} = 0.05 \text{ m}$; $V = 15 \text{ m/s}$ and $v = 6 \text{ m/s}$.

(a) Force exerted by the jet

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.05)^2 = 1.964 \times 10^{-3} \text{ m}^2$$

and force exerted by the jet, $F = \frac{waV(V-v)}{g} = \frac{9.81 \times (1.964 \times 10^{-3}) \times 15(15-6)}{9.81} \text{ kN}$
 $= 0.265 \text{ kN} = 265 \text{ N Ans.}$

(b) Work done by the jet

We know that work done by the jet

$$W = \text{Force} \times \text{Distance} = 265 \times 6 = 1590 \text{ N-m} = 1590 \text{ J Ans.}$$

(c) Efficiency of the jet

We also know that efficiency of the jet,

$$\eta = \frac{2(V-v)v}{V^2} = \frac{2(15-6) \times 6}{(15)^2} = 0.48 = 48\% \text{ Ans.}$$

29-7 Force of Jet Impinging on a Fixed Curved Vane

Consider a jet of water entering and leaving a fixed curved vane tangentially as shown in Fig. 29-9.

- Let
- V = Velocity of the jet,
 - a = Cross-sectional area of the jet,
 - α = Inlet angle of the jet, and
 - β = Outlet angle of the jet.

The jet, while moving through the vane, will exert some force on vane. This force may be determined by the finding out the components of the force along the normal and perpendicular to the normal of the vane.

We know that the force of the jet along normal to the vane

$$= \text{Mass of water flowing per second} \\ \times \text{Change of velocity along normal to the vane} \\ = \frac{waV}{g} (V \cos \alpha + V \cos \beta) \quad \dots (i)$$

and force of the jet along perpendicular to the normal to the vane

$$= \frac{waV}{g} (V \sin \alpha - V \sin \beta)$$

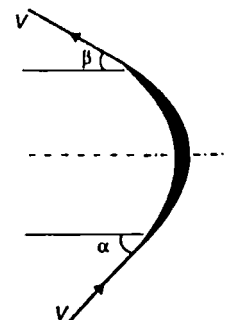


Fig. 29-9. Jet impinging on a fixed curved vane.

Note : Sometimes the total angle through which the jet is deflected is given instead of the inlet and outlet angles of the jet. In such a case, assuming $\alpha = \beta$ and $\theta = 180^\circ - (\alpha + \beta)$, the force of the jet along X-X axis,

$$F_x = \text{Mass of water flowing per second} \times \text{Change of velocity}$$

$$= \frac{waV}{g} \times 2V \sin \frac{\theta}{2} = \frac{2waV^2}{g} \times \sin \frac{\theta}{2}$$

Example 29-12. Calculate the magnitude of the force exerted by a jet of cross-sectional area 2000 mm^2 and of velocity 25 m/s on a fixed smooth curved vane which deflects the jet by 120° .

Solution. Given : $a = 2000 \text{ mm}^2 = 0.002 \text{ m}^2$; $V = 25 \text{ m/s}$ and $\theta = 120^\circ$.

We know that magnitude of the force exerted by the jet,

$$F = \frac{2waV^2}{g} \times \sin \theta = \frac{2 \times 9.81 \times 0.02 \times (25)^2}{9.81} \times \sin 60^\circ \text{ kN}$$

$$= 25 \times 0.8660 = 21.65 \text{ kN Ans.}$$

Example 29-13. A jet of water 40 mm diameter enters a fixed curved vane with a velocity of 50 m/s at an angle 20° to the horizontal. Find the normal and tangential forces exerted by the jet, if it leaves the vane at angle 15° to the horizontal.

Solution. Given : $d = 40 \text{ mm} = 0.04 \text{ m}$; $V = 50 \text{ m/s}$ and $\alpha = 20^\circ$ and $\beta = 15^\circ$.

