34

Performance of Turbines

1. Introduction. 2. Characteristics of Turbines. 3. Unit Power. 4. Unit Speed. 5. Unit Discharge. 6. Significance of Unit Power, Unit Speed and Unit Discharge. 7. Specific Speed of a Turbine. 8. Significance of Specific Speed. 9. Selection of Turbines. 10. Selection Based on Specific Speed. 11. Selection Based on Head of Water. 12. Relation between Specific Speed and Shape of Reaction Turbine Runner. 13. Characteristic Curves of Turbine. 14. Characteristic Curves for Pelton Wheel. 15. Characteristic Curves for Francis' Turbine. 16. Cavitaiton.

34·1 Introduction

In the last two chapters (*i.e.*, in Impulse Turbines and Reaction Turbines) we have assumed that, in general, a turbine will work under a constant head, speed and output. But in actual practice, these assumptions rarely prevail. It is thus essential to review the nature of such variations, which generally take place. Though there are many types of variations, yet the following are important from the subject of point of view:

1. Keeping the discharge contant, the head of water and output may vary. In such cases, the speed should be adjusted, so that there is no appreciable change in efficiency.
2. Keeping the head of water and speed constant, the output may vary. In such cases, the discharge of the turbine should be adjusted.
3. In turbines, working under low heads, the head of water and speed may vary. Although the speed is allowed to fluctuate within narrow permissible limits, yet the head may vary up to 50%.
4. Keeping the head of water and discharge constant, the speed may vary by adjusting the load on the turbine. This is usually done in a laboratory.

34·2 Characteristics of Turbines

Sometimes, we have to compare the performances of turbines, of different outputs and speeds, working under different heads. This comparison will be much convenient, if we calculate the outputs of the turbines when the head of water is reduced to unity. *i.e.*, 1 metre. We always study the following three characteristics of a turbine under a unit head.

1. Unit power, 2. Unit speed, and 3. Unit discharge.

34·3 Unit Power

The power developed by a turbine, working under a head of 1 metre, is known as unit power. Now consider a turbine whose unit power is required to be found out.

Let H = Head of water, under which the turbine is working

P = Power developed by the turbine under a head of water H,

Q = Discharge through the turbine under a head of water H, and

P_u = Power developed by the same turbine, under a unit head.

We know that the velocity of water (assuming C_v as unity), and discharge, $$Q = aV = a\sqrt{2gH}$$

We also know that the power developed by a turbine,

$$P = wQH = w(a\sqrt{2gH})H$$
$$\propto H^{3/2}$$
$$= P_u.H^{3/2}$$

or $$P_u = \frac{P}{H^{3/2}}$$

34·4 Unit Speed

The speed of a turbine, working under a head of 1 metre, is known as unit speed. Now consider a turbine whose unit speed is required to be found out.

Let H = Head of water under, which the turbine is working.

v = Tangential velocity of the runner,

N = Speed of the turbine runner under a head of water H, and

N_u = Speed of the same turbine, under a unit head,

We know that the velocity of water (assuming C_v as unity),

$$V = \sqrt{2gH}$$

and tangential velocity of the runner,

$$v \propto \text{Velocity of water } (V)$$
$$\propto \sqrt{H} \qquad ...(V = C_v\sqrt{2gH})$$

We also know that the speed of the turbine runner,

or $$N = \frac{60v}{\pi D} \qquad ...\left(\because v = \frac{\pi DN}{60}\right)$$
$$\propto v$$
$$\propto \sqrt{H} \qquad ...(\because v \propto H)$$
$$= N_u.\sqrt{H}$$

$\therefore$ $$N_u = \frac{N}{\sqrt{H}}$$

34·5 Unit Discharge

The discharge of a turbine, working under a head of 1 metre, is known as unit discharge. Now consider a turbine whose unit discharge is required to be found out.

TLet H = Head of water, under which the turbine is working,

Q = Discharge through the turbine under a head of water H, and

Q_u = Discharge through the same turbine, under a unit head.

We know that the velocity of water (assuming C_v as unity),

$$V = aV = a\sqrt{2gH}$$
$$\propto \sqrt{H}$$
$$= Q_u.\sqrt{H}$$

$\therefore$ $$Q_u = \frac{Q}{\sqrt{H}}$$

Example 34·1 *A Pelton wheel develops 1750 kW under a head of 100 metres while running at 200 r.p.m. and discharging 2500 litres of water per second. Find the unit power, unit speed and unit discharge of the wheel.*

Solution. Given, $P = 1750$ kW; $H = 100$ m; $N = 200$ r.p.m. and $Q = 2500$ litres/s $= 2{\cdot}5$ m^3/s

Unit power of the wheel

We know that unit power of the wheel,

$$P_u = \frac{P}{H^{3/2}} = \frac{1750}{(100)^{3/2}} = \frac{1750}{1000} = 1{\cdot}75 \text{ kW} \quad \textbf{Ans.}$$

Unit speed of the wheel

We know that unit speed of the wheel,

$$N_u = \frac{N}{\sqrt{H}} = \frac{200}{\sqrt{100}} = \frac{200}{10} = 20 \text{ r.p.m.} \quad \textbf{Ans.}$$

Unit discharge of the wheel

We also know that unit discharge of the wheel,

$$Q_u = \frac{Q}{\sqrt{H}} = \frac{2{\cdot}5}{\sqrt{100}} = \frac{2{\cdot}5}{10} = 0{\cdot}25 \text{ m}^3/\text{s} \quad \textbf{Ans.}$$

34·6 Significance of Unit Power, Unit speed and Unit Discharge

In the last articles, we have discussed the characteristics of turbines. *i.e.*. unit power, unit speed and unit discharge. As a matter of fact, the significance of unit power, unit speed and unit discharge is of much importance in the field of Hydraulic Machines. It helps us in finding out the behaviour of a turbine, when it is put to work under different heads of water as discussed as follows:

1. *Significance of unit power*

Let H = Head of water, under which the turbine is working.

P = Power developed by the turbine, under head of water H, and

P_1 = Power developed by the turbine under head of water H_1.

We have seen in Art 34·3 that

$$P \propto H^{3/2}.$$

$$\therefore \quad P_1 = H_1^{3/2}$$

or

$$\frac{P}{P_1} = \frac{H^{3/2}}{H_1^{3/2}}$$

$$\therefore \quad P_1 = P \times \left(\frac{H_1}{H}\right)^{3/2} \qquad \text{...}(i)$$

2. *Significance of unit speed*

Let H = Head of water, under which the turbine is working,

Q = Discharge of the turbine under a head of water H, and

Q_1 = Discharge of the turbine under head of water H_1.

