STRENGTH OF MATERIALS

Material

- Here materials are referred as structural material.

Structural materials are generally selected on the basis of mechanical properties rather than its electronics, chemical, optical, magnetic etc.

In general on the basis of physical properties all the engineering materials are classified as-

1. Elastic material
2. Plastic material
3. Ductile material
4. Brittle material.

Material

Elastic materials- If a material regains its original position, on removal of the external forces acting on it then it is called elastic material.

Plastic materials- If a material does not regains its original position, on removal of the external forces acting on it then it is called plastic material.

Ductile materials- If a material can undergo a considerable deformation without rupture, it is called a ductile material. (e.g If a material can drawn into wires)

Brittle material – If a material cannot undergo any deformation when some external forces act on it and it fails by rupture it is called a brittle material.

IMPORTANT DEFINATIONS

- **Elasticity :-** Elasticity is the property of certain materials of returning their original position after removing the external force .

- **Stress :-** Whenever some external system of force acts on a body , it undergoes some deformation and set up some resistance to deformation . This resistance per unit area to deformation is known as stress .

Mathematically , p or σ or f = P / A

Where , p or σ or f = Intensity of stress .

P = Load or force acting on the body.

A = Cross sectional area .

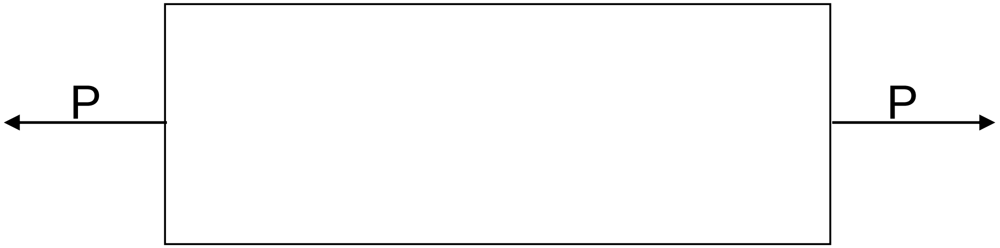
**Unit of stress :- If the load or force is in Newton(N) and the area is square metre(m) then the unit of stress will be N/m^2 or Pascal(P).

Types Of Stress

- **1. Tensile stress :- When a section is subjected to two equal and opposite pulls , as a result the body tends to lengthen , the stress induced is called tensile stress . The stress per cross sectional area of the body is known as intensity of tensile stress .This is denoted by p_t or σ_t or f_t .**

- **Now if the 'P' is the force and the cross sectional area is 'A'**

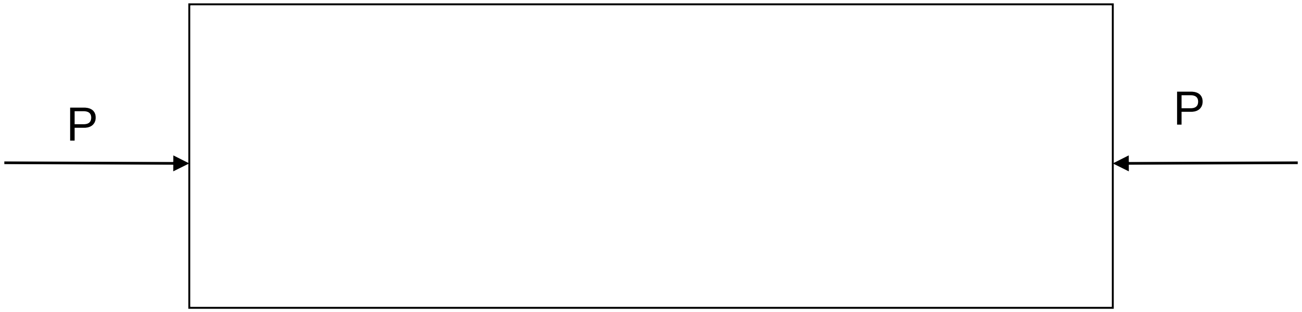
  then f_t . = P / A



2. Compressive Stress

- **When a section is subjected to two equal and opposite pushes , as a result the body tends to shorten and thus the stress induced is called compressive stress .The stress per unit cross sectional area of the body is known as intensity of compressive stress. It is opposite in sign of tensile stress.**

This is denoted by 'σ_c' , now if the compressive load is 'P' kg and the cross sectional area 'A' , then $\sigma_c = P/A$.



3. Shear Stress

- When a body is subjected to a vertical force , the system of internal forces develop within the body is known as shear stress and the stress per unit cross sectional area of the body is known as intensity of shear stress .

This is denoted by σ_S. If the vertical force (push) is 'P' kg and the cross sectional area is 'A' then $\sigma_S = P / A$

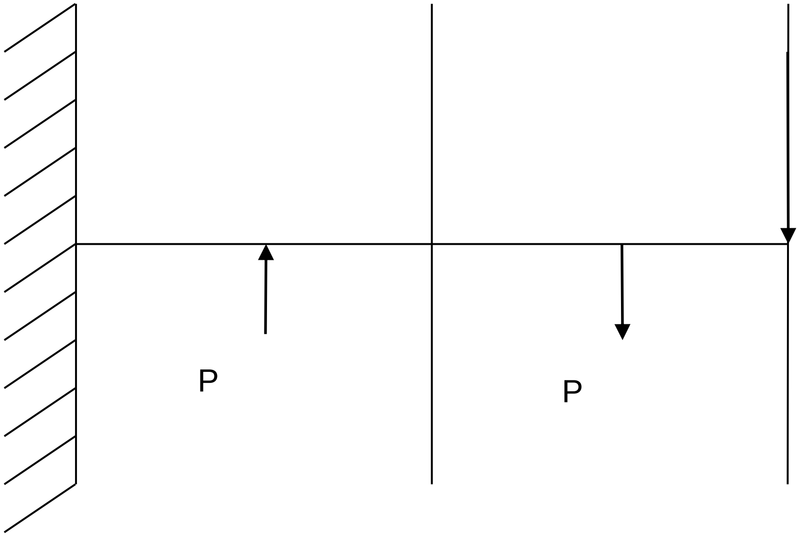
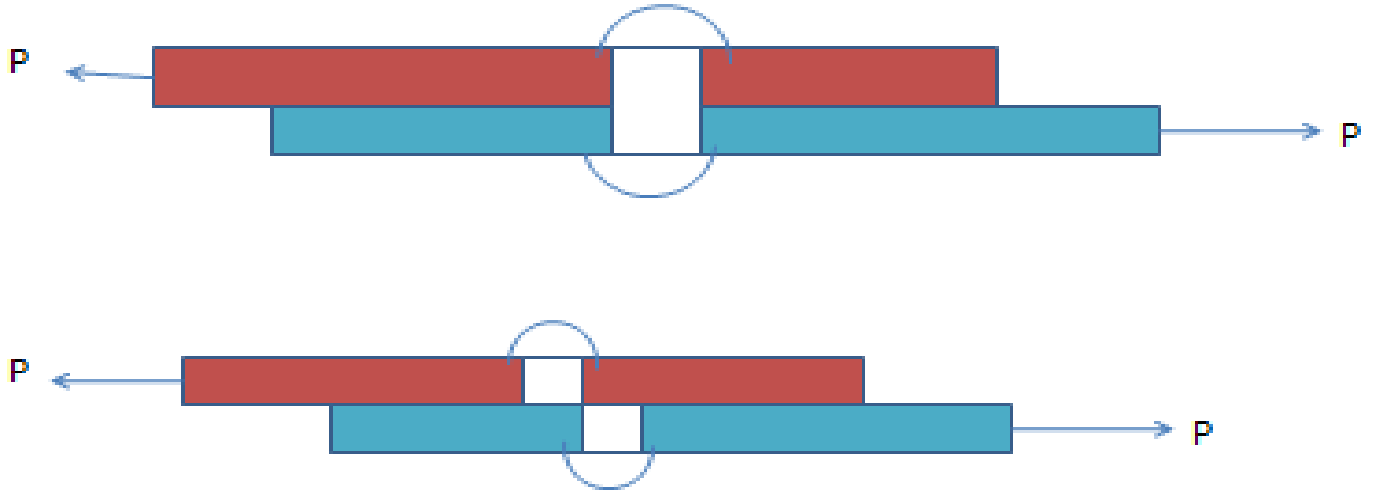


FIGURE SHOWS HOW SHEARING OCCURS IN A RIVETTED JOINT



STRAIN

- Whenever a single force or a system of force acts on a body, it undergoes some deformation. The deformation per unit length is known as strain.

Mathematically , $e = \delta l / l$ (δ = delta)

Where , e = strain

δl = change of length of the body .

l = Original length of the body .