Normal force exerted by the jet

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.04)^2 = 1.257 \times 10^{-3} \text{ m}^2$$

and normal force exerted by the jet,

$$F_n = \frac{waV}{g} (V \cos \alpha + V \cos \beta)$$

$$= \frac{9.81 \times (1.257 \times 10^{-3}) \times 50}{9.81} \times [(50 \cos 20^\circ + 50 \cos 15^\circ)] \text{ kN}$$

$$= 0.063 \times [(50 \times 0.9397) + (50 \times 0.9659)] = 6.0 \text{ kN Ans.}$$

Tangential force exerted by the jet

We also know that tangential force exerted by the jet,

$$F_t = \frac{waV}{g} (V \sin \alpha - V \sin \beta)$$

$$= \frac{9.81 \times (1.257 \times 10^{-3}) \times 50}{9.81} \times [(50 \sin 20^\circ) - (50 \sin 15^\circ)] \text{ kN}$$

$$= 0.063 \times [(50 \times 0.3420) - (50 \times 0.2588)] \text{ kN}$$

$$= 0.262 \text{ kN Ans.}$$

EXERCISE 29-2

1. A jet of 150 mm diameter is moving with a velocity of 30 m/s . Find the force exerted by the jet when the plate is moving with a velocity of 12 m/s in the direction of the jet. [Ans. 5.73 kN]
2. A jet of water of 100 mm diameter impinges with a velocity of 25 m/s on a plate moving with a velocity of 10 m/s in the direction of the jet. Find the force exerted by the jet.
If the plate is now replaced with a series of vanes moving with the same velocity as that of the plate, find the force exerted by the jet on the vanes. [Ans. 1.77 kN ; 2.95 kN]

3. A jet of 100 mm diameter, moving with a velocity of 12 m/s impinges on a series of vanes moving with a velocity of 8 m/s. Determine (i) force on the plate, (ii) work done per second, and (iii) efficiency
[Ans. 377 N; 3.02 kJ; 44.4%]
4. A jet of water of 80 mm diameter moving with a velocity of 20 m/s enters tangentially a fixed curved vane and is deflected through 135° . Find the work done by the jet.
[Ans. 1.42 kN]

29.8 Force of Jet Impinging on a Moving Curved Vane

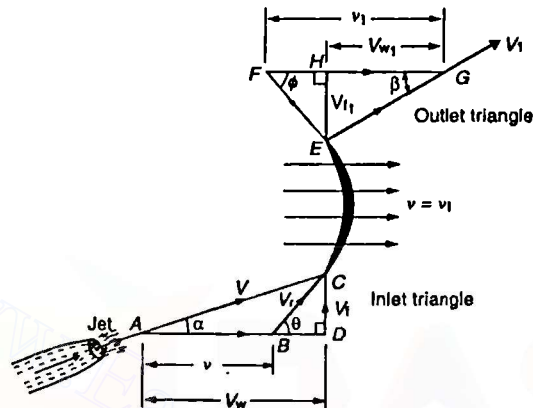


Fig. 29.10. Jet impinging on a moving curved vane.

Consider a jet of water entering and leaving a moving curved vane as shown in Fig. 29.10.

Let

V = Velocity of the jet (AC), while entering the vane,

V_1 = Velocity of the jet (EG) while leaving the vane,

v_1, v_2 = Velocity of the vane (AB, FG),

α = Angle with the direction of motion of the vane, at which the jet enters the vane,

β = Angle, with the direction of motion of the vane, at which the jet leaves the vane,

V_r = Relative velocity of the jet and the vane (BC) at entrance
(It is the vectorial difference between V and v),

V_{r1} = Relative velocity of the jet and the vane (EF) at exit (It is the vectorial difference between v_1 and v_1),

θ = Angle, which V_r makes with the direction of motion of the vane at inlet (known as vane angle at inlet),

ϕ = Angle, which V_{r1} makes with the direction of motion of the vane at outlet (known as vane angle at outlet),

V_w = Horizontal component of V (AD , equal to $V \cos \alpha$). It is a component parallel to the direction of motion of the vane (known as velocity of whirl at inlet),

V_{w1} = Horizontal component of V_1 (HG , equal to $V_1 \cos \beta$). It is a component parallel to the direction of motion of the vane (known as velocity of whirl at outlet),

V_f = Vertical component of V (DC , equal to $V \sin \alpha$). It is a component at right angles to the direction of motion of the vane (known as velocity of flow at inlet).