We have seen in Art. 34·4 that,

$$N \propto \sqrt{H}$$

$$\therefore \quad N_1 \propto \sqrt{H_1}$$

$$N \quad \sqrt{H} \quad \left(H\right)^{1/2}$$

$$\therefore \qquad N_1 = N \times \left(\frac{H_1}{H}\right)^{1/2} \qquad \ldots(ii)$$

3. *Significance of unit discharge*

Let H = Head of water under which the turbine is working,

Q = Discharge of the turbine under a head of water H, and

Q_1 = Discharge of the turbine under head of water H_1.

We have seen in Art. 34·5 that

$$Q \propto \sqrt{H}$$

$$\therefore \qquad Q_1 \propto \sqrt{H_1}$$

or
$$\frac{Q}{Q_1} = \frac{\sqrt{H}}{\sqrt{H_1}} = \left(\frac{H}{H_1}\right)^{1/2}$$

$$\therefore \qquad Q_1 = Q \times \left(\frac{H_1}{H}\right)^{1/2} \qquad \ldots(iii)$$

Example 34·2 *An impulse turbine develops 4500 kW under a head of 200 metres. The turbine runner has a speed of 200 r.p.m. and discharges 0·8 cubic metre of water per second. If the head on the same turbine falls during summer season to 184·3 metres, find the new discharge, power and speed of the turbine.*

Solution. Given : P = 4500 kW; H = 200 m; N = 200 r.p.m.; Q = 0·8 m^3/s; and H_1 = 184·3 m

New discharge of the turbine

We know that new discharge of the turbine,

$$Q_1 = Q \times \left(\frac{H_1}{H}\right)^{1/2} = 0{\cdot}8 \times \left(\frac{184{\cdot}3}{200}\right)^{1/2} \text{ m}^3\text{/s}$$

$$= 0{\cdot}8 \times 0{\cdot}96 = 0{\cdot}768 \text{ m}^3\text{/s} \quad \textbf{Ans.}$$

New power of the turbine

We know that new power of the turbine,

$$P_1 = P \times \left(\frac{H_1}{H_2}\right)^{3/2} = 4500 \times \left(\frac{184{\cdot}3}{200}\right)^{3/2} \text{ kW}$$

$$= 4500 \times 0{\cdot}88 = 3960 \text{ kW} \quad \textbf{Ans.}$$

New speed of the turbine

We also know that new speed of the turbine,

$$N_1 = N \times \left(\frac{H_1}{H_2}\right)^{1/2} = 200 \times \left(\frac{184{\cdot}3}{200}\right)^{1/2} \text{ r.p.m.}$$

$$= 200 \times 0{\cdot}96 = 192 \text{ r.p.m.} \quad \textbf{Ans.}$$

Example 34·3 *A reaction turbine, at best speed, produces 125 kW under a head of 64 metres. By what percent should the speed be increased for a head of 81 metres.*

Solution. Given : P = 125 kW; H = 64 m and H_1 = 81 m.

Let N = Speed of the turbine under a head of 64 metres.

We know that speed of the turbine under a head of 81 metres.

$$N_1 = N \times \left(\frac{H_1}{\ldots}\right)^{1/2} = N \times \left(\frac{81}{\ldots}\right)^{1/2} = \frac{9N}{\ldots} \text{ r.p.m.}$$

$\therefore$ Increase in speed $$= \frac{N_1 - N}{N} = \frac{\frac{9N}{8} - N}{N} = 0{\cdot}125 = 12.5\% \textbf{ Ans.}$$

EXERCISE 34·1

1. A turbine working under a head of 25 metres develops 2000 kW at 250 r.p.m. Find the unit power and unit speed of the turbine. **(Ans. 16 kW; 50 r.p.m.)**
2. A turbine develops 1000 kW under a head of 16 m at 200 r.p.m. while discharging 9 cubic metres of water per second. Find the unit power and unit discharge of the wheel. **(Ans. 15·6 kW; 2·25 m^3/s)**
3. A turbine running at 150 r.p.m. discharges 3·5 cubic metres of water per second under a head of 40 metres and produces 1000 kW. Find the normal speed and power developed by the turbine under a head of 62·5 metres. **(Ans. 187·5 r.p.m.; 1953 kW)**

34·7 Specific Speed of a Turbine

After studying the behaviour of a turbine, under unit conditions, the next step is to know the characteristics of an imaginary turbine identical* with the actual turbine, but reduced to such a size so as to develop a unit power under a unit head (*i.e.*, 1 kW under a head of 1 metre). This imaginary turbine is called the specific turbine and its speed is known as specific speed. Thus the specific speed of a turbine may be defined as the speed of an imaginary turbine, identical with the given turbine, which will develop a unit power under a unit head.

Let N_s = Specific speed of turbine,

D = Diameter of the turbine runner,

N = Speed of the runner in r.p.m.

v = Tangential velocity of the runner,

V = Absolute velocity of the water.

We know that the tangential velocity of the runner,

$$v \propto V$$

$$\therefore \quad \propto \sqrt{2gH} \qquad ...(\because V = \sqrt{2gH})$$

$$\propto \sqrt{H}$$

We also know that the tangential velocity of the runner,

$$v = \frac{\pi DN}{60}$$

$$\therefore \quad D \propto \frac{v}{N}$$

$$\propto \sqrt{H} \qquad ...(\because v \propto \sqrt{H})$$

and $$D \propto \frac{\sqrt{H}}{N} \qquad ...(i)$$

Now let Q = Discharge through the turbine,

b = Width of the turbine runner,

V_f = Velocity of flow, and

D = Diameter of the turbine runner

We know that the discharge of a turbine,

$$Q = \pi D b V_f$$

But $$b \propto D$$

and $$V_f \propto \sqrt{2gH}$$

$$\propto \sqrt{H}$$

$\therefore$ $$Q \propto \pi D.D.\sqrt{2gH}$$

$$\propto D^2\sqrt{H}$$

Substituting the value of D^2 from equation (*i*), in the above equation,

$$Q \propto \left(\frac{\sqrt{H}}{N}\right)^2 \times \sqrt{H}$$

$$\propto \frac{H^{3/2}}{N^2} \quad \ldots(ii)$$

Now let P = Power produced by the turbine,

We know that the power,

$$P = wQH$$

$$\propto QH$$

Substituting the value of Q from equation (*ii*)

$$P \propto \frac{H^{3/2}}{N^2} \times H$$

$$\propto \frac{H^{5/2}}{N^2}$$

or $$N^2 \propto \frac{H^{5/2}}{P}$$

$\therefore$ $$N \propto \frac{H^{5/4}}{\sqrt{P}}$$

$$= N_s \times \frac{H^{5/4}}{\sqrt{P}}$$

or $$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

In the above relation for specific speed, it is useful to express P in kW, H in metres and N in r.p.m.