Types of strain

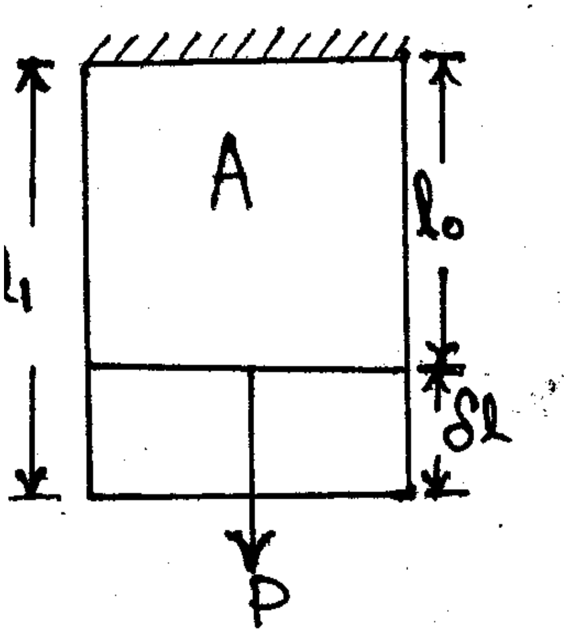
- **1. Tensile Strain :- (e_t) Tensile strain is the deformation produced in a body per unit length due to some tensile force . In the figure let 'A' be a bar of length l_0 . Now an external force 'P' is applied at the bottom end of the bar keeping the top ed fixed . As a result the bar elongated by δl .**

i.e., tensile strain (e_t) =

$$e_t = \frac{\text{Deformation (elongation)}}{\text{Initial length}}$$

$$= \frac{\delta l}{l_0}$$

$$= \frac{l_1 - l_0}{l_0}$$

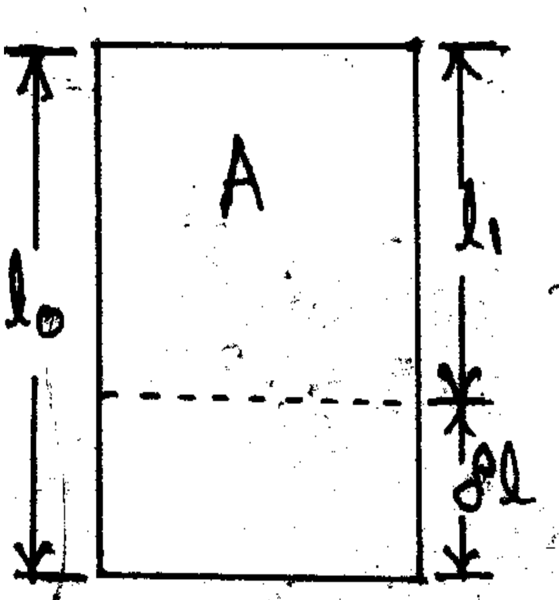


2. Compressive Strain (e_c)

- **Compressive strain is the deformation produced in a body per unit length due to some compressive force.**

  In the figure let ‘A’ be a bar of length l0 . Now an external force ‘P’ is applied at the bottom end of the bar keeping the top end fixed . As a result the bar is compressed by δl .

$$\text{compressive strain ,ec} = \frac{\text{Deformation (compression)}}{\text{Initial length}} = \frac{\delta l}{l0} = \frac{l0 - l1}{l0}$$



3. Shear strain

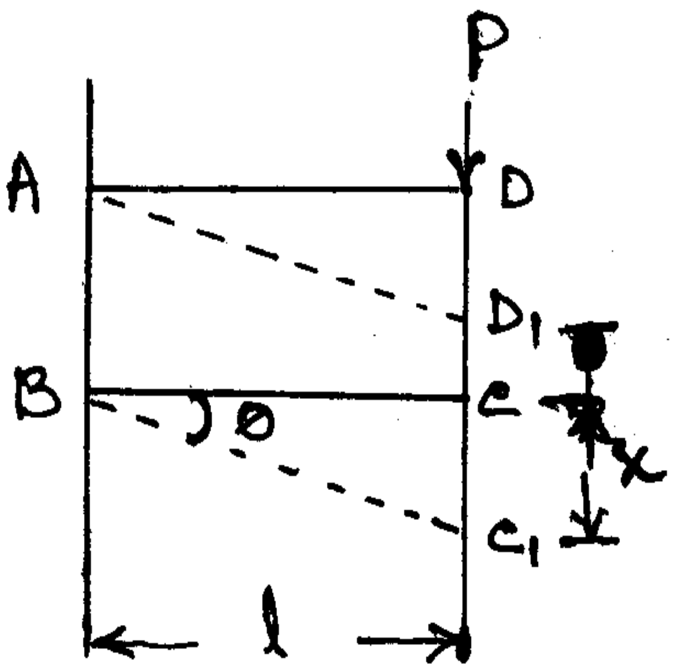
- Shear strain (es) is the angular deformation (distortion) in a body per unit length due to some external vertical load.

  Let ABCD be a rectangular block whose AB side is fixed . Now a vertical force 'P' is applied at the bottom end of the bar keeping the top end fixed . As a result the block ABCD is distorted to ABC 1D 1 through an angle Ø . Let CC 1 be X.

  Then , Tan Ø = CC 1 / BC = x / l

  i.e, Tan Ø = Ø , Since Ø is very small , therefore Tan Ø = Ø in radian ,

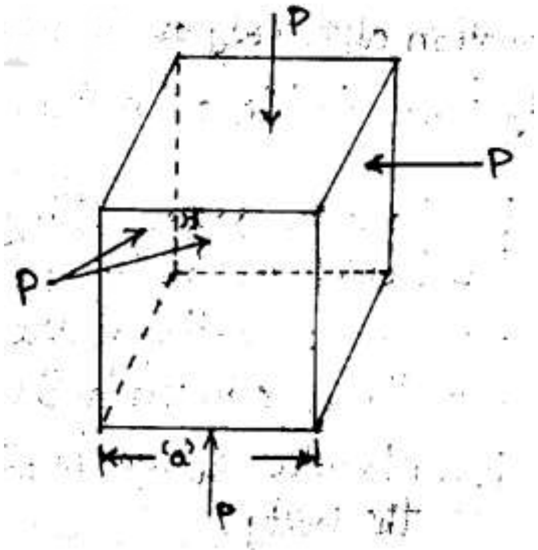
  this angular deformation is called shear strain.



4. Volumetric Strain

Volumetric Strain (ev) is the deformation produced in a body per unit per volume due to some external load (either tensile or compressive) in the figure cube having side ‘a’ is shown so the volume of the cube is a^3.

Now an external force (here compressive) P is applied on all places of the cube. As a result inside of the cube becomes a - δa , where δa is the deformation (compression) in each side.



$$\text{Volumetric strain (ev)} = \frac{\text{Change in volume}}{\text{Initial volume}}$$

Volumetric strain ev = 3 δa / a ,

ev = 3. Linear strain(Tensile or compressive)

- **Hookes Law :-** It states when a material is loaded within its elastic limit the stress is proportional to the strain .

  Mathematically, Stress $\propto$ Strain

  Stress / Strain = E = a constant

  It may be noted that Hookes law equally holds good for tension as well as compression .

- **Modulus Of Elasticity :- (Youngs Modulus)** Whenever a material is loaded within its elastic limit the stress is proportional to the strain .

  $P \propto e$

  therefore $\sigma / e = E$

where , p or f or σ = Stress

e = Strain

E = A constant of proportional known as Modulus of Elasticity or Young Modulus .

Deformation

- Deformation of a body due to force acting on it :- Consider a body subjected to a tensile stress .

Let ,

p = Load or force acting on the body .

l = Length of the body

A = Cross sectional area of the body

f = Stress induced in the body

E = Modulus of elasticity for the material of the body .

e = Strain

δl = Deformation of the body .

We know that the stress , $p = P / A$ And $p / e = E$

Therefore , strain , $e = P / AE$ [since , $f = P / A$]

Or , $\delta l / l = P / AE$ [since $e = \delta l / l$]

Therefore , $\delta l = Pl / AE$

Principle Of Superposition

- Whenever a body is subjected to a number of forces acting on its outer edges as well as some other sections along the length of the body in such the forces are split up and their effects are considered on individual sections . The principle of finding out the resultant deformation is called the principle of super position .

The relation for the resulting deformation is modified as ,

$$\delta l = \frac{Pl}{AE} = \frac{1}{AE} \left[P_1 l_1 + P_2 l_2 + P_3 l_3 + \text{--------} \right)$$

Where : P_1, P_2 , P_3 -- , Forces acting on the section 1 , 2 , 3 --

l_1 , l_2 , l_3 ---, length of the section 1 ,2 ,3 , ----

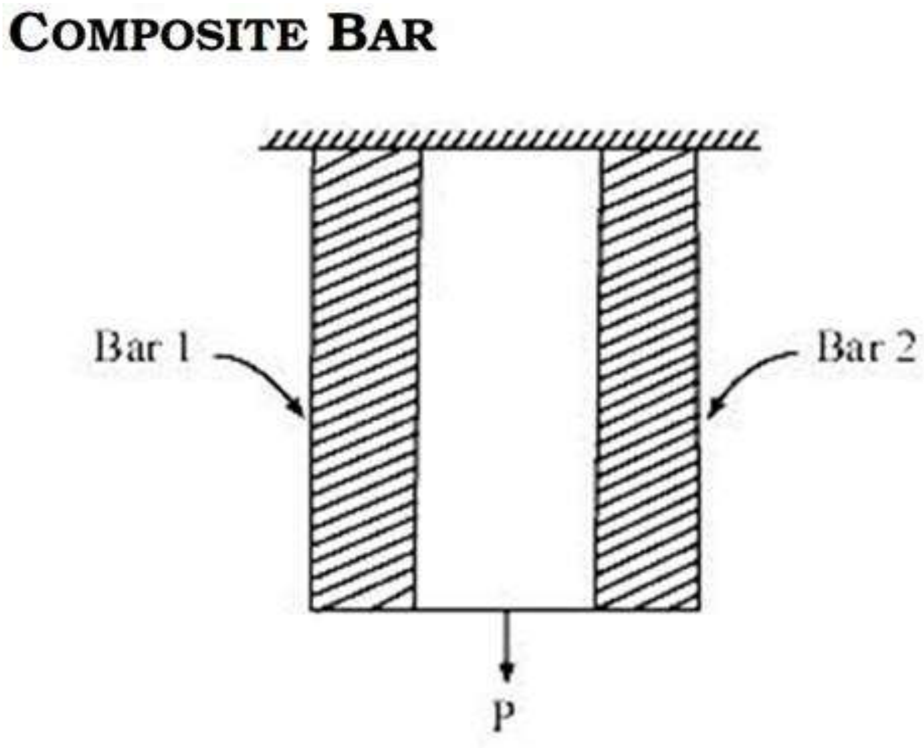
STRESSES IN COMPOSITE BARS

A composition may be defined as a bar made up of two or more different materials joined together in such a manner that the system extends or contracts as one unit , equally ,when subjected to tension or compression . Following two points should be considered while solving numerical problems.