V_{f1} = Vertical component of V_1 (EH , equal to $V_1 \sin \beta$). It is a component at right angles to the direction of motion of the vane (known as velocity of flow at outlet), and

a = Cross-sectional area of the jet.

It may be seen, from the above, that original notations (*i.e.*, V , V_r , V_w , V_f) stand for the inlet triangle. The notations with suffix (*i.e.*, V_1 , V_{r1} , V_{w1} , V_{f1}) stand for the outlet triangle.

A little consideration will show, that as the jet of water enters and leaves the vanes tangentially, therefore shape of the vanes will be such that V_r and V_{r1} will be along with tangents to the vanes at inlet and outlet. It is thus obvious, that the shape of the vanes is always designed according to the given data (*i.e.*, first the triangle is drawn with the given data and then the vane is drawn in such a way, that V_r and V_{r1} are along the tangents to the vanes at inlet and outlet).

The relations between the inlet and outlet triangles (until and unless given) are :

$$(i) \quad v = v_1, \text{ and}$$

$$(ii) \quad V_r = V_{r1}.$$

We know that the force of jet, in the direction of motion of the vane,

$$\begin{aligned} F_x &= \text{Mass of water flowing per second} \\ &\quad \times \text{Change of velocity of whirl} \\ &= \frac{waV}{g} (V_w - V_{w1}) \end{aligned} \quad \dots(i)$$

and work done in the direction of motion of the vane

$$= \text{Force} \times \text{Distance} = \frac{waV}{g} (V_w - V_{w1}) \times v \quad \dots (ii)$$

and work done per kN of water

$$= (V_w - V_{w1}) \times v \quad \dots(iii)$$

Notes : 1. It is very important to draw the correct shape of inlet and outlet triangle of velocities. Lot of patience and understanding is required for this. But, once the correct shape of the two triangles is drawn, then the example can be solved very easily.

2. The direction of V_{w1} plays an important role for finding out the work done. If v_1 is greater than $(V_{r1} \cos \phi)$, the value of V_{w1} is positive. Otherwise it is negative.

3. Power developed by the vane may be found out as usual.

Example 29-14. A jet of water, moving at 60 m/s is deflected by a vane moving at 25 m/s in a direction at 30° to the direction of the jet. The water leaves the blades normally to the motion of the vanes.

Draw inlet and outlet triangles of velocities, and find the vane angles for no shock at entry and exit. Take relative velocity at outlet to be 0.85 of the relative velocity at inlet.

Solution. Given : $V = 60$ m/s; $v = 25$ m/s; $\alpha = 30^\circ$ and $V_{r1} = 0.85 V_r$.

Inlet and outlet triangles of velocities

The inlet and outlet triangles of velocities may be drawn as shown in Fig. 29-11 and as discussed below :

1. First of all, draw a horizontal line (*i.e.*, in the direction of motion of the vanes) and cut off AB equal to 25 m/s to some suitable scale, to represent the velocity of vanes (v).

2. Draw a line at an angle of 30° (i.e., angle at which the jet enters the vanes) and cut off AC equal to 60 m/s to the scale to represent the velocity of jet (V).
3. Join BC , which gives the relative velocity (V_r) to the scale. Now extend AB to D , such that DC is perpendicular to AB . From the geometry of the figure, we find that the lengths AD and DC give the velocity of whirl (V_w) and velocity of flow (V_f) respectively to the scale. Measure these two lengths.
4. Now draw the curved vane, such that V_r is tangential to the vane at the inlet tip.
5. Then draw the outlet triangle of velocities with V_{r1} tangential to the vane. Now cut off (EF) i.e., $V_{r1} = 0.85 V_r$.

... (As given)

and (EG) i.e., $V_{f1} = V_f$ at right angles to v and v_1 (because it is given that the water leaves the blades normally to the motion of the blades).

6. Now take (FG) i.e., $v_1 = v$ and complete the outlet triangle of velocities.

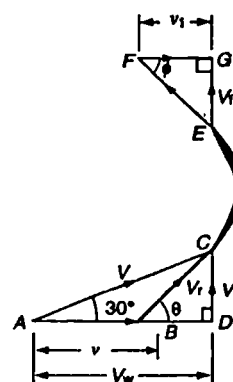


Fig. 29-11.

