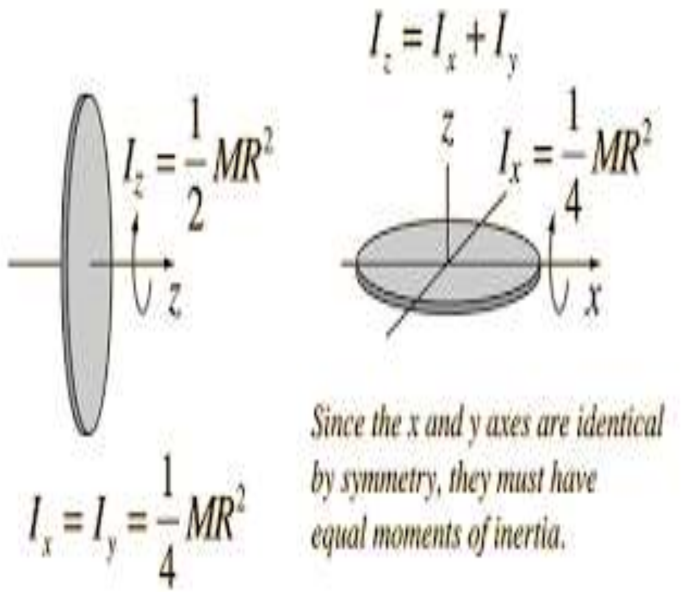
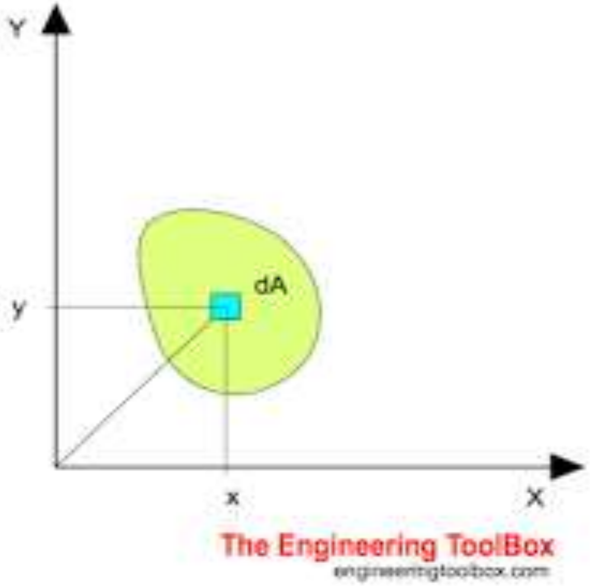
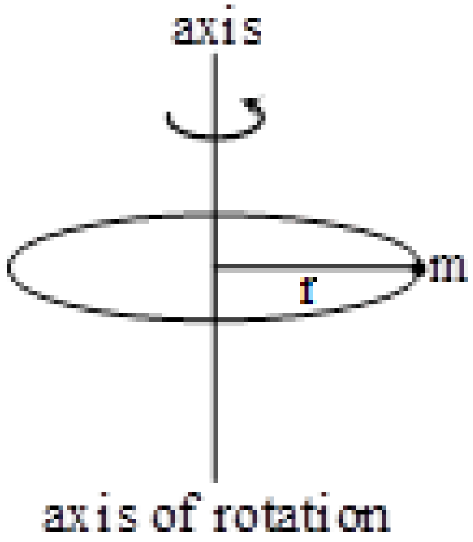
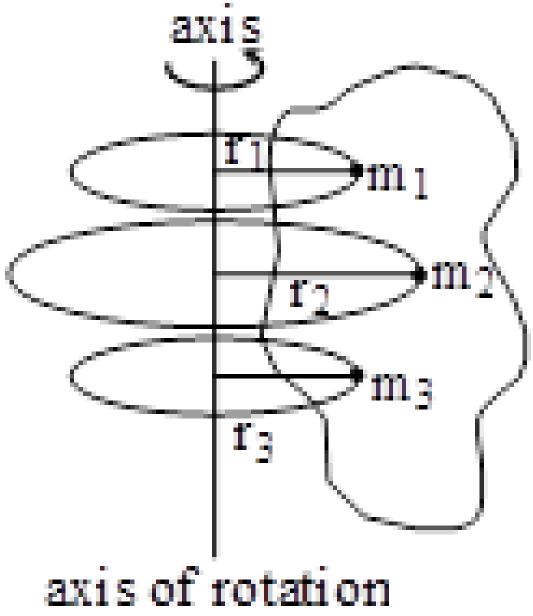
MOMENT OF INERTIA:

When examinee by itself feels that there is no physical significance for Moment of Inertia. It is just a mathematical expression & usually denoted by I. Generally instead of force, the area or mass of a figure or body is taken into consideration, then the second moment is termed as second moment of area or second moment of mass, but all such second moments are broadly called Momentum of Inertia.