1.Extension or contraction of the bar being equal the strain i.e deformation per unit length is also equal.

2. The total external load on the bar is equal to the sum of the loads carried by the different materials.

Composite bar



Composite bar

Let

P = total load on the bar.

$A1$ = area of the bar 1

$E1$ = modulus of elasticity of bar 1

$P1$ = load shared by bar 1

$A2, E2, P2$ = corresponding values for bar 2

l = length of the composite bar.

δl = elongation of composite bar.

We know that total load on the bar-

$$P= P1 + P2 \text{------------------ (i)}$$

Stress in bar1, $\sigma 1 = P1/A1$

Strain in bar 1 $e1= \sigma 1/E1$

$$\Rightarrow e1= P1/A1E1$$

Elongation in the bar $\delta l = e.l = P1l/A1E1$ ------------ (ii)

Similarly elongation of bar 2 $= P2l/A2E2$ ----------(iii)

Composite bar

Since both the elongations are equal so equqting (ii) and (iii),

$P1l/A1E1 = P2l/A2E2$ or $P1/A1E1 = p2/A2E2$ ----------(iv)

or $P2 = P1 \times A2E2/A1E1$

But $P = P1 + P2 = P1 + P1 \times A2E2/A1E1$

$= P1(1 + A2E2/A1E1) = P1 \dfrac{(A1E1 + A2E2)}{A1E1}$

Or, $P1 = P \times \dfrac{A1E1}{A1E1 + A2E2}$ ------------ (v)

$P1 = P \times \dfrac{A2E2}{A1E1 + A2E2}$ ------------- (vi)

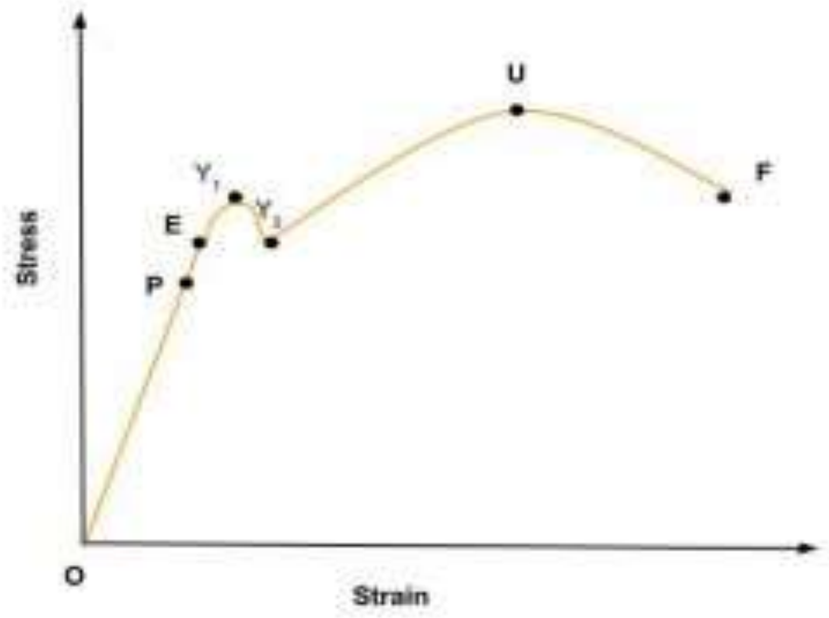
From these equqtions we can find out the loads shared by the different materials.

we have also seen in equqtion (iv) that $P1/A1E1 = P2/A2E2$

or $\sigma 1/E1 = \sigma 2/E2$ ($P/A = \sigma$)

Stress -Strain Diagram

- **The diagram which shows the relation between stress & strain showing the different values of stress along y axis, is called stress strain diagram .**



Stress strain diagram for mild steel.

The various terms related with the stress-strain diagram

- **Limit of Proportionality(point P) :- It is the greatest stress up to which the strain is proportional to the stress producing it. This limit does not exist for brittle material.**
- **Elastic limit (point E) :- It is the greatest stress up to which the material recovers its original length or dimension as soon as the stress causing the strain is removed. For many materials the numerical values of the elastic limit and the proportional limit are almost identical and the terms are sometimes used synonymously. If distinction prevails between these tow limits then elastic limit is always posses greater value than proportionality limit.**

- **Permanent Set** :- If a material body is loaded beyond elastic limit , it does not fully recover its original length or dimension when the load is removed . This permanent deformation of the material is known as permanent set.

- **Yield point(point y1 and y2)** :- The stress which the deformation of the material body grows without further increase in the load is called yield point of the material. At this point material attains permanent set. In mild steel generally two distinct yield point is seen as shown in figure. One point is called upper yield point and another is called lower yield point.
- **Ultimate Stress** :- The maximum stress up to which there is no deformation in the cross sectional area of a material body (Just before starting the formation of waist) is called ultimate stress .

SOME ADDITIONAL TERMS RELATED WITH STRESS- STRAIN

- 1. Working Stress :- The greatest stress to which a structure subjected in actual practice and design is known as working stress . It is always well below the elastic limit.

- 2. Proof Stress :- The largest stress repeatedly applied , which the material body withstand without taking a permanent set is known as proof stress and corresponding load is known as proof load .

- 3. Factor Of Safety :- The ratio of ultimate stress and the working stress is called factor of safety . Now a days the general practice followed that for structural steel work subjected to gradually increasing load the factor of safety is taken as the ratio of elastic limit to the working stress whose value is taken as 2 to 2.5.

But in case of structural steel work subjected to sudden load the factor of safety is taken as the ratio of ultimate stress to the working stress. Its value varies from 4 to 6.

SOME ADDITIONAL TERMS RELATED WITH STRESS- STRAIN

Percentation elongation- Let Lo = Initial length of the specimen ,

L = length at fracture. Then

$$\frac{L - Lo}{Lo} \times 100$$ is called percentage elongation.

Percentation reduction of area : - Let

Ao = Initial cross section of the specimen

A = Area at neck at fracture.

Then

$$\frac{Ao - A}{Ao} \times 100$$ is called percentage reduction of area

SOME ADDITIONAL TERMS RELATED WITH STRESS- STRAIN

Fluctuating stress : -When a material stressed (tensile or compressive) within a range then the stress is termed as fluctuating.

Repeated stress : - When a material stressed (tensile or compressive) between zero and some other specified value then it is known as repeated stress.

Alternating or reversed stress :- When amaterial is stressed alternately tensile and compressive, it is called an alternating or reversed stress.

Fatigue :- Experimentaly , it has been established that a material will fail at a stress considerably below its ultimate strength if that stress is repeated at sufficiently large number of times and the number of cycles necessary to cause failure would be considerably less if the same stress be reversed stress. This phenomenon is called fatigue.

Endurance limit :- If a material stressed within some range doesnot fail even when the cycle is repeated indefinite times, then this range of stress is known as endurance limit.

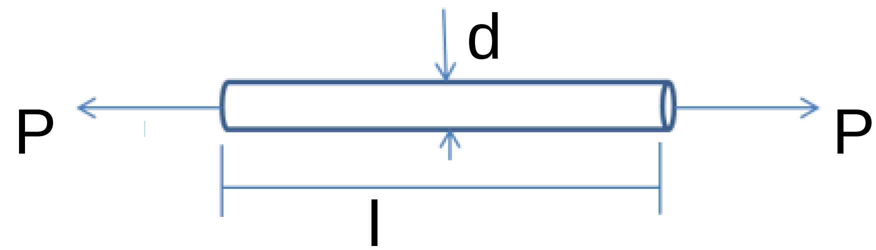
Elastic constants

Modulus Of elasticity:- When a material is under direct stress then the ratio between stress to the strain is called modulus of elasticity or Young's modulus. It is denoted by 'E' and the unit is N/sq.m.

Modulus Of rigidity :- During shear if the material is subjected to a shear stress producing an angular deformation i.e shear strain then the ratio between shear stress to the shear strain is called modulus of rigidity. It is denoted by C or G or N and its unit is N/sq.m.

Elastic constants

Linear strain:-

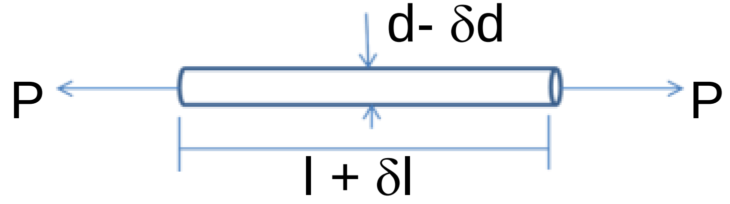


When external force acts on a body it undergoes deformation. The deformation of the body per unit length in the direction of the force is known as primary or linear strain .

Linear strain = $\delta l/l$.