Example 34·3 *A reaction turbine is working under a head of 9 metres and average discharge of 11 200 litre/s for a generator speed of 200 r.p.m. What is its specific speed? Assume overall efficiency of the turbine = 92%.*

Solution. Given: H = 9 m; Q = 11 200 litre/s = 11·2 m³/s; N = 200 r.p.m. and $\eta_0 = 92\%$ = 0·92

We know that overall efficiency of the turbine (η_0),

$$0.92 = \frac{P}{wQH} = \frac{P}{9.81 \times 11.2 \times 9} = \frac{P}{988.8}$$

$$\therefore \quad P = 0.92 \times 988.8 = 909.7 \text{ kW}$$

and specific speed of the turbine,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{200 \times \sqrt{909.7}}{(9)^{5/4}} = \frac{6032}{15.6} = 386.7 \text{ r.p.m. } \textbf{Ans.}$$

Example 34·4 *A turbine develops 10 000 kW under a head of 25 metres at 135 r.p.m. What is its specific speed? What would be its normal speed and output under a head of 20 metres?*

Solution. Given : $P = 10\,000$ kW; $H = 25$ m; $N = 135$ r.p.m.; and $H_1 = 20$ m.

Specific speed of the turbine

We know that specific speed of the turbine,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{135 \times \sqrt{10\,000}}{(25)^{5/4}} = \frac{13\,500}{55.9} = 241.5 \text{ r.p.m. } \textbf{Ans.}$$

Normal speed and output of the turbine.

We also know that normal speed of the turbine,

$$N_1 = N \times \left(\frac{H_1}{H_2}\right)^{1/2} = 135 \times \left(\frac{20}{25}\right)^{1/2} \text{ r.p.m.}$$

$$= 135 \times 0.894 = 120.7 \text{ r.p.m. } \textbf{Ans.}$$

and normal output of the turbine,

$$P_1 = P \times \left(\frac{H_1}{H_2}\right)^{3/2} = 10\,000 \times \left(\frac{20}{25}\right)^{3/2} \text{ kW}$$

$$= 10\,000 \times 0.716 = 7160 \text{ kW } \textbf{Ans.}$$

Example 34·5 *One of the Kaplan turbines, installed at Ganguwal Power House (Bhakra Dam Project) is rated at 25 000 kW when working under 30 m of head at 180 r.p.m. Find the diameter of the runner, if overall efficiency of the turbine is 0·91. Assume flow ratio of 0·65 and diameter of runner hub equal to 0·3 times the external diameter of runner. Also find specific speed of the turbine.*

Solution. Given : $P = 25\,000$ kW; $H = 30$ m; $N = 180$ r.p.m.; $\eta_0 = 0.91$; $\dfrac{V_f}{\sqrt{2gH}} = 0.65$

or $V_f = 0.65 \times \sqrt{2gH} = 0.65 \times \sqrt{2 \times 9.81 \times 30} = 15.8$ m/s and $D_b = 0.3D$

Diameter of the runner.

Let D = Diamter of the runner, and

Q = Discharge of the turbine.

We know that overall efficiency of the turbine (η_0),

$$0.91 = \frac{P}{wQH} = \frac{25\,000}{9.81 \times Q \times 30} = \frac{84.9}{Q}$$

$$\therefore \quad Q = 84.9/0.91 = 93.3 \text{ m}^3/\text{s}$$

We also know that discharge of the turbine (Q),

$$93.3 = V_f \times \frac{\pi}{4}(D^2 - D_b^2) = 15.8 \times \frac{\pi}{4}\left[D^2 - (0.3D)^2\right] = 11.3D^2$$

Specific speed of the turbine

We also know that specific speed of the turbine,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{180 \times \sqrt{25\,000}}{(30)^{5/4}} = \frac{28\,460}{70 \cdot 2} = 405 \text{ r.p.m. } \textbf{Ans.}$$

Example 34·6 *In a hydroelectric station, the water is available under a head of 15 m at the rate of 100 m³/s. Calculate the number of turbines with a speed of 65 r.p.m. and 82% efficiency. The specific speed of the turbines is not to exceed 125 r.p.m. Also calculate the power produced by each turbine.*

Solution. Given : H = 15 m; Water available = 100 m³/s; N = 65 r.p.m.; η_0 = 82% = 0·82 and N_s = 125 r.p.m.

Power produced by each turbine

Let P = Power produced by each turbine.

We know that specific speed of the turbine (N_s),

$$125 = \frac{N\sqrt{P}}{H^{5/4}} = \frac{65\sqrt{P}}{(15)^{5/4}} = \frac{65\sqrt{P}}{29 \cdot 5} = 2 \cdot 2\sqrt{P}$$

$$\therefore \quad \sqrt{P} = 125/2 \cdot 2 = 56 \cdot 8 \text{ or } P = 3226 \text{ kW } \textbf{Ans.}$$

Number of turbines

Let Q = Discharge of each turbine.

We know that efficiency of the turbine (η_0),

$$0 \cdot 82 = \frac{P}{wQH} = \frac{3226}{9 \cdot 81 \times Q \times 15} = \frac{21 \cdot 9}{Q}$$

or $$Q = 21 \cdot 9/0 \cdot 82 = 26 \cdot 7 \text{ m}^3/\text{s}$$

∴ Number of turbines

$$= \frac{\text{Water available}}{\text{Discharge of each turbine}} = \frac{100}{26 \cdot 7} = 3 \cdot 7 \text{ say 4 } \textbf{Ans.}$$

34·8 Significance of Specific Speed

The significant feature of the specific speed, of a turbine, is that it is independent of the dimensions or size of the both actual and specific turbines. It is thus obvious, that all the turbines, geometrically similar, working under the same head and having the same values of speed ratio and flow ratio will have the same specific speed.

In actual practice, the conception of specific speed is of the utmost utility. The mere value of specific speed helps us in predicting the performance of a turbine, which will be discussed in the following pages.

34·9 Selection of Turbines

An engineer is often required to select the type of turbine, which he should employ for his project. It is a highly technical job and requires great experience and patience. The selection of turbine is, generally, basd upon the following two factors:

1. Selection based on the specific speed, and
2. Selection based on the head of water.

The former (*i.e.*, selection based on the specific speed) is a scientific method, and gives a precise information, whereas the later (*i.e.*, selection based on the head of water) is based on experience and observational factors only.