Vane angle at inlet

Let θ = Vane angle at inlet.

From the geometry of the inlet triangle (ACD) we find that the velocity of whirl at inlet,

$$V_w = V \cos 30^\circ = 60 \times 0.866 = 51.96 \text{ m/s}$$

and velocity of flow at inlet, $V_f = V \sin 30^\circ = 60 \times 0.5 = 30 \text{ m/s}$

$$\text{and} \quad \tan \theta = \frac{V_f}{V_w - v} = \frac{30}{51.96 - 25} = 1.1128$$

$$\text{or} \quad \theta = 48.1^\circ \quad \text{Ans.}$$

Vane angle at outlet

Let ϕ = Vane angle at outlet.

From the geometry of the inlet triangle, we also find that the relative velocity of jet and vane,

$$V_r = \frac{V_f}{\sin 48.1^\circ} = \frac{30}{0.7447} = 40.31 \text{ m/s}$$

Now in the outlet triangle (EFG) relative velocity of jet and vane,

$$V_{r1} = 0.85 \times V_r = 0.85 \times 40.31 \text{ m/s} \quad \dots (\text{given})$$

$$= 34.26 \text{ m/s}$$

In the outlet triangle, we find that

$$\cos \phi = \frac{v_1}{V_{r1}} = \frac{25}{34.26} = 0.7297$$

$$\therefore \quad \phi = 43.1^\circ \quad \text{Ans.}$$

Example 29-15. A jet of water, having a velocity of 30 m/s impinges on a series of vanes with a velocity of 15 m/s. The jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120° .

Sketch velocity triangles at entrance and exit, and determine the vane angles, so that the water enters and leaves without shock.

Solution. Given : $V = 30 \text{ m/s}$; $v = 15 \text{ m/s}$; $\alpha = 30^\circ$ and $\beta = 120^\circ$.

Velocity triangles at entrance and exit

The velocity triangles at entrance and exit may be drawn as shown in Fig. 20-12 and as discussed below :

1. First of all, draw a horizontal line (*i.e.*, in the direction of motion of the vanes) and cut off AB equal to 15 m/s to some suitable scale, to represent the velocity of vanes (v).
2. Draw a line at an angle of 30° (*i.e.*, the angle at which the jet enters the vane) and cut off AC equal to 30 m/s to the scale to represent the velocity of jet (V).
3. Join BC , which gives the relative velocity (V_r) to the scale. Now extend AB to D , such that DC is perpendicular to AB . From the geometry of the figure, we find that the lengths of AD and DC give the velocity of whirl (V_w) and velocity of flow V_f respectively to the scale. Measure these two lengths.
4. Now draw the curved vane, such that V_r is tangential to the vane at the entrance tip.
5. Then draw a velocity triangle at exit with V_{r1} tangential to the vane. Now cut off (EF) *i.e.*, $V_{r1} = V_r$ = Through F , draw a horizontal line and cut off FH equal to 15 m/s to the scale to represent the velocity of vanes at exit (*i.e.*, v_1).
6. Join EH which will make an angle of 120° with FH (as the jet leaves the vane at an angle of 120°). Now draw EG perpendicular to FH .

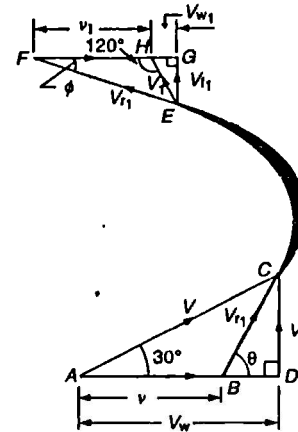


Fig. 29-12.

Vane angles

Let θ = Vane angle at inlet, and
 ϕ = Vane angle at outlet.

