STRENGTH OF MATERIALS

MOMENT OF INERTIA

- The moment of inertia of a body about a line is the product of its mass and the square of its distance from that line.

- Mathematically moment of inertia of a body may be expressed.

$$I = M.r.^2$$

Where, I = Moment of inertia

M = Mass of the body

r = Distance between the C.G. of the body and the line about which moment of Inertia is required to be finding out.

MOMENT OF INERTIA OF A PLANE AREA

- **Consider a plane area, whose moment of inertia is required to be finding out about a line. AB as shown in figure.**

  Split up the plane surface in to a number of small strips as shown in figure.

  Let, *a1, a2, a3* , ----------- be the area of the strips.

  ***r1, r2, r3* , ----------- be the distance between the fixed line and C.G. of strips.**

Than moment of inertia of the area,

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 \text{-------------}$$

$$= \sum a r^2$$

METHODS FOR FINDING OUT MOMENT OF INERTIA

- The moment of inertia of a body may be find out the following methods:-

1. By using ROUTH'S rule
2. By integration

MOMENT OF INERTIA OF RECTANGULAR SECTION *(BY INTRIGRATION)*

- A rectangular section is consider where

  'l' = Length of the rectangular area

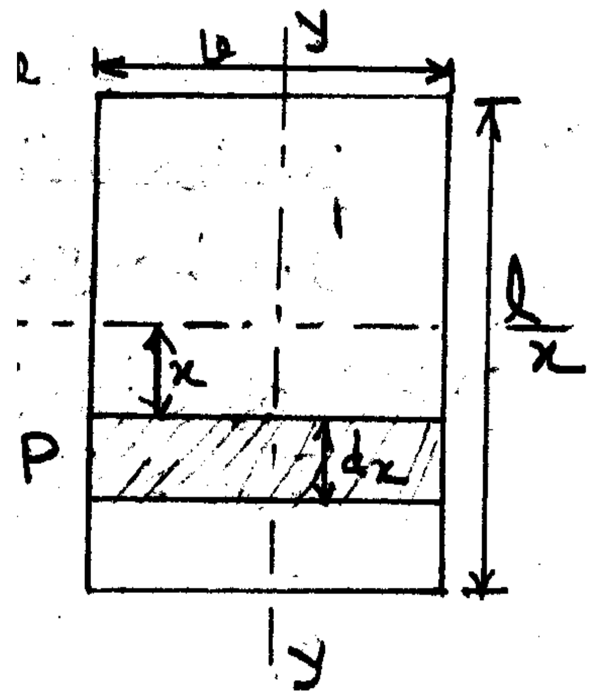
  '*b'* = Breadth of the rectangular area

  A strip 'PQ' of thickness '*dx'* parallel to *x – x* and at a distance x from it.

  Therefore, area of the strip 'PQ'= *b.dx*

  Moment of inertia of the strip about *y – y* axis = area × distance ^{2}

  = *bdx.x²*



By integration *:-*

- $$I_{xx} = \frac{bl^3}{12} \quad \& \quad I_{yy} = \frac{lb^3}{12}$$

MOMENT OF INERTIA OF A HOLLOW RECTANGULAR SECTION

- $$I_{xx} = \frac{bl^3}{12} - \frac{b_1 l_1^3}{12} \quad \& \quad I_{yy} = \frac{lb^3}{12} - \frac{l_1 b_1^3}{12}$$

Where,

b = Breadth of the out line rectangle

l = Length of the out line rectangle

b_1= Breadth of the cut rectangle

l_1 = Length of cut out rectangle

MOENT OF INERTIA OF CIRCULAR SECTION

- M.I. of circular section

$$I_{xx} = \frac{\pi}{64} D^4$$

$$I_{yy} = \frac{\pi}{64} D^4$$

Where, D = Diameter of the circle.

MOENT OF INERTIA OF HOLLOW CIRCULAR SECTION

- M.I. of hollow circular section

$$I_{xx} = \frac{\pi}{64} (D^4 - d^4)$$

$$I_{yy} = \frac{\pi}{64} (D^4 - d^4)$$

Where, D = Outer diameter

d = Inner diameter

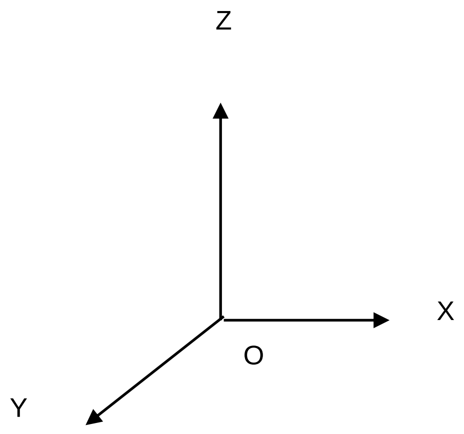
THEOREM OF PERPENDICULAR AXIS

- It states, if Ixx and Iyy be the moment of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia, I_{zz} about the axis z-z perpendicular to the plane and passing through the inter section of x–x and y-y axis is given by the relation

  $I_{zz} = I_{xx} + I_{yy}$.

THEOREM OF PARALLEL AXIS

- It states if the M.I. of a plane are about an axis through it's C.G. be denoted by IG, the M.I. of the area about an axis, AB, parallel to the first and at a distance 'h' from the C.G.is given by,



I_{AB} = M.I. of the area about the line AB
I_G = M.I. of the area about the line C.G.
A = area of the section
h = Distance between C.G. of the section and the axis AB.

$$I_{AB} = I_G + A h^2$$

RADIUS OF GYRATION

- Radius of gyration of an area about any axis is defined as the distance, square of which, when multiply by the whole area gives the moment of inertia of the area about that axis. It is generally denoted by the letter 'A'. It's unit is cm.

Therefore, by definition,

$$K^2 \times A = I$$

Therefore, $K = \sqrt{\dfrac{I}{A}}$

Where,

K = Radius of gyration about any axis

I = Moment of inertia of the whole section of an area about the same axis.

A = Area of the whole section.

EMPIRICAL FORMULA

- Euler as formula is valid or long column. It does not take into consideration the direct compressive stress. In order to fill up this lacuna. Many more formula were proposed by different centrist all over the world. The following empirical formulas are important.

Rankin’s formula, 2. Johnson’s formula, 3. Indian standard coad.