Elastic constants

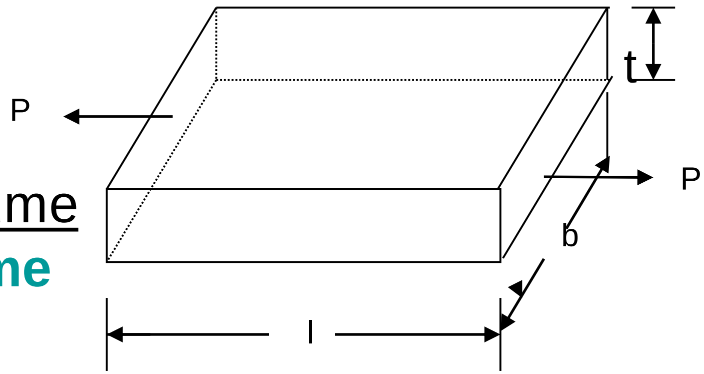
Lateral strain :- When a circular bar is subjected to a tensile or compressive force the length of the bar is extended or contracted accordingly. Subsequently the diameter of the bar is also decreased or increased .



Thus it obvious that every direct stress is always accompanied by a strain in its own direction, and an opposite kind of strain in every direction at right angles to it. This strain is called lateral strain.

Poissons ratio : - If a body is stressed within elastic limit the lateral strain bears a constant ratio to the linear strain. It is called poissons ratio. Denoted by 1/ m or μ.

VOLUMETRIC STRAIN OF A RECTANGULAR SECTION , SUBJECTED TO AN AXIAL LOAD



Volumetric Strain **ev =** $\dfrac{\text{Change in volume}}{\textbf{Original volume}}$

= $\delta V/V$

Bulk modulus :- The ratio of the direct stress intensity to the volumetric strain within the elastic limit is known as bulk modulus. It is denoted by ‘K’.

$$K = \frac{\sigma}{\delta V/V}$$

Its unit is N/sq.m.

RELATION BETWEEN
MODULUS OF ELASTICITY & MODULUS OF RIGIDITY

$$C = \frac{mE}{2(m+1)}$$

Where,

C = Modulus Rigidity

$\frac{1}{m}$ = Poisson's Ratio

E = Modulus of Elasticity

RELATION BETWEEN BULK MODULUS & YOUNG'S MODULUS

$$\mathbf{K} = \frac{mE}{3\,(m - 2)}$$

- Where, K = Bulk Modulus

  $\frac{1}{m}$ = Poisson's Ratio

  E = Young's Modulus

Elastic constant

Relation between modulus of elasticity , modulus of rigidity and bulk modulus

$$E = \frac{9\,G\,K}{3\,K + G}$$

CENTER OF GRAVITY

- The center of gravity of a body is the point through, which the line of the total weight at the body always passes in whatever position the body is held. It is generally denoted by the letter ‘G’.

- Methods for finding out the center of gravity of simple figure.

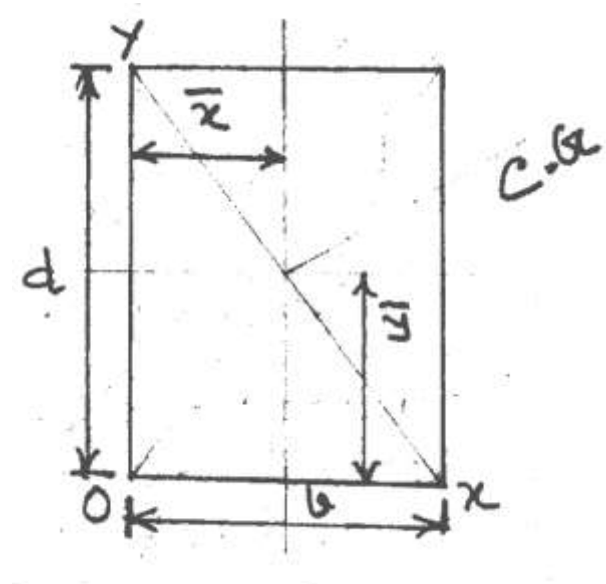
- Geometrical Consideration.
- By the method of moments.

CENTER OF GRAVITY
BY THE GEOMETRICAL CONSIDERATION

- The center of gravity of a uniform thin rod is its middle point.

CENTER OF GRAVITY
BY THE GEOMETRICAL CONSIDERATION

- **Center of gravity of a rectangle is at a point where the two diagonals meet each other. It is also the middle point of the length and breadth of the rectangle.**

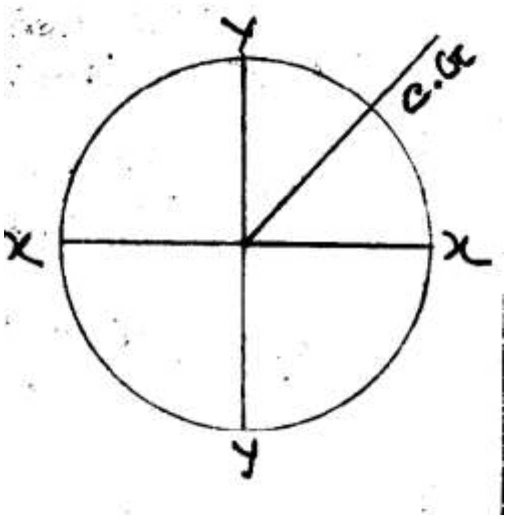


$\bar{x}$ = Distance of C.G. from '*y*' axis = $\frac{b}{2}$

$\bar{y}$ =Distance of C.G. from '*x*' axis = $\frac{d}{2}$

CENTER OF GRAVITY
BY THE GEOMETRICAL CONSIDERATION

- Center of gravity of a circular area.



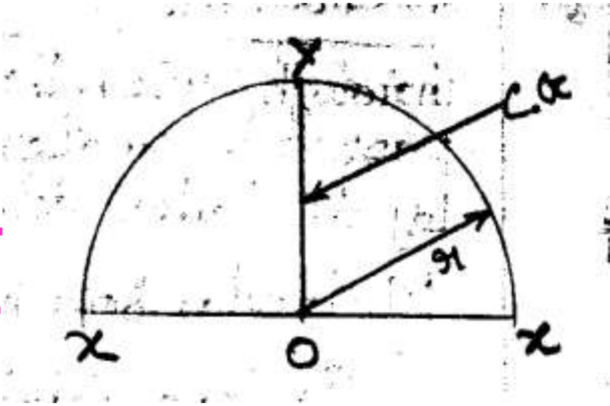
$\bar{x}$ = Distance of C.G. from $\bar{y}$ axis = 0

$\bar{y}$ = Distance of C.G. from $\bar{x}$ axis = 0

CENTRE OF GRAVITY OF A SEMI CIRCULAR AREA

$\bar{x}$ = Distance of C.G. from '*y*' axis = 0

$\bar{y}$ = Distance of C.G. from '*x*' axis = $\dfrac{4r}{3\pi}$



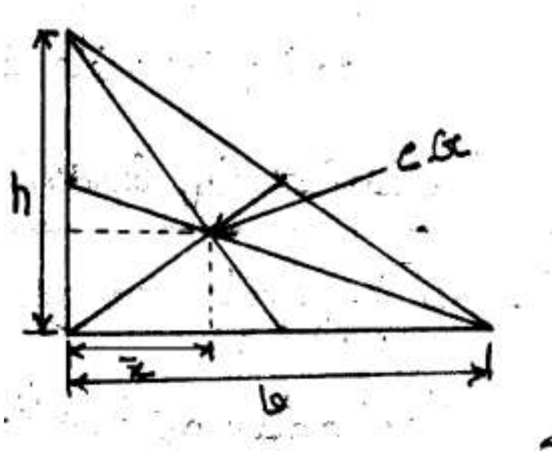
CENTRE OF GRAVITY OF A TRIANGULAR AREA

- The center of gravity of a triangle where the three medias meet.

  $\bar{x}$ = Distance of C.G. from 'y' axis = $\frac{1}{3}$ x b

  $\bar{y}$ = Distance of C.G. from '*x*' axis = $\frac{1}{3}$ x h

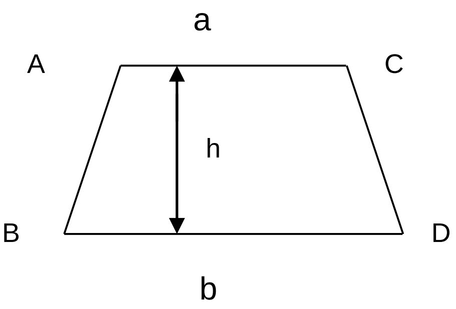
- (b = Base, h = height,)



CENTRE OF GRAVITY OF A TRAPIZODIAL AREA

- The center of gravity with parallel side A & B is at a distance of h

$$= \frac{h}{3}\left(\frac{b + 2a}{b + a}\right)$$



CENTROID

- The plain geometrical figures (like triangle, quadrilaterals, circle etc.) have only areas and no mass. The center of area no mass. The center of area of such figure will coincide with that of the area and is know as centroied.

The centroied of a plain area or that of a body of uniform thickness may be found by the formula.

$$\bar{x} = \frac{a1\,x1 + a2\,x2 + a3\,x3}{a1 + a2 + a3}$$

$$\bar{y} = \frac{a1\,y1 + a2\,y2 + a3\,y3}{a1 + a2 + a3}$$

MOMENT OF INERTIA

- The moment of inertia of a body about a line is the product of its mass and the square of its distance from that line.

- Mathematically moment of inertia of a body may be expressed.

$$I = M.r.^2$$

Where, I = Moment of inertia

M = Mass of the body

r = Distance between the C.G. of the body and the line about which moment of Inertia is required to be finding out.