Moment of Inertia Defined

- The moment of inertia measures the resistance to a change in rotation.
  - Change in rotation from torque
  - Moment of inertia $I = mr^2$ for a single mass
- The total moment of inertia is due to the sum of masses at a distance from the axis of rotation.

$$I = \sum_{i=1}^{N} m_i r_i^2$$

Rigid Body Rotation

- The moments of inertia for many shapes can found by integration.
  - Ring or hollow cylinder: $I = MR^2$
  - Solid cylinder: $I = (1/2)\ MR^2$
  - Hollow sphere: $I = (2/3)\ MR^2$
  - Solid sphere: $I = (2/5)\ MR^2$

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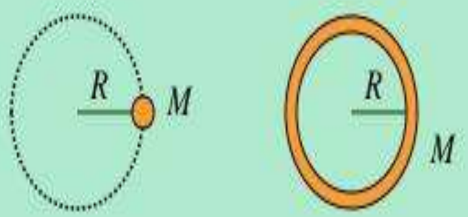
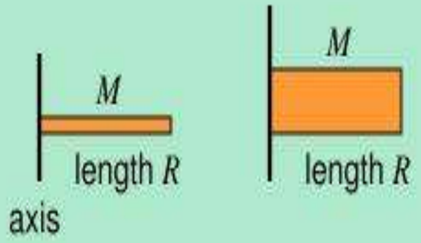
Point and Ring

- The point mass, ring and hollow cylinder all have the same moment of inertia.
  - $I = MR^2$
- All the mass is equally far away from the axis.



- The rod and rectangular plate also have the same moment of inertia.
  - $I = (1/3)\ MR^2$
- The distribution of mass from the axis is the same.



Understanding the Mass Moment of Inertia

a) This ballerina is rotating about an axis passing through her center of mass.

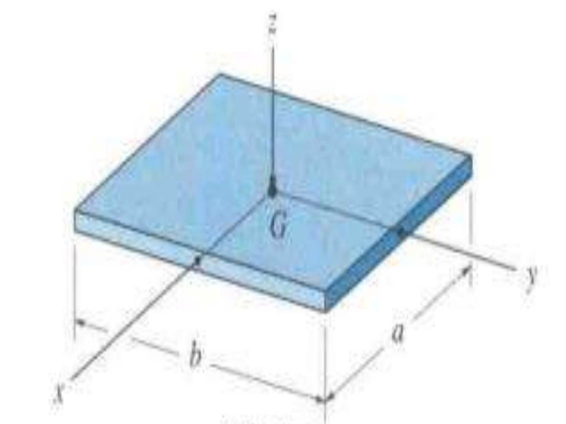
b) This ballerina is rotating about an axis which is not passing through her center of mass.

In which case the rotation is easier and there is less resistance against angular acceleration? Case a

MOMENT OF INERTIA

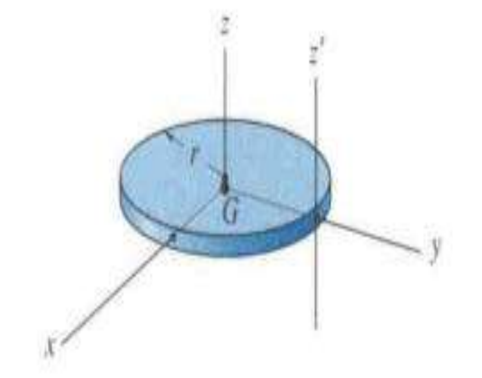
(continued)

The figures below show the mass moment of inertia formulations for two flat plate shapes commonly used when working with three dimensional bodies. The shapes are often used as the differential element being integrated over the entire body.



Thin plate

$$I_{xx} = \frac{1}{12}mb^2 \quad I_{yy} = \frac{1}{12}ma^2 \quad I_{zz} = \frac{1}{12}m(a^2 + b^2)$$



Thin circular disk

$$I_{xx} = I_{yy} = \frac{1}{4}mr^2 \quad I_{zz} = \frac{1}{2}mr^2 \quad I_{z'z'} = \frac{3}{2}mr^2$$

THEOREM OF PERPENDICULAR AXIS:

It states " If I_{xx} and I_{yy} be the Moments of Inertia of a plane section about to perpendicular axis meeting at "O" point, the Moment of Inertia I_{zz} about the axis z-z perpendicular to the plane and passing through the intersection of x-x and y-y axes is given by the relation, $I_{zz} = I_{xx} + I_{yy}$.