34·10 Selection Based on Specific Speed

First of all, specific speed of a turbine is found out, as usual. Then the type of turbine is selected. Following table shows the type of turbine to be selected, for the corresponding specific speeds.

TABLE 34·1

S. No.	*Specific speed*	*Type of turbine*
1.	8 to 30	Pelton wheel with one nozzle.
2.	30 to 50	Pelton wheel with 2 or more nozzles.
3.	50 to 250	Francis' turbine.
4.	250 to 1000	Kaplan turbine.

34·11 Selection Based on Head of Water

Following table shows the type of turbine, to be used, for the corresponding head of water.

TABLE 34·2

S. No.	Head of water in metres	*Type of turbine*
1.	0 to 25	Kaplan or Francis' (preferably Kaplan)
2.	25 to 50	Kaplan or Francis (preferably Francis)
3.	50 to 150	Francis
4.	150 to 250	Francis or Pelton (preferably Francis)
5.	250 to 300	Francis or Pelton (preferably Pelton)
6.	above 300	Pelton

Example 34·7 *Find the type of turbine, which should be used under a head of 150 metres to develop 1500 kW, while running at 300 r.p.m.*

Solution. Given : $H = 150$ m; $P = 1500$ kW and $N = 300$ r.p.m.

We know that specific speed of the turbine,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300 \times \sqrt{1500}}{(150)^{5/4}} = \frac{11\,610}{525} = 22{\cdot}1 \text{ r.p.m.}$$

Since the specific speed of the turbine lies between 8 and 30, therefore Pelton wheel with one nozzle should be used. **Ans.**

Example 34·8 *Fnd the specific speed and the type of a water turbine developing 7 000 kilowatt under a head of 20 metres when running at 100 r.p.m. Calculate its normal speed and output under 25 metres head.*

Solution. Given : $P = 7000$ kW; $H = 20$ m; $N = 100$ r.p.m. and $H_1 = 25$ m

Specific speed of the turbine and type of the turbine

We know that specific speed of the turbine,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{100 \times \sqrt{7000}}{(20)^{5/4}} = \frac{8370}{42{\cdot}3} = 197{\cdot}8 \text{ r.p.m. Ans.}$$

Since the specific speed of the turbine lies between 50 and 250, therefore Francis turbine should

Normal speed and output of the turbine

We also know that normal speed of the turbine

$$N_1 = N\left(\frac{H_1}{H}\right)^{1/2} = 100 \times \left(\frac{25}{20}\right)^{1/2} = 100 \times 1{\cdot}12 = 120 \text{ r.p.m. } \textbf{Ans.}$$

and normal output of the turbine,

$$P = P\left(\frac{H_1}{H_2}\right)^{1/2} = 7000 \times \left(\frac{25}{20}\right)^{1/2} = 7000 \times 1{\cdot}4 = 9800 \text{ kW } \textbf{Ans.}$$

Example 34·9 *A single jet Pelton wheel develops 2500 kW under a head of 70 metres. Find the maximum and minimum speeds of the turbine.*

Solution. Given : $P = 2500$ kW and $H = 70$ m

Maximum speed of turbine

Let N_1 = Maximum Speed of the wheel.

From table 34 1, we find that the maximum specific speed of a Pelton wheel with single nozzle, (N_s) is equal to 30.

We know that specific speed of the turbine (N_s),

$$30 = \frac{N_1\sqrt{P}}{H^{5/4}} = \frac{N_1\sqrt{2500}}{(70)^{5/4}} = \frac{50\,N_1}{202{\cdot}5} = 0{\cdot}247\,N_1$$

$\therefore$ $N_1 = 30/0{\cdot}247 = 121{\cdot}5$ r.p.m. **Ans.**

Minimum speed of the turbine

Let N_2 = Minimum speed of the turbine

From the same table, we also find that minimum specific speed of a Pelton wheel with single nozzle (N_s) in equal to 8.

We know that specific speed of the turbine (N_s),

$$8 = \frac{N_2\sqrt{P}}{H^{5/4}} = \frac{N_2\sqrt{2500}}{(70)^{5/4}} = \frac{50\,N_2}{202{\cdot}5} = 0{\cdot}247\,N_2$$

$\therefore$ $N_2 = 8/0{\cdot}247 = 32{\cdot}4$ r.p.m. **Ans.**

Example 34 10 *The total power generated in a hydroelectric station is 18 000 kW under a head of 16 m, while the turbines run with a speed of 192 r.p.m. Find the minimum number of turbines of the same size required in case of*

(i) Francis' turbines with maximum specific speed of 210 r.p.m. and

(ii) Kaplan turbines with maximum specific speed of 300 r.p.m.

Solution. Given : $P = 18\,000$ kW; $H = 16$ m and $N = 192$ r.p.m.

(i) Number of Francis turbines with maximum specific speed of 210 r.p.m.

Let P_1 = Power generated by each Francis' turbine

We know that maximum specific speed of the Francis' turbine (N_s).

$$210 = \frac{N\sqrt{P_1}}{H^{5/4}} = \frac{192\sqrt{P_1}}{(16)^{5/4}} = \frac{192\sqrt{P_1}}{32} = 6\sqrt{P_1}$$

or $\sqrt{P_1} = 210/6 = 35$ or $P_1 = (35)^2 = 1225$ kW

$\therefore$ Number of Francis' turbines

$$= \frac{\text{Total power generated}}{\text{Power generated by each turbine}}$$

$$= \frac{18\,000}{1225} = 14{\cdot}7 \text{ say } 15 \textbf{ Ans.}$$

(ii) Number of Kaplan turbines with maximum specific speed of 300 r.p.m.

Let P_2 = Power generated by each Kaplan turbine.

We also know that maximum specific speed of the Kaplan turbine (N_s),

$$300 = \frac{N\sqrt{P_2}}{H^{5/4}} = \frac{192\sqrt{P_2}}{(16)^{5/4}} = \frac{192\sqrt{P_2}}{32} = 6\sqrt{P_2}$$

or $\sqrt{P_2} = 300/6 = 50$ or $P_2 = (50)^2 = 2500$

$\therefore$ Number of Kaplan turbines

$$= \frac{\text{Total power generated}}{\text{Power generated by each turbine}}$$

$$= \frac{18\,000}{2500} = 7{\cdot}2 \text{ say } 8 \textbf{ Ans.}$$

34·12 Relation between Specific Speed and Shape of Reaction Turbine Runner

We have already discussed in Art. 34·7 that the specific speed of a turbine,

$$N_s = \frac{N\sqrt{P}}{(H)^{5/4}}$$

Since the power produced by a turbine and the available head of water is more or less constant. for a power house, therefore specific speed is directly proportional to the speed of the turbine runner.