MOMENT OF INERTIA OF A PLANE AREA

- Consider a plane area, whose moment of inertia is required to be finding out about a line. AB as shown in figure.

  Split up the plane surface in to a number of small strips as shown in figure.

  Let, *a1, a2, a3* , ----------- be the area of the strips.

  r1, r2, r3 , ----------- be the distance between the fixed line and C.G. of strips.

Than moment of inertia of the area,

$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2$ -------------

$= \sum ar^2$

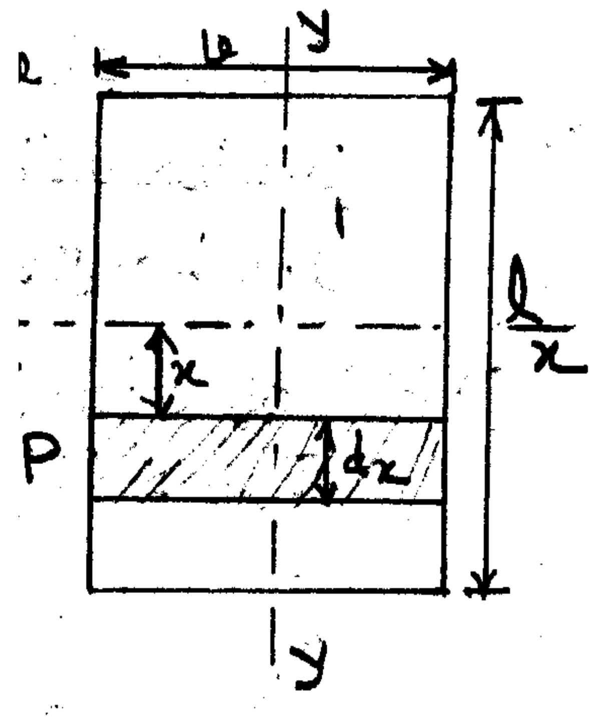
METHODS FOR FINDING OUT MOMENT OF INERTIA

- The moment of inertia of a body may be find out the following methods:-

1. By using ROUTH'S rule
2. By integration

MOMENT OF INERTIA OF RECTANGULAR SECTION *(BY INTRIGRATION)*

- **A rectangular section is consider where**
  'l' **= Length of the rectangular area**
  'b' **= Breadth of the rectangular area**
  A strip 'PQ' of thickness '*dx'* parallel to *x – x* and at a distance x from it.
  Therefore, area of the strip 'PQ'= *b.dx*
  Moment of inertia of the strip about
  y – y **axis = area × distance ^{2}**
  = *bdx.x²*



By integration *:-*

- $$I_{xx} = \frac{bl^3}{12} \quad \& \quad I_{yy} = \frac{lb^3}{12}$$

MOMENT OF INERTIA OF A HOLLOW RECTANGULAR SECTION

- $$I_{xx} = \frac{bl^3}{12} - \frac{b_1 l_1^3}{12} \quad \& \quad I_{yy} = \frac{lb^3}{12} - \frac{l_1 b_1^3}{12}$$

Where,

b = Breadth of the out line rectangle

l = Length of the out line rectangle

b_1= Breadth of the cut rectangle

l_1 = Length of cut out rectangle

MOENT OF INERTIA OF CIRCULAR SECTION

- M.I. of circular section

$$I_{xx} = \frac{\pi}{64} D^4$$

$$I_{yy} = \frac{\pi}{64} D^4$$

Where, D = Diameter of the circle.

MOENT OF INERTIA OF HOLLOW CIRCULAR SECTION

- M.I. of hollow circular section

$$I_{xx} = \frac{\pi}{64} (D4 – d4)$$

$$I_{yy} = \frac{\pi}{64} (D4 – d4)$$

Where, D = Outer diameter

d = Inner diameter

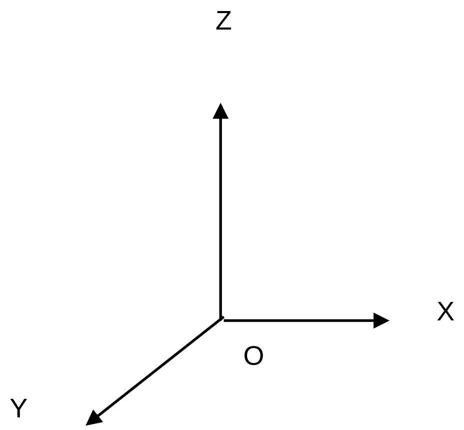
THEOREM OF PERPENDICULAR AXIS

- It states, if Ixx and Iyy be the moment of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia, I*zz* about the axis *z-z* perpendicular to the plane and passing through the inter section of *x –x* and y-y axis is given by the relation

  $I_{zz} = I_{xx} + I_{yy}$.

THEOREM OF PARALLEL AXIS

- It states if the M.I. of a plane are about an axis through it's C.G. be denoted by IG, the M.I. of the area about an axis, AB, parallel to the first and at a distance 'h' from the C.G.is given by,



I_{AB} = M.I. of the area about the line AB
I_G = M.I. of the area about the line C.G.
A = area of the section
h = Distance between C.G. of the section and the axis AB.

$$I_{AB} = I_G + A h^2$$

RADIUS OF GYRATION

- **Radius of gyration of an area about any axis is defined as the distance, square of which, when multiply by the whole area gives the moment of inertia of the area about that axis. It is generally denoted by the letter 'A'. It's unit is cm.**

Therefore, by definition,

$$K^2 \times A = I$$

$$\text{Therefore, } K = \sqrt{\frac{I}{A}}$$

Where,

K = Radius of gyration about any axis

I = Moment of inertia of the whole section of an area about the same axis.

A = Area of the whole section.

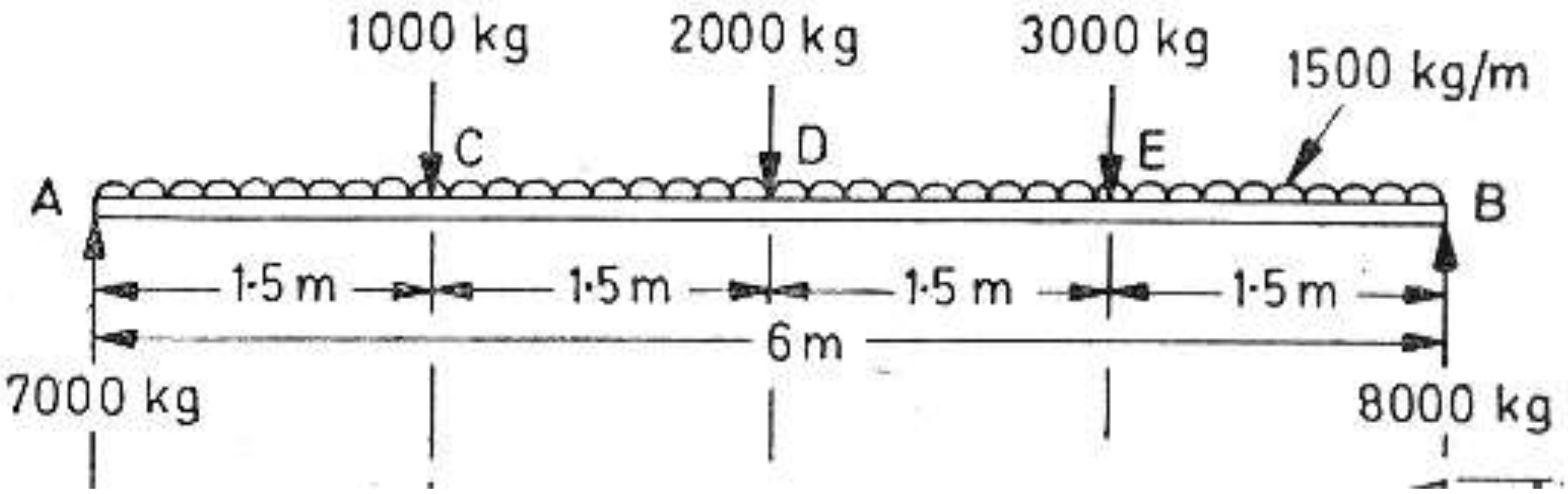
EMPIRICAL FORMULA

- Euler as formula is valid or long column. It does not take into consideration the direct compressive stress. In order to fill up this lacuna. Many more formula were proposed by different centrist all over the world. The following empirical formulas are important.

Rankin’s formula, 2. Johnson’s formula, 3. Indian standard coad.