From the inlet triangle, we find that the velocity of whirl at inlet,

$$V_w = V \cos 30^\circ = 30 \times 0.866 = 25.98 \text{ m/s}$$

and velocity of flow at inlet, $V_f = V \sin 30^\circ = 30 \times 0.5 = 15 \text{ m/s}$

$$\therefore \tan \theta = \frac{V_f}{V_w - v} = \frac{15}{25.98 - 15} = 1.366$$

$$\text{or } \theta = 53.8^\circ \text{ Ans.}$$

From the inlet triangle, we also find that the relative velocity of jet and vane,

$$V_r = \frac{V_f}{\sin 53.8^\circ} = \frac{15}{0.8070} = 18.59 \text{ m/s}$$

From the outlet triangle, we find from the sine rule,

$$\frac{v_1}{\sin (60^\circ - \phi)} = \frac{V_{r1}}{\sin 120^\circ}$$

$$\text{or } \sin (60^\circ - \phi) = \frac{v_1 \sin 120^\circ}{V_{r1}} = \frac{15 \times \sin 60^\circ}{18.59} = \frac{15 \times 0.866}{18.59} = 0.6988$$

$$\therefore (60^\circ - \phi) = 44.3^\circ$$

$$\text{or } \phi = 15.7^\circ \text{ Ans.}$$

Example 29-16. A circular jet delivers water at the rate of 60 litres/s, with a velocity of 24 m/s. The jet impinges tangentially on the vane moving in the direction of the jet, with a velocity of 12 m/s. The vane is so shaped that, if stationary, it would deflect the jet through an angle 45° . Through what angle will it deflect the jet? Also find the work done/s.

Solution. Given : $Q = 60 \text{ litres/s} = 0.06 \text{ m}^3/\text{s}$; $V = 24 \text{ m/s}$; $v = 12 \text{ m/s}$; $\alpha = 0$ (because the jet impinges tangentially on the vane moving in the direction of the jet) and $\phi = 45^\circ$.

Angle through which the jet is deflected

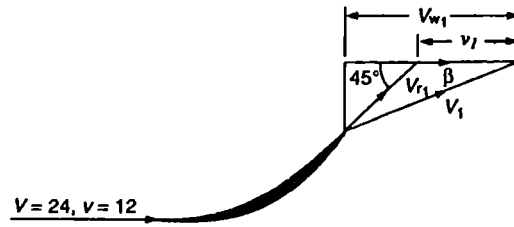


Fig. 29-13.

Since the jet of water moves in the same direction, as that of the vanes, therefore the inlet triangle will be a straight line as shown in Fig. 29-13. Thus velocity of whirl at inlet,

$$V_{w1} = 24 \text{ m/s}$$

and relative velocity of the jet and vane,

$$V_r = 24 - 12 = 12 \text{ m/s}$$

Since the velocity of vanes at outlet (v_1) and relative velocity of the jet and vane at outlet (V_{r1}) are equal to 12 m/s, therefore the outlet triangle of velocities will be an isosceles triangle as shown in Fig. 29-13.

\therefore Angle through which the jet is deflected,

$$\beta = \frac{\phi}{2} = \frac{45}{2} = 22.5^\circ \text{ Ans.}$$

Work done per second

From the outlet triangle of velocities, we find that velocity of whirl at outlet,

$$V_{w1} = 12 + 12 \cos 45^\circ = 12 + (12 \times 0.707) = 20.5 \text{ m/s}$$

and work done per second $W = \frac{waV}{g} (V_w - V_{w1}) \times v$

$$= \frac{9.81 \times 0.06}{9.81} (24 - 20.5) = 2.52 \text{ kN-m} = 2.52 \text{ kJ Ans.}$$

Example 29-17. A jet of water 100 mm in diameter, moving with a velocity of 25 m/s in the direction of the vanes, enters the vanes moving with a velocity of 12.5 m/s. If the jet leaves the vanes at an angle of 60° with the direction of motion of the vanes, find (i) force on the vanes in the direction of their motion, and (ii) work done per second.

Solution. Given : $d = 100 \text{ mm} = 0.1 \text{ m}$; $V = 25 \text{ m/s}$; $v = 12.5 \text{ m/s}$; $\alpha = 0$ (because the jet is moving in the direction of the vanes) and $\beta = 60^\circ$.