THEOREM OF PARALLEL AXIS:

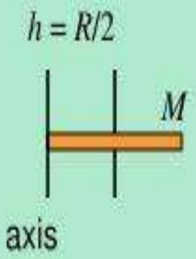
It states " If the Moment of Inertia of a plane area or section about an axis through its centre of gravity, be denoted by I_G, the Moment of Inertia of the area about an axis AB and parallel to the base at a distance h from the Centre of Gravity is given by $I_{AB} = I_G + ah^2$, where I_{AB} = Moment of Inertia of the required area about AB, I_G = Moment of Inertia of the area through centre of gravity, a = area of the required section and h=distance between centre of gravity of the given section and the axis AB.

Parallel Axis Theorem

- Some objects don't rotate about the axis at the center of mass.
- The moment of inertia depends on the distance between axes.

$$I = I_{CM} + Mh^2$$

- The moment of inertia for a rod about its center of mass:

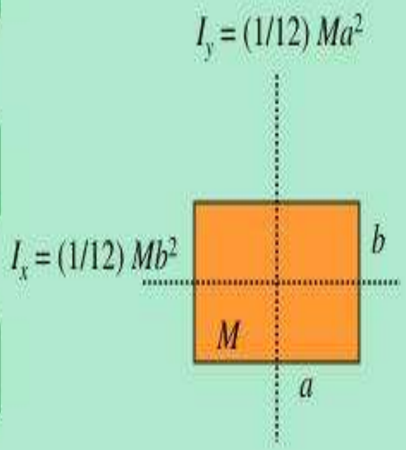


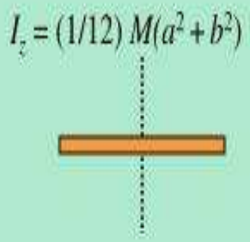
$$(1/3)MR^2 = I_{CM} + M(R/2)^2$$

$$I_{CM} = (1/3)MR^2 - (1/4)MR^2$$

$$I_{CM} = (1/12)MR^2$$

Perpendicular Axis Theorem



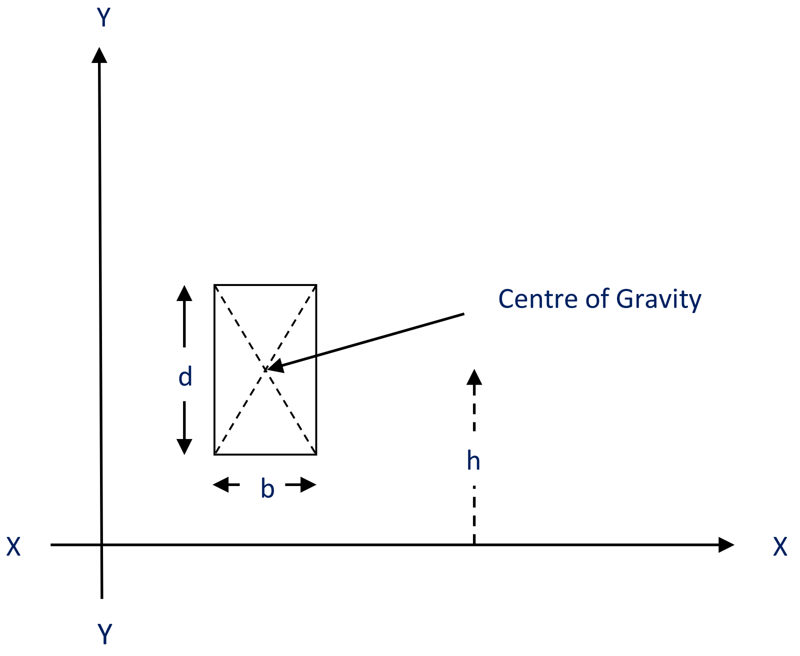


- For flat objects the rotational moment of inertia of the axes in the plane is related to the moment of inertia perpendicular to the plane.