We have also discussed that power produced by a turbine,

$$P = wQH$$

Since the value of w is constant, therefore power produced is directly proportional to Q (*i.e* discharge) and H (head of water). Moreover, the head of water is also fixed for every power house. It is thus obvious, that the power produced by a turbine, in a power house, depends upon the discharge A little consideration will show that under a low head, if the same power is to be produced, then more flow is required. This can be achieved either by increasing the area of flow or velocity of water. In a reaction turbine, this is achieved by increasing the height of gates and velocity of flow.

Fig. 34·1 (*a*) to (*d*) shows the changes in the shape of blades of the ...

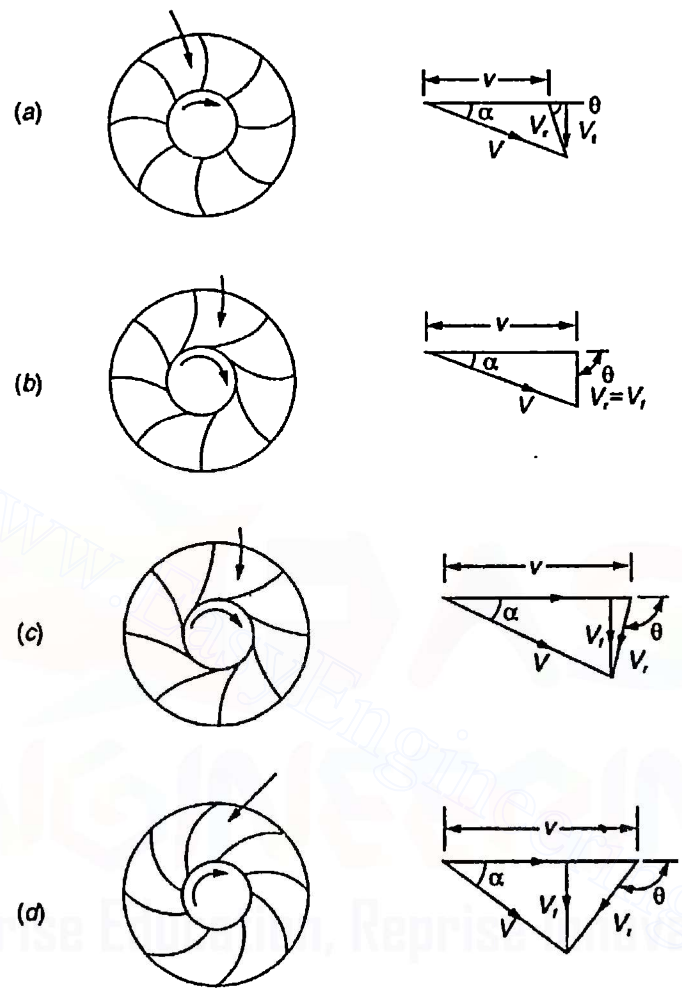

Fig. 34·1 Effect of specific speed on the shape of reaction turbine runner

1. *Fig. 34·1* (*a*)

It is a general case, in which the inlet triangle of velocities is shown for a slow speed reaction turbine. In this case, the common features are:

$$N_s = 60 \text{ to } 120 \text{ r.p.m.}$$

$$\alpha = 10° \text{ to } 20°$$

$$\theta = 60° \text{ to } 90°$$

2. *Fig. 34·2* (*b*)

In this case, the power developed (*P*) and diameter of the turbine runner (*D*) are the same as in the first case. Due to reduction of the available head of water, specific speed of the turbine and its discharge will increase. A little consideration will show that:

---

*The specific speed of a turbine and its discharge are inversely proportional to the available head of water, because specific speed of a turbine,

$N\sqrt{P}$ ... P

(*i*) The reduction in the available head of water will reduce the velocity of water ($\because V = \sqrt{2gH}$).

(*ii*) The increase in specific speed will increase the speed of the turbine runner, and ultimately the tangential velocity of the wheel at inlet $\left(\because v = \dfrac{\pi DN}{60}\right)$

(*iii*) The increase in discharge of water will increase its velocities. In this case, the common features are :

$$N_s = 120 \text{ to } 180 \text{ r.p.m.}$$
$$\alpha = 20° \text{ to } 30°$$
$$\theta = 90°$$

3. *Fig. 34·1* (*c*)

In this case, the power developed (*P*) and diameter of turbine runner (*D*) are the same as in the previous cases. Due to further reduction in the available head of water, specific speed of the turbine and its discharge will further increase. A little consideration will show, that this phenomenon will (*i*) reduce the velocity of water, (*ii*) increase the tangential velocity of wheel at inlet, and (*iii*) increase the velocity of flow.

The above mentioned changes will further change the shape of inlet triangle of velocities. In this case, the common features are :

$$N_s = 180 \text{ to } 240 \text{ r.p.m.}$$
$$\alpha = 30° \text{ to } 45°$$
$$\theta = 90° \text{ to } 120°$$

4. *Fig. 34·1* (*d*)

In this case, the power developed (*P*) and diameter of the turbine runner (*D*) are the same as in previous cases. Due to further reduction in the available head of water, specific speed of the turbine and its discharge will further increase. This phenomenon will further reduce the velocity of water, increase the tangential velocity of wheel at inlet and increase the velocity of flow. These changes will further change the shape of inlet triangle of velocities. In this case, the common features are :

$$N_s = 240 \text{ to } 300 \text{ r.p.m.}$$
$$\alpha = 45° \text{ to } 60°$$
$$\theta = 120° \text{ to } 135°$$

It will be interesting to know, that the above mentioned theory led to the development of Kaplan turbine.

34·13 Characteristics Curves of Turbines

We have discussed in the chapters 32 and 33 the various types of impulse turbines and reaction turbines. As a matter of fact, a turbine is always designed and manufactured to work under a given set of conditions (or a limited range of conditions) such as discharge, head of water, speed, power generated, efficiency etc. (at full speed or unit speed). But a turbine may have to be used under conditions, other than those for which it has been designed. It is therefore essential, that the exact behaviour of the turbine under varied conditions should be predetermined. This is represented graphically by means of curves, known as characteristic curves.