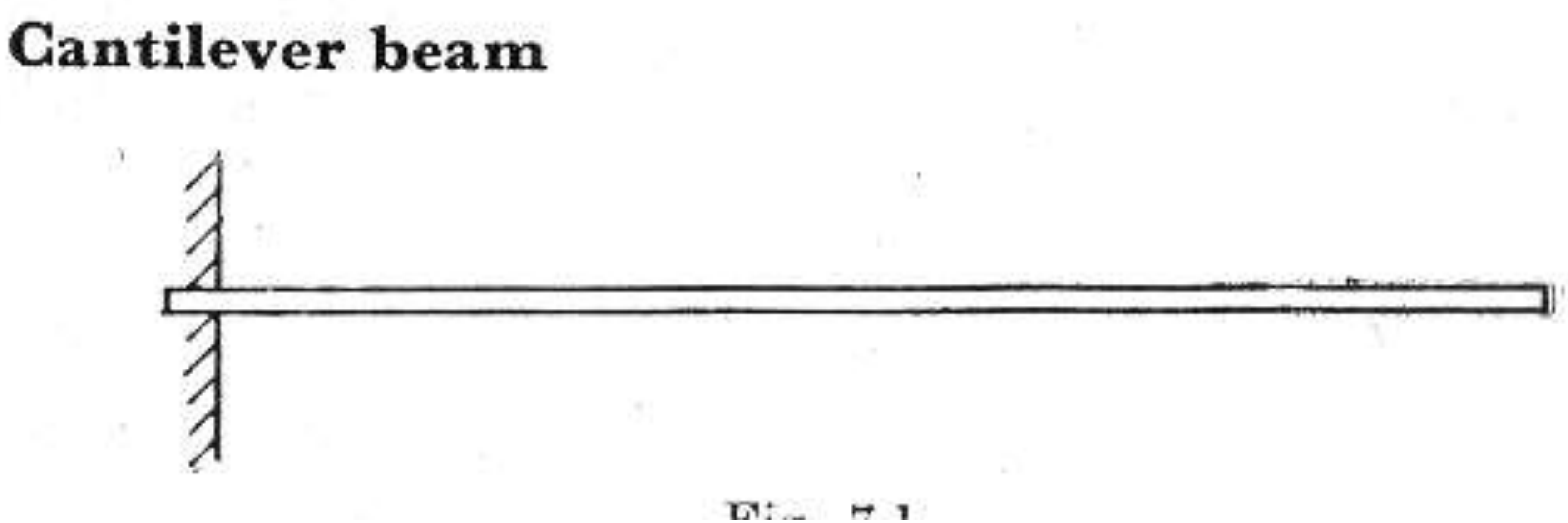
BENDING MOMENT SHEAR FORCE

BEAM: - A beam is a structure member which is subjected to transverse load.



TYPES OF BEAM

- **CANTILIVER BEAM**: - A beam fixed at one and free at other is known as a cantilever beam.



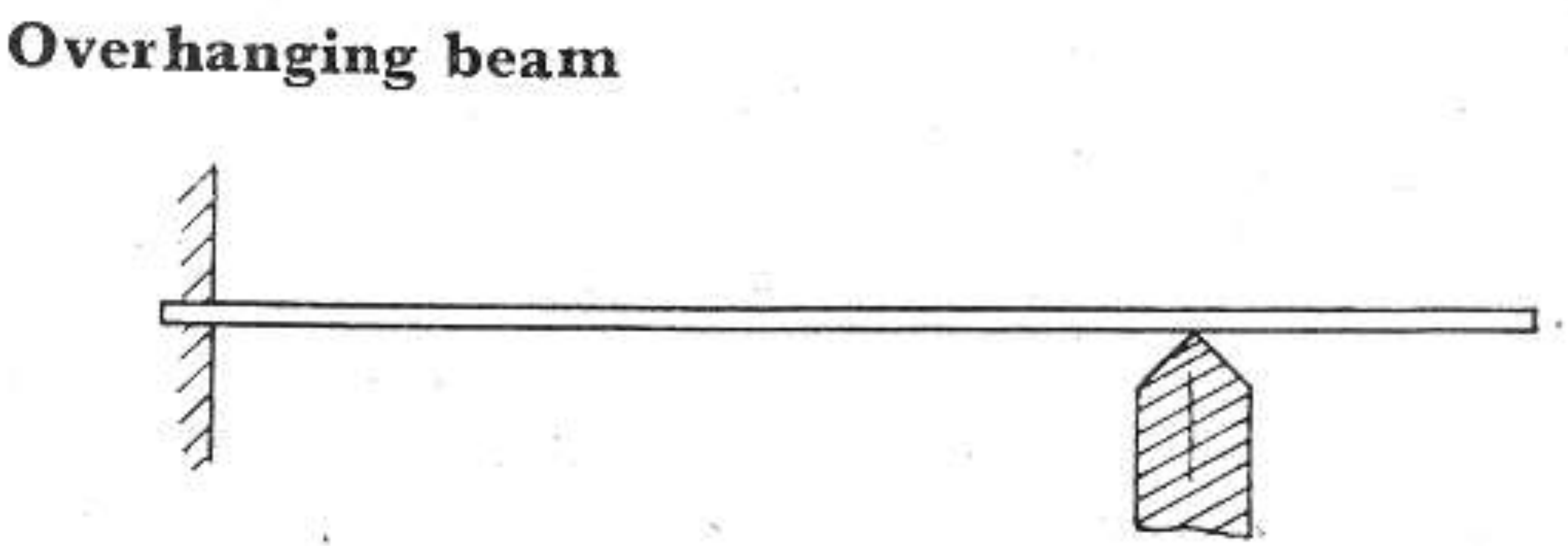
SIMPLY SUPPORTED BEAM

- A beam supported or resting freely on the walls or column at it’s both end is known as simply supported beam.

Simply supported beam

OVERHANGING BEAM

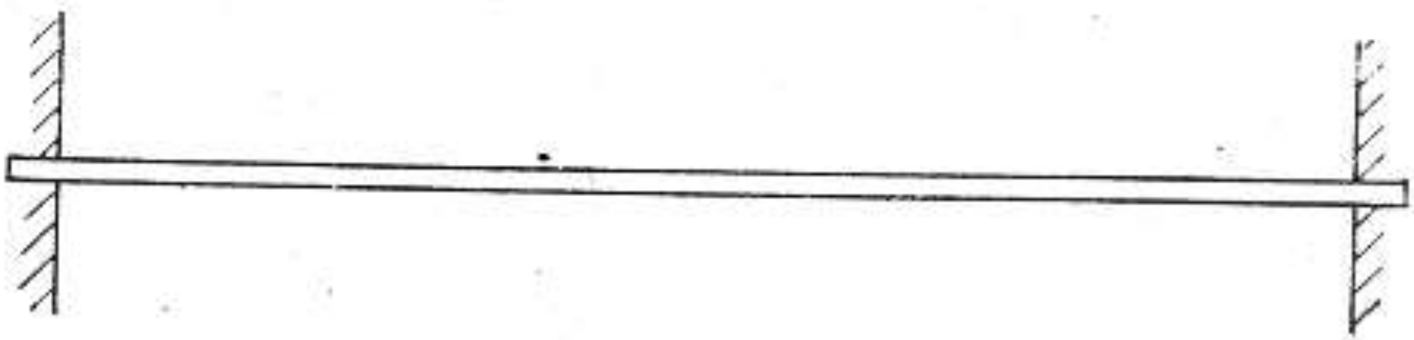
- A beam having it’s end portion extended beyond the support is known as overhanging beam. A beam may be overhanging on one side or on both sides.



RIGIDLY FIXED BEAM

- A beam whose both end are rigidly fixed or built in walls is known as rigidly fixed beam.

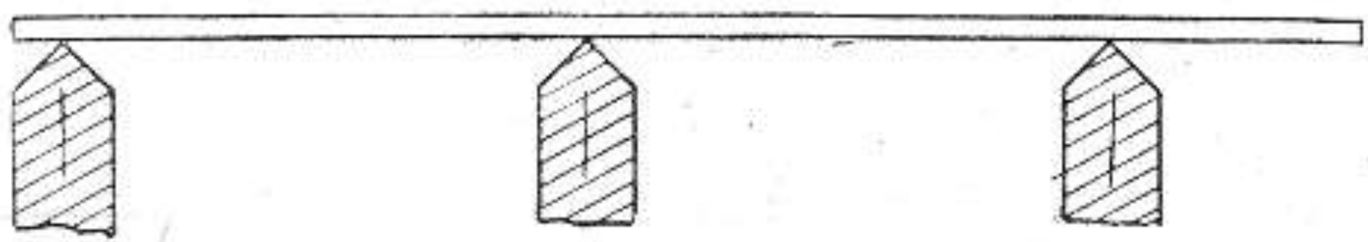
Rigidly fixed beam



CONTINUOUS BEAM

- A beam supported on more than two supports is known as continuous beam.

Continuous beam

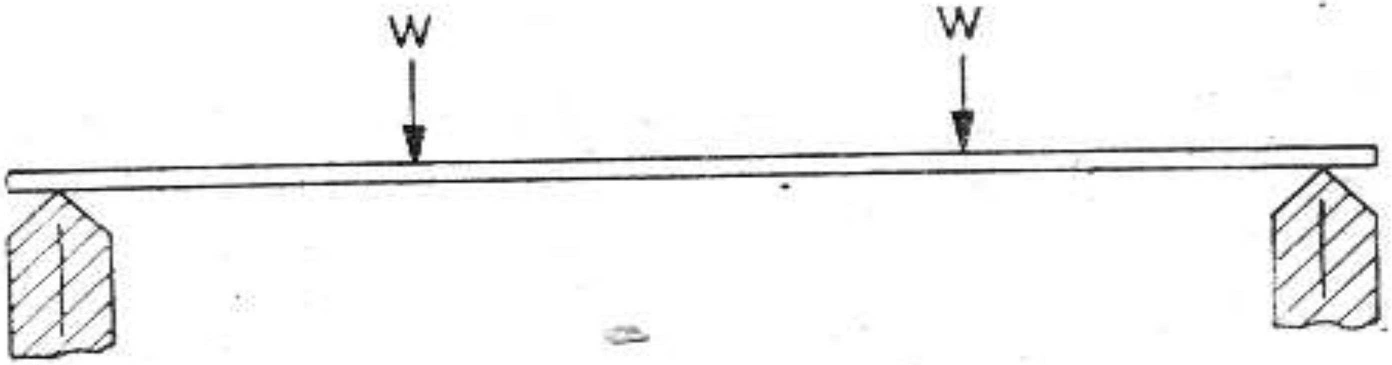


TYPES OF LOADING

- **CONCENTRATED OR POINT LOAD: -**

  A load acting at a point on a beam is known as a concentrated or point load.