$$I_z = I_x + I_y$$

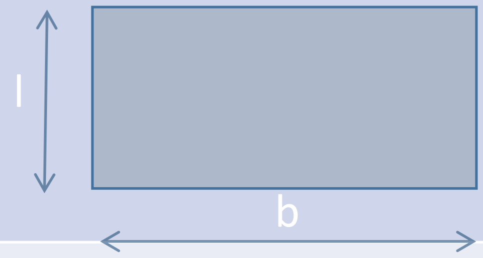
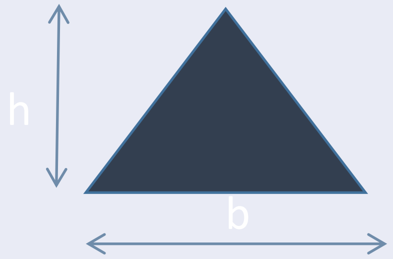
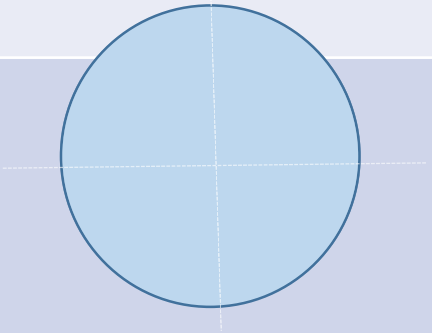
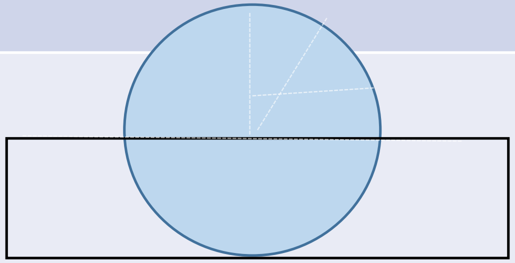
Moment of Inertia of a Composite Section is find out as below :

- Split up the given section into plane areas (i.e. rectangular, trangular, circular etc.)
- Find out the Moment of Inertia of these areas about their respective Centre of Gravity.



Now transfer these Moments of Inertia about the required axis by the theorem of Parallel axis, [i.e. $I_{AB} = I_G + ah^2$].

- The Moment of Inertia of a given section may now be obtained by the ∑ (Algebraic Sum) of the Moments of the required axes.

Sl. No.	Basic Shape	Moment of Inertia	Diagram
1.	Rectangle	$I_{xx} = \frac{1}{12} bl^3$ $I_{YY} = \frac{1}{12} lb^3$	
2.	Triangle	$I_{xx} = 1/36 * bh^3$ (Through CG) (Through Base $1/12 * bh^3$)	
3.	Circle	$I_{xx} = I_{yy} = \pi r^4/4$	**πr^2 *1/4** 
4.	Semi Circle	$I_{xx} = 0.11 r^4$ $I_{yy} = \pi r^4/8$	**3r/8** 

PARALLEL-AXIS THEOREM
(continued)

Radius of Gyration

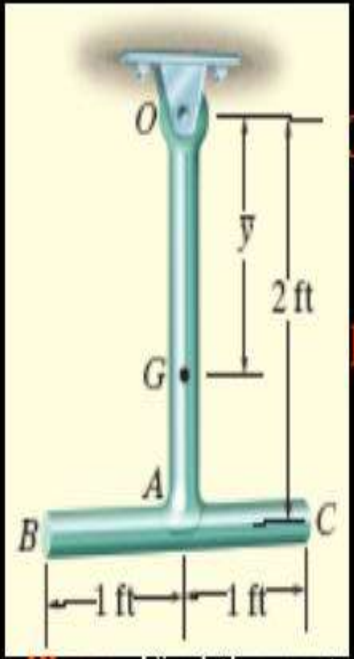
The mass moment of inertia of a body about a specific axis can be defined using the radius of gyration (k). The radius of gyration has units of length and is a measure of the distribution of the body's mass about the axis at which the moment of inertia is defined.

$$I = m k^2 \quad \text{or} \quad k = \sqrt{(I/m)}$$

Composite Bodies

If a body is constructed of a number of simple shapes, such as disks, spheres, or rods, the mass moment of inertia of the body about any axis can be determined by algebraically adding together all the mass moments of inertia, found about the same axis, of the different shapes.

EXAMPLE II



Given: Two rods assembled as shown, with each rod weighing 10 lb.

Find: The location of the center of mass G and moment of inertia about an axis passing through G of the rod assembly.

Plan: Find the centroidal moment of inertia for each rod and then use the parallel axis theorem to determine I_G.

Solution: The center of mass is located relative to the pin at O at a distance $\bar{y}$, where

$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i} = \frac{1(\frac{10}{32.2}) + 2(\frac{10}{32.2})}{\frac{10}{32.2} + \frac{10}{32.2}} = 1.5 \text{ ft}$$