The characteristic curves are generally drawn for constant head or constant speed of the turbine runner. Sometimes, these curves are also drawn for various gate openings (briefly written as G.O.) *i.e.* when the gate is fully open, 0·75 open, 0·5 open etc. Though

34·14 Characteristic Curves for Pelton Wheels

The following curves have been drawn by the various engineers, working in hydraulic research laboratories all over the globe. Though there is a little variation in the characteristic curves drawn by them, yet the most accepted and important curves are given below:

1. Characteristic curves for constant head

(*a*) Speed ratio versus percentage of maximum efficiency

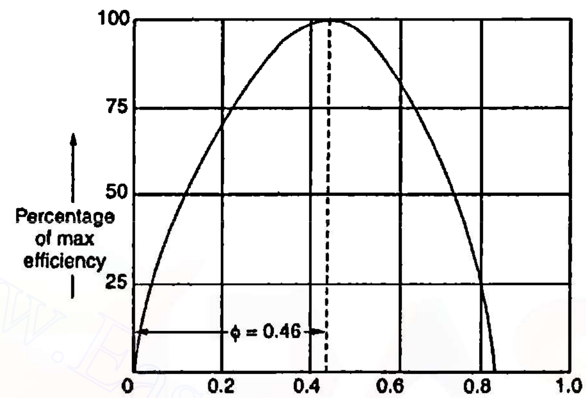

Fig. 34·2 Characteristic curve for speed ratio *vs* percentage of maximum efficiency

Fig. 34·2 shows the performance of a Pelton wheel under a constant head and discharge. It is a parabolic curve, which shows that the efficiency increases from zero, and beyond the value of ϕ = 0·46, the efficiency decreases.

(*b*) Power versus efficiency

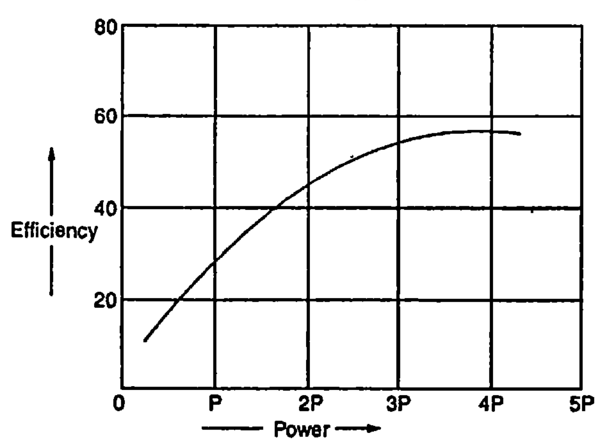

Fig. 34·3. Characteristic curve for power *vs* efficiency

Fig. 34·3 shows the performance of a Pelton wheel under a constant head and speed. It is a parabolic curve, which shows that the efficiency increases with the increase in power.

2. Characteristic curves for varying gate opening

(*a*) Speed versus power

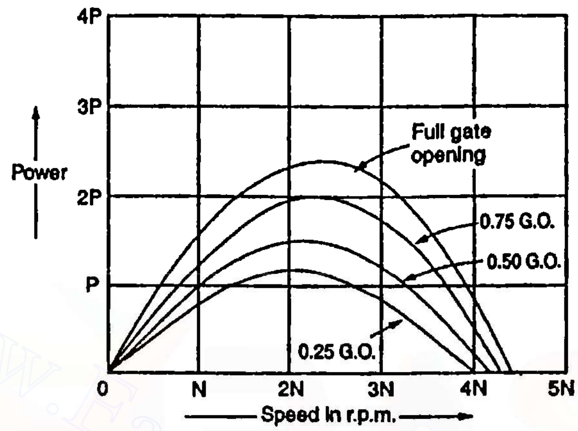

Fig. 34·4. Characteristic curves for speed *vs* power

Fig. 34·4 shows the performance of a Pelton wheel under a constant head. These are parabolic curves, which show that the power developed increases with the gate opening.

(*b*) Speed versus efficiency

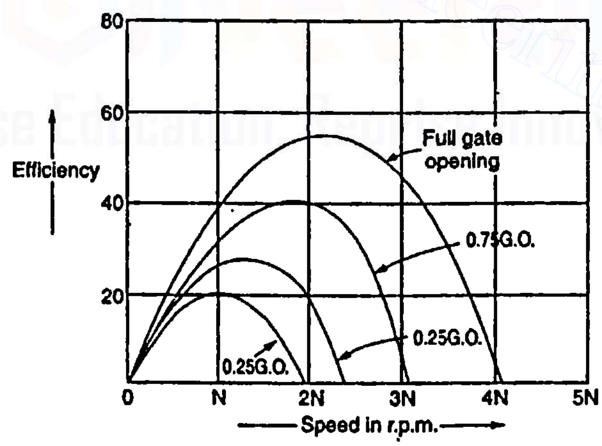

Fig. 34·5. Characteristic curves for speed *vs* efficiency·

Fig. 32·5 shows the performance of a Pelton wheel under a constant head. These are parabolic curves, which show that the efficiency increases with the gate opening.

34·15 Characteristic Curves of Francis' Turbines

Like characteristic curves of Pelton wheel, the following curves have also been drawn by the various engineers. In general, the characteristic ...

1. Characteristic curves for unit speed,
2. Characteristic curves for speed, and
3. Characteristic curves for varying gate opening.

1. Characteristic curves for unit speed

(*a*) Unit speed versus discharge

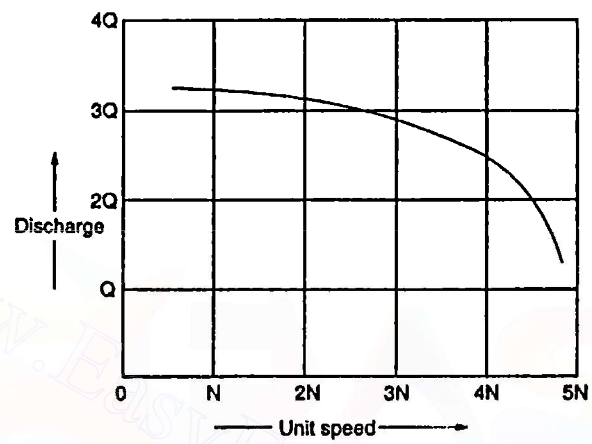

Fig. 34·6. Characteristic curves for unit speed *vs* discharge

Fig. 34·6 shows the performance of a reaction turbine. It is a parabolic curve which shows that the discharge decreases with the unit speed.