(i) Force on the vanes in the direction of their motion

We know that cross-sectional area of the jet,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.1)^2 = 7.854 \times 10^{-3} \text{ m}^2$$

Since the jet of water moves in the same direction as that of the vanes, therefore the inlet triangle will be a straight line as shown in Fig. 29-14. Thus velocity of whirl at inlet,

$$V_w = 25 \text{ m/s}$$

and relative velocity of the jet and vane,

$$V_r = 25 - 12.5 = 12.5 \text{ m/s}$$

Since the velocity of vane at outlet (v_1) and relative velocity of the jet and vane at outlet (V_{r1}) are equal, and the jet leaves the vanes at 60° (given), therefore the outlet triangle will be an equilateral triangle as shown in Fig. 29-14.

Thus, from the outlet triangle, we find that the velocity of whirl at outlet,

$$V_{w1} = 6.25 \text{ m/s}$$

Fig. 29-14.

∴ Force on the vanes in the direction of their vanes

$$\begin{aligned} &= \frac{waV}{g} (V_w - V_{w1}) \\ &= \frac{9.81 \times (7.854 \times 10^{-3}) \times 25}{9.81} (25 - 6.25 \text{ kN}) = 3.68 \text{ kN} \end{aligned}$$

(ii) Work done per second

We also know that work done per second

$$\begin{aligned} W &= \frac{waV}{g} (V_w - V_{w1}) \times v = \frac{9.81 \times (7.854 \times 10^{-3}) \times 25}{9.81} (25 - 6.25) \times 12 \\ &= 44.2 \text{ kN-m/s} = 44.2 \text{ kJ Ans.} \end{aligned}$$

Example 29-18. A jet of water moving at 12 m/s impinges on a concave vane shaped to deflect the jet through 120° when stationary. If the vane is moving at 5 m/s, find the angle of jet so that there is no shock at outlet. What is the absolute velocity of jet at exit and the work done per kN of water? Assume that the vane is smooth.

Solution. Given : $V = 12 \text{ m/s}$; Angle through which the jet is deflected = 120° . Therefore $\phi = 180^\circ - 120^\circ = 60^\circ$ and $v = 5 \text{ m/s}$.

Angle of the jet at outlet

Let β = Angle of the jet at outlet.

Since no data is given for the inlet triangle of velocities, therefore the jet of water will be assumed to move in the same direction as that of the vanes. It is thus obvious, that the inlet triangle will be a straight line as shown in Fig. 29-15.

∴ Velocity of whirl at inlet,

$$V_w = V = 12 \text{ m/s}$$

and relative velocity of the jet and vane,

$$V_r = 12 - 5 = 7 \text{ m/s}$$

With the velocity of vanes at outlet (v_1) equal to 5 m/s, relative velocity ($V_{r1} = V_r$) equal to 7 m/s and vane angle at outlet (ϕ) equal to 60° , draw the outlet triangle of velocities as shown in Fig. 29-15.

From the geometry of outlet triangle of velocities, find that the velocity of flow

$$V_{r1} = V_{r1} \sin 60^\circ = 7 \times 0.866 = 6.06 \text{ m/s}$$

and velocity of whirl, $V_{w1} = 5 - V_{r1} \cos 60^\circ = 5 - (7 \times 0.5) = 1.5 \text{ m/s}$

$$\therefore \tan \beta = \frac{V_n}{V_{w1}} = \frac{6.06}{1.5} = 4.04$$

$$\text{or } \beta = 76.1^\circ \text{ Ans.}$$

Absolute velocity of the jet at exit

From the geometry of outlet triangle of velocities, we also find that absolute velocity of the jet at exit,

$$V_1 = \frac{V_n}{\sin \beta} = \frac{6.06}{\sin 76.1^\circ} = \frac{6.06}{0.9707} = 6.24 \text{ m/s Ans.}$$

Work done per kN of water

We also know that work done per kN of water

$$\begin{aligned} &= (V_w - V_{w1}) \times v = (12 - 1.5) \times 5 = 52.5 \text{ kN-m/s} \\ &= 52.5 \text{ kJ/s Ans.} \end{aligned}$$

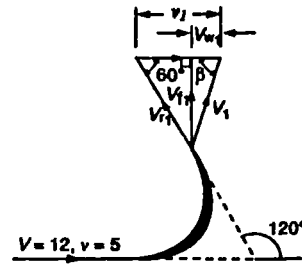


Fig. 29-15.