Concentrated or point load



UNIFORMLY DISTRIBUTED LOAD

- A load which is spread over a beam in such a manner that each unit length is loaded to the same extent is known as a uniformly distributed load.

Uniformly distributed load

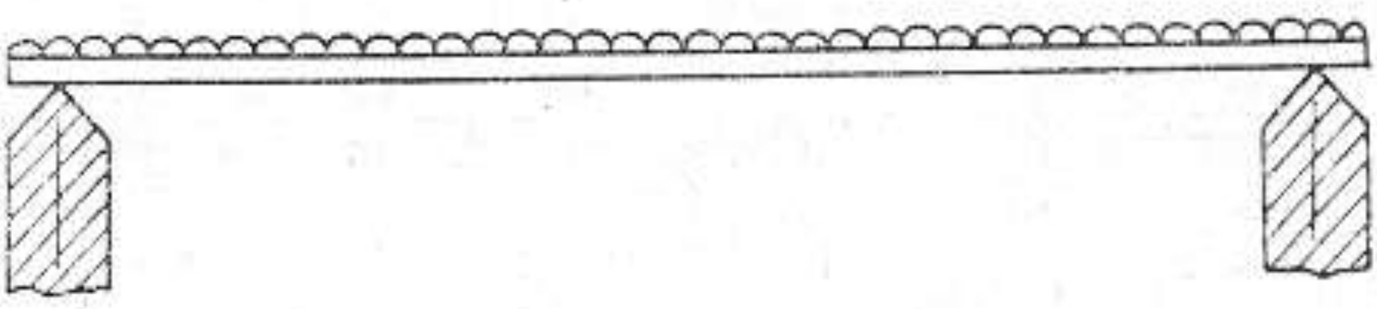


Fig. 7.7

UNIFORMLY VARYING LOAD

- A load which is spread in such a manner that it’s extent varies uniformly on unit length is known as a uniformly varying load.

Uniformly varying load

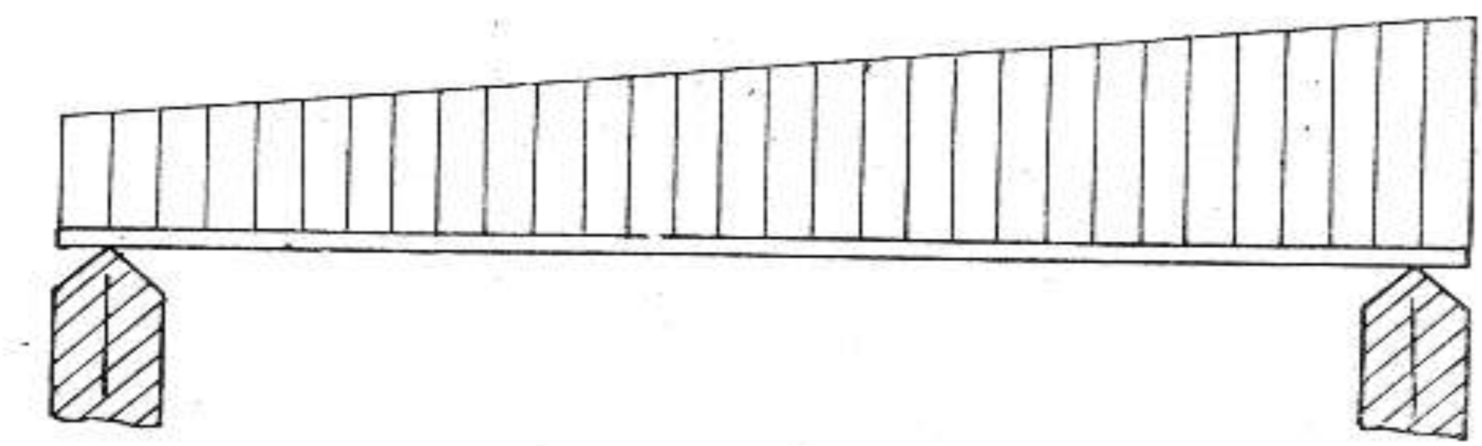
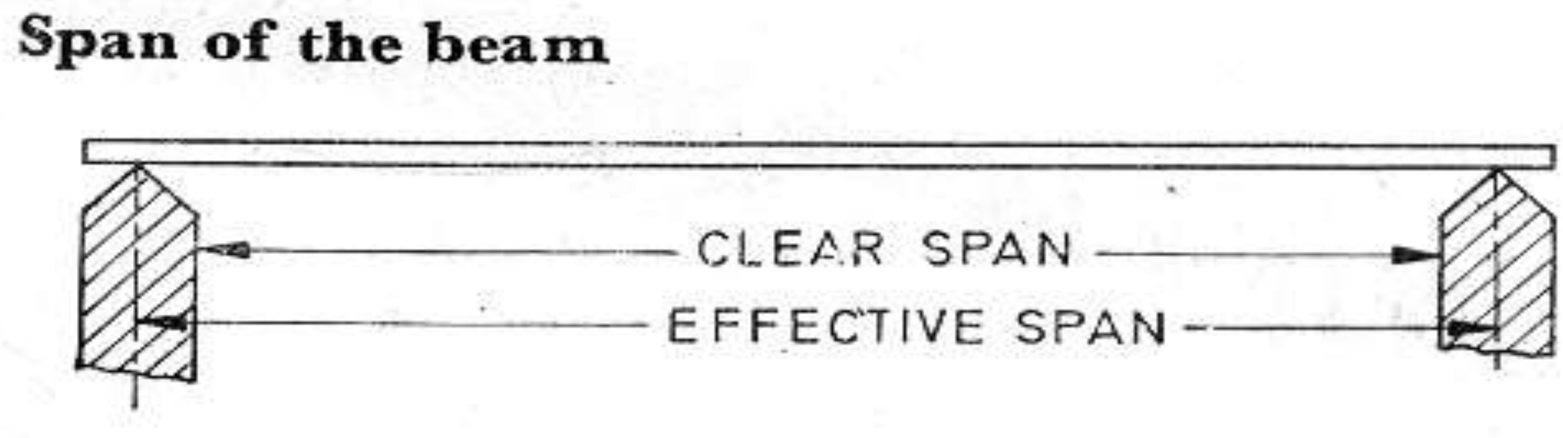


Fig. 7.8

SPAN OF THE BEAM

- The horizontal distance between the supporting walls is known as clear span of the beam. The horizontal distance between the lines of acting of the supporting walls is known as effective span. For calculation purpose we shall always consider the effective span.

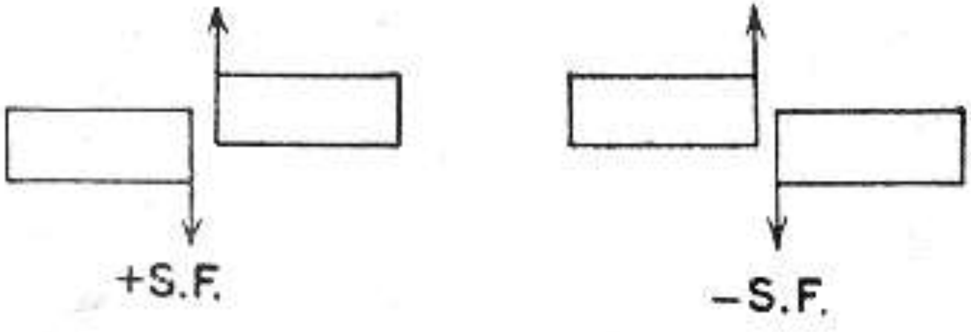


- **BENDING MOMENT (B.M.):-** The bending moment at the cross section of a beam may be defined as the algebraic sum of the moments of the forces to the right or left of the section.

- **SHEAR FORCE (S.F.):-** The shear force at the cross section of a beam may be defined as the unbalance vertical force to the right or left of the section.

SIGN CONVENTION

- **SHEAR FORCE (S.F.):-** **The shear force is the unbalanced vertical force, therefore it tends to slit one portion of the beam, upward or downward with respect to the order. The shear force is said to be positive, at a section, when the right hand portion tents to slight upwards with respect to the left hand portion, or in other wards all the upward forces to the right of the section cause positive shear and those acting downwards cause negative shear.**



BENDING MOMENT (B.M.)

- At section where the bending moment is such that it tents to bend the beam at the point to a curvature having concavity at the top, is taken as positive on the other hand where the bending moment is such that it tents to bend the beam at curvature having concavity at the top.

In other wards the bending moment is said to be positive at a section when it is acting is an anti clock wise direction to the right and negative, when acting in a clock wise direction. On the other hand (B.M.) is said to be negative when it is acting in a clock wise direction to the left and positive when it is acting in an anti clock wise direction.