(*b*) Unit speed versus power

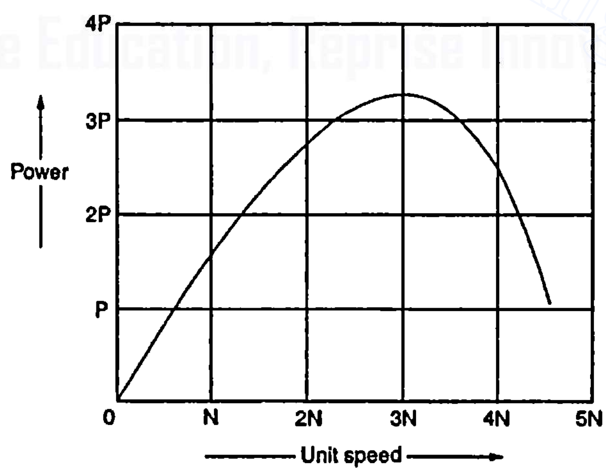

Fig. 34·7. Characteristic curves for unit speed *vs* power

Fig. 34·7 shows the performance of a reaction turbine. It is a parabolic curve, which shows that the power increases with the unit speed, and beyond a certain speed the power decreases.

(*c*) Unit speed versus efficiency

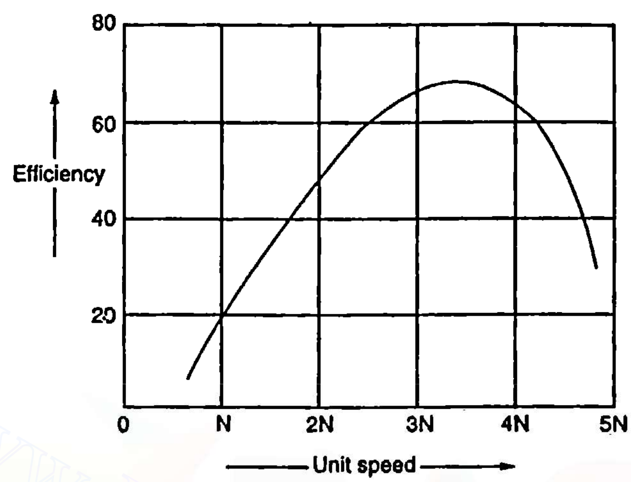

Fig. 34·8. Characteristic curves for unit speed *vs* efficiency

Fig. 34·8 shows the performance of a reaction turbine. It is a parabolic curve, which shows that the efficiency increases with the unit speed, and beyond a certain speed the efficiency decreases.

2. *Characteristic curves for speed with varying heads*

(*a*) Speed versus discharge

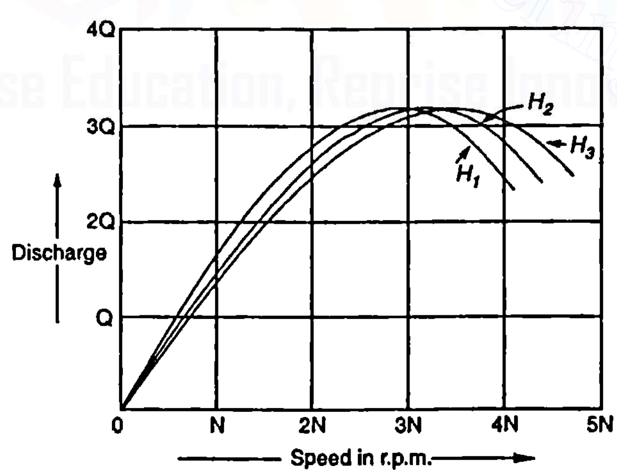

Fig. 34·9. Characteristic curves for speed *vs* discharge

Fig. 34·9 shows the performance of a Francis' turbine (or any other reaction turbine) under variable heads but constant discharge. It is a parabolic curve, which shows th . . .

(*b*) Speed versus power

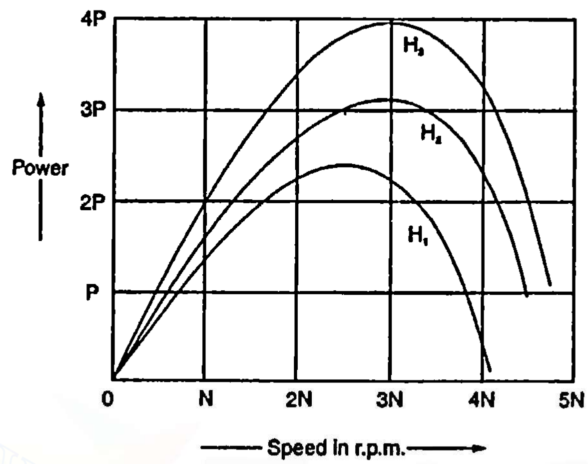

Fig. 34·10. Characteristic curves for speed *vs* power

Fig. 34·10 shows the performance of a Francis' turbine (or any other reaction turbine) under variable heads but constant discharge. It is a parabolic curve, which shows that for a given head the power increases with the speed from zero, and beyond a certain speed the power decreases.

(*c*) Speed versus efficiency

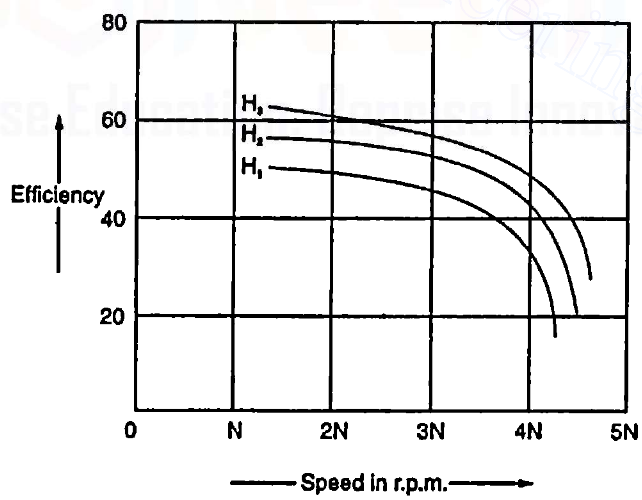

Fig. 34·11. Characteristic curves for speed *vs* efficiency

Fig. 34·11 shows the performance of a Francis' turbine (or any reaction turbine) under variable heads but constant discharge. It is a parabolic curve, which shows that for a given head the efficiency

3. *Characteristic curves for varying gate opening*

(*a*) Speed versus power

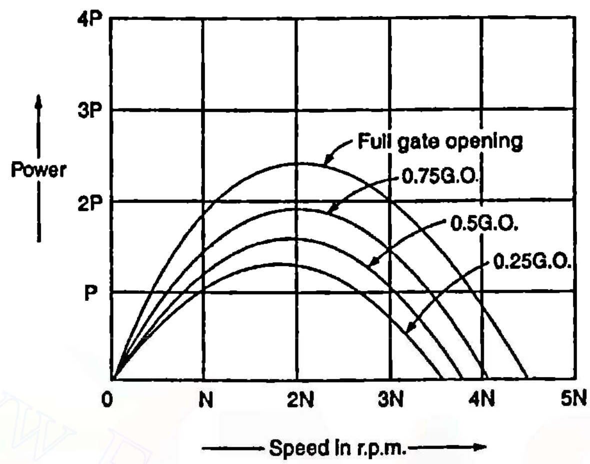

Fig. 34·12. Characteristic curves for speed *vs* power

Fig. 34·12 shows the performance of a Francis' turbine (or any other reaction turbine) under a constant head. These are parabolic curves, which show that the power developed increases with the gate opening.