EXERCISE 29.3

1. A jet of water having a velocity of 40 m/s enters a curved vane which is moving with a velocity of 20 m/s. The jet makes an angle of 30° with the direction of motion of the vane at inlet and leaves at angle of 90° with the direction of motion at outlet. Draw the velocity triangles and determine the vane angles at inlet and outlet, so that the water enters and leaves without shock. [Ans. 53.8° ; 36.2°]
2. A jet of water moving with a velocity of 20 m/s strikes curved vanes moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vanes at inlet and leaves at an angle of 130° with the direction of motion of the vanes. Calculate the vane angles at inlet and outlet, so that the water has shockless entry and exit. [Ans. 37.9° ; 6.6°]
3. A jet of water moving with a velocity of 12 m/s impinges on concave shaped vanes to deflect the jet through 120° when stationary. If the vane is moving with a velocity of 5 m/s, find the angle of jet, so that there is no shock at inlet. What is the absolute velocity of the jet at exit both in magnitude and direction? Assume the vane to be symmetrical. [Ans. 17.98° ; 3.96 m/s; 69.1°]
[Hint : Since the vane is symmetrical, therefore $\theta = \phi = 30^\circ$]
4. A 150 mm diameter jet of water moving at 30 m/s impinges on a series of vanes moving at 15 m/s in the direction of the jet and leaves at 60° with the direction of motion of the jet. Calculate (i) force exerted by the jet in the direction of motion of the vanes; and (ii) work done by the jet. [Ans. 11.95 kN; 179.25 kJ]

QUESTIONS

1. What do you understand by the term 'jet of water'? Derive an expression for the force of jet on a fixed plate.
2. Show that the normal force exerted by a jet of water on an inclined plate is given by the relation,

$$F = \frac{waV^2 \sin \theta}{g}$$

where

a = Area of jet,

V = Velocity of the jet, and

θ = Inclination of the plate with the jet.

3. Derive an equation between the angle, through which a hinged plate will swing, in terms of force of the jet and weight of the plate.
4. Derive an expression for the force of jet impinging on a moving plate and compare it with the force, when the same jet is impinging on a series of moving vanes.

5. Derive an expression for the force, work done and efficiency of a moving curved vane.
6. Define velocity of flow and velocity of whirl and explain their significance.

OBJECTIVE TYPE QUESTIONS

1. The force exerted by a jet of water impinging normally on a fixed plate
 (a) $\frac{waV}{2g}$ (b) $\frac{waV}{g}$ (c) $\frac{waV^2}{2g}$ (d) $\frac{waV^2}{g}$
 where w = Specific weight of water,
 a = Cross-sectional area of the jet, and
 V = Velocity of the jet.
2. The force exerted by a jet of water impinging on a fixed plate inclined at an angle θ with jet.
 (a) $\frac{waV}{2g} \times \sin \theta$ (b) $\frac{waV}{g} \times \sin \theta$ (c) $\frac{waV^2}{2g} \sin \theta$ (d) $\frac{waV^2}{g} \sin \theta$
3. The force exerted by a jet of water impinging normally with a velocity V on a plate, which moves in the direction of the jet with a velocity v is
 (a) $\frac{wa(V-v)^2}{2g}$ (b) $\frac{wa(V-v)^2}{g}$ (c) $\frac{wa(V-v)}{2g}$ (d) $\frac{wa(V-v)}{g}$
4. The efficiency of a jet of water impinging normally with a velocity V on a series of vanes moving with a velocity (v) is given by the relation
 (a) $\frac{2(V-v)}{V^2}$ (b) $\frac{(V-v)}{V^2}$ (c) $\frac{(v-v)^2}{2V}$ (d) $\frac{(V-v)^2}{V}$

ANSWERS

1. (d) 2. (c) 3. (b) 4. (a)