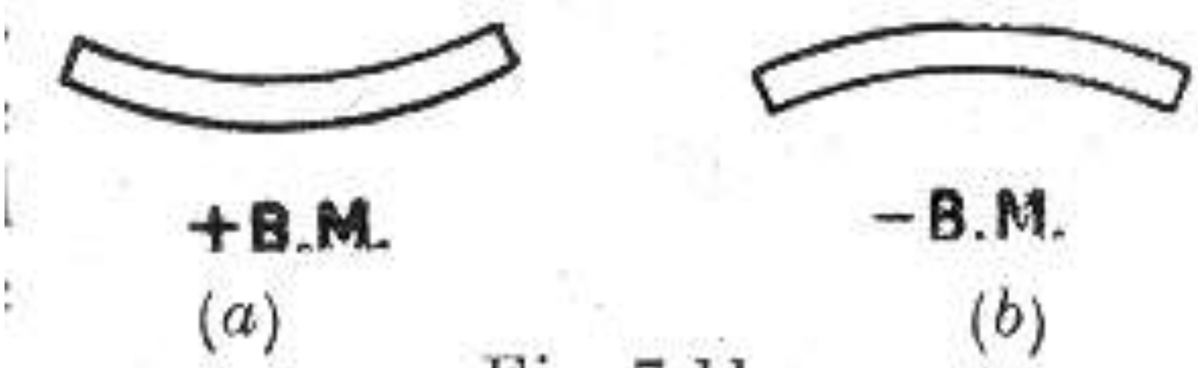


Fig. 7·11

BENDING STRESSES IN BEAMS

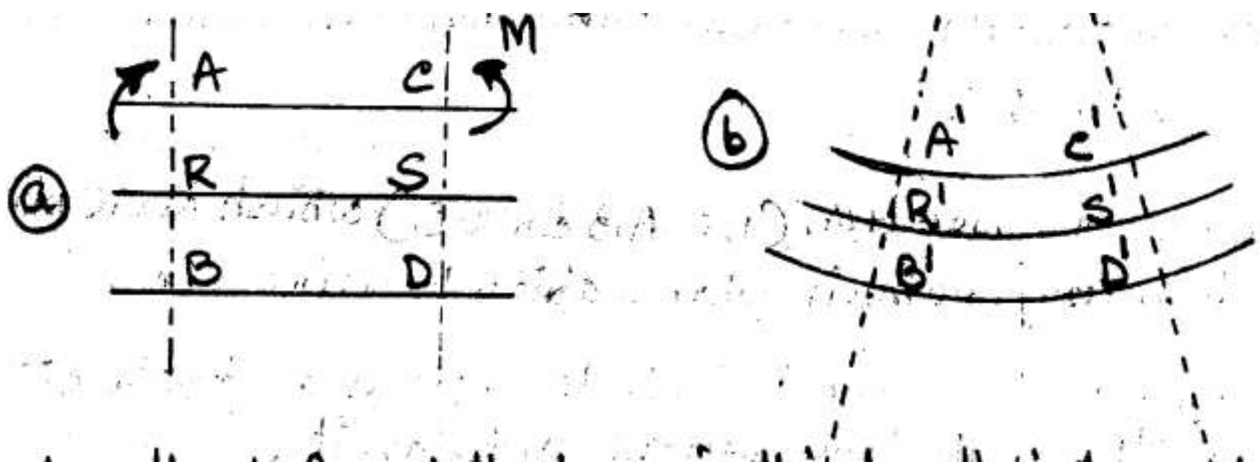
- The bending moment at a section tents to bend or deflect the beam and the internal stresses resists it’s bending. The process of bending stops, when every cross section sets up full resistance to the bending moment. The resistance offered by the internal stresses to the bending is known as bending stress.

THEORY OF SIMPLE BENDING

- **Consider a small length ‘δ*x*’ of a simply supported beam to bending moment ‘M’ as shown in figure (a)**

The section ‘AB’ and ‘CD’ are normal to axis of the beam due to the action of the bending moment, the beam as a whole will bend as shown figure (b).

We see that the layers above ‘RS’ have been compressed and those below, have been stretched. The amount, by which a layer is compressed or stretched, depends upon the position of the layer with reference to ‘RS’. This layer ‘RS’ which is neither compressed none stretched is known as neutral plane or neutral layer.



ASSUMPTIONS IN THE THEORY OF SIMPLE BENDING

- **The following assumptions are made in the theory of simple bending:-**
  - **The material of the beam is perfectly homogeneous (i.e. at the same kind elastic properties in all direction.)**
  - **The beam material is stressed within its elastic limit and thus obeys Hooke’s law.**
  - **The transverse section (i.e. AB or CD) which were plane before bending, remain plane after bending also.**
  - **Each layer of the beam is free to expend or contract independently of the layer, above or below it.**
  - **The value of ‘E’ (Young’s modulus of elasticity) is the same tension and compression.**

POSITION OF NEUTRAL AXIS

- ✓ The line of inter section of the neutral layer with any normal cross section of a beam is known as neutral axis of that section. On one side of the neutral axis there are compressive stresses and on the other there are tensile stresses. At the neutral axis there is no stress of any kind.

MOMENT OF RESISTANCE

- We know that on one side of neutral axis there are compressive stress and the other there are tensile stresses. These stresses from a couple, whose moment must be equal to the external moment 'M'. The moment of this couple which resists the external bending moment is known as amount of resistance.

- Mathematically, $\frac{M}{I} = \frac{f}{Y} = \frac{E}{R}$
- Where,
- M = Moment
- I = Moment of inertia
- f = Bending stress
- Y = Distance of the layer from neutral axis
- E = Modulus of elasticity
- R = Radius of curvature

BENDING STRESS
IN SYMMETRICAL SECTION

- In a symmetrical section (i.e. Circular, Square, and Rectangular) the C.G. of the section lies at the geometrical center of section.

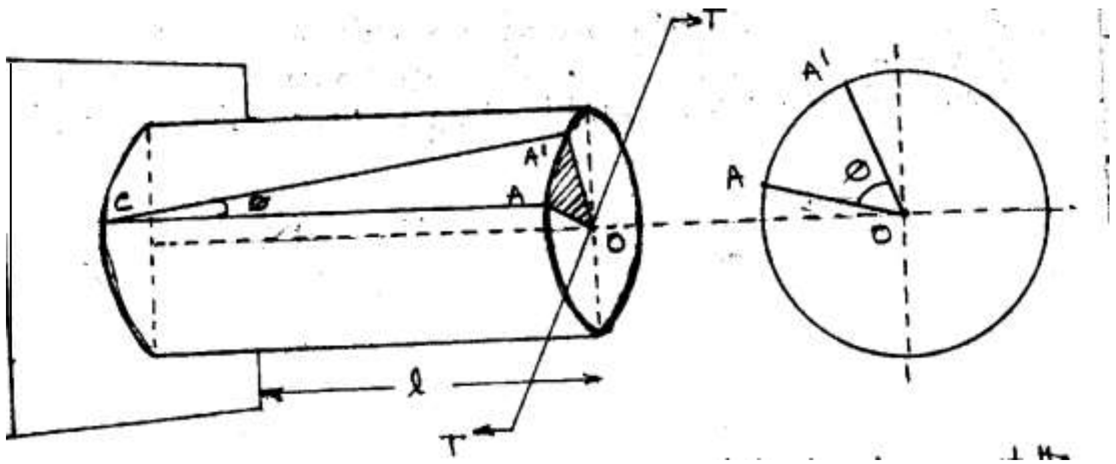
Mathematically,

$$y = \frac{d}{2}$$

Where ,'d' is the diameter (in a circular section) or depth (in square or rectangular section).

TORSION

- In workshop and factories, a turning force is always applied to transmit energy by rotation. This turning force is applied either rim of a pulley, keyed to the shaft, or to any other suitable point at some distance from the axis of the shaft. The product of this turning force and the distance between the point of application of the force and the axis of the shaft is known as torque, turning moment or twisting moment. The shaft is said to be subjected to torsion. Due to this torque every cross section is subjected to a shaft stress.



ASSUMTION FOR FINDING OUT SHEAR STRESS IN A CIRCULAR SHAFT SUBJECTED TO TORSION

- Following assumption are made while finding out shear stress in a circular shaft subjected to torsion.
- The material of the shaft is uniform through out.
- The twist along the shaft is uniform.
- Normal cross section of the shaft which were plane and circular before twist, remain plane and circular after twist.
- All diameters of the normal cross section which were straight before twist, remain straight with there magnitude unchanged after twist.
- A little consideration will show that the above assumption is justified. If the torque applied is small and the angle of twist is also small.

STRENGTH OF A SHAFT

- By the strength of shaft is meant, the maximum torque or horse power it can transmit.

If 'T' is the torque which is subjected on a shaft

Then,

$$T = \frac{\pi}{16} f_s D^3$$

Where, fs = Shear stress

D = Diameter of the shaft

STRENGTH OF HOLLOW SHAFT

- If 'T' is the torque which is subjected to a hollow shaft

Then,

$$T = \frac{\pi}{16} \cdot \frac{fs\,(D4 - d4)}{D}$$

Where, fs = Shear stress

D = External diameter of the hollow shaft.

d = Internal diameter of the hollow shaft.

POLAR MOMENT OF INERTIA

- The moment of inertia of a plane area with respect to and axis perpendicular to the plane of the figure is called polar moment of inertia with respect to the point where the axis intersect the plane the plane. In a circular plane this point is always the center of the circle.

Mathematically represented by 'J'

$$J = \frac{\pi}{32} D^4$$

: for solid shaft ,

[where D = Diameter of the shaft]

$$J = \frac{\pi}{32} (D^4 - d^4)$$

: for hollow shaft

where, D = Outer dia of the shaft.

d = Inner dia. of the shaft]

HORSE POWER TRASMITED BY A SHAFT

- A rotating shaft is considered which transmit power from one of it's end to another.

Let, N= Number of revolution per minute.

T = Average torque in Kg.m.

Therefore,

Work done per minute

= Force × Distance

= Average torque × Angular displacement.

= T × π N

= 2 π N T Kg.m.

- Now we know horse power,

$$= \frac{\text{Work done per minute}}{4500} = \frac{2\pi NT}{4500} \text{ H.P.}$$