(*b*) Speed versus efficiency

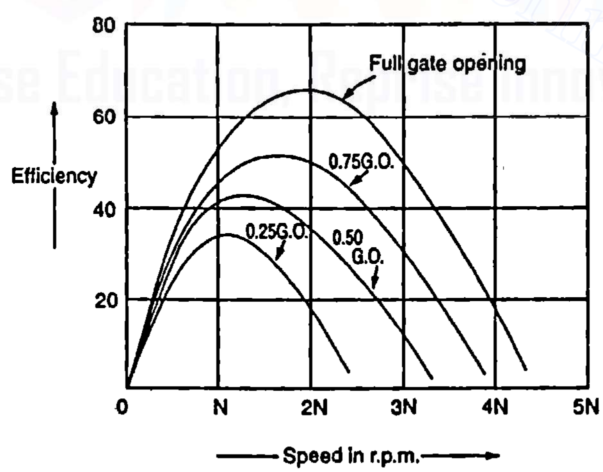

Fig. 34·13. Characteristic curves for speed *vs* efficiency

Fig. 34·13 shows the performance of a Francis' turbine (or any other reaction turbine) under a constant head. These are parabolic curves, which show that the efficiency increases with the

34·16 Cavitation

The cavitation may be broadly defined as the formation of bubbles, filled with vapours, within the body of a moving liquid. It has been observed, that the vapour cavities begin to appear, whenever the pressure at any point in a flow falls to the vapour pressure of the liquid at that temperature. These bubbles, which are formed on account of low pressure, are carried by the stream to the zones of high pressure. In these zones, the vapours condense and the bubbles collapse. The space, previously occupied by the bubbles, is filled up by the surrounding liquid. As a result of this, some noise occurs and vibrations are produced. The pressure, which makes the bubbles to collapse, is generally of the order of 100 times the atmospheric pressure.

A little consideration will show, that when the cavities collapse on the surface of a body, due to repeated hammering action of surrounding liquid, the metal particles give way, which ultimately cause a great deal of erosion of the metal. This erosion of material is called pitting. The cavitation effects a hydraulic machine in the following three ways:

1. Irregular collapse of cavities cause noise and vibration of various parts.
2. As a result of pitting, there is a loss of material, which makes the surfaces rough.
3. As a result of cavities, there is always a reduction in the discharge of a turbine. The reduction in discharge causes sudden drop in the power output and efficiency.

Prof. D. Thoma of Germany, after carrying out a series of experiments, suggested a cavitation factor σ (sigma) to find out the zone, where a reaction turbine can work, without the effect of cavitation. The critical value of this factor is given by:

$$\sigma_{crit} = \frac{H_b - H_s}{H} = \frac{(H_a - H_v - H_s)}{H}$$

where
H_b = Barometric pressure in metres of water,
H_s = Suction pressure head in metres of water,
H_a = Atmospheric pressrue head in metres of water,
H_v = Vapour pressure head in metres of water, and
H = Working head of a turbine in metres.

But in actual practice, the cavitation, in reaction turbines, is avoided to a great extent by the following methods:

1. By installing the turbine below the tail race level.
2. By providing a cavitation-free runner of the turbine.
3. By using stainless steel runner of the turbine.
4. By providing highly polished blades to the runner.
5. By running the turbine runner at the designed speed.

EXERCISE 34·2

1. Find the specific speed of a turbine developing 625 kW under a head of 20 metres of 150 r.p.m.
   (Ans. 88·7 r.p.m)
2. A turbine develops 1225 kW under a head of 64 metres while running at 120 r.p.m. Find the type of turbine suited for the project. **(Ans. N_s = 23·2 r.p.m.: Peltan wheel with one nozzle)**
3. Find the type of turbine you would employ to develop 1700 kW under a head of 150 metres at 350 r.p.m., while discharging 1·75 cubic metres of water per second.
   (Ans. N_s = 34·5; Peltan wheel with two nozzles)

QUESTIONS

1. Define the terms 'unit power' 'unit speed' and 'unit discharge' with reference to a hydraulic turbines.
2. What do you understand by the terms 'specific turbine' and specific speed?
3. Derive an expression for the specific speed of a turbine in terms of power (P), head (H) and speed in r.p.m. (N).
4. Explain the significance of specific speed of water turbines.
5. Give the range of specific speed values of the Kaplan, Francis' and Pelton wheels.
6. What factors decide whether a Kaplan, Francis' or a Pelton type turbine would be used in a hydro-electric project.
7. Draw the following characteristic curves of a turbine:
   (*i*) Power *versus* efficiency with constant speed.
   (*ii*) Discharge, power and efficiency *versus* speed.
8. Draw the characteristic curves of water turbine showing rate of flow and power *versus* unit speed.

OBJECTIVE TYPE QUESTIONS

1. The value of unit power is equal to
   (*a*) $\frac{P}{H}$ (*b*) $\frac{P}{\sqrt{H}}$ (*c*) $\frac{P}{H^2}$ (*d*) $\frac{P}{H^{3/2}}$
2. The value of unit speed is equal to
   (*a*) $\frac{N}{H}$ (*b*) $\frac{N}{\sqrt{H}}$ (*c*) $\frac{N}{H^2}$ (*d*) none of these
3. The value of unit discharge is equal to
   (*a*) $\frac{Q}{H}$ (*b*) $\frac{Q}{\sqrt{H}}$ (*c*) $\frac{Q}{42}$ (*d*) $\frac{Q}{H^3}$
4. The specific speed of a turbine is given by the relation
   (*a*) $\frac{NP}{H}$ (*b*) $N_s = \frac{N\sqrt{P}}{H}$ (*c*) $\frac{N\sqrt{P}}{H^{5/4}}$ (*d*) $\frac{N\sqrt{P}}{H^{3/2}}$

Answers

1. (*d*) 2. (*b*) 3. (*b*) 4. (*c*)