C H A P T E R 3

Learning Objectives

- Laws of Magnetic Force
- Magnetic Field Strength (H)
- Magnetic Potential
- Flux per Unit Pole
- Flux Density (*B*)
- Absolute Parmeability (m) and Relative Permeability (m_r)
- Intensity of Magnetisation (I)
- Susceptibility (K)
- Relation Between *B, H, I* and *K*
- Boundary Conditions
- Weber and Ewing's Molecular Theory
- Curie Point. Force on a Current-carrying Conductor Lying in a Magnetic Field
- Ampere's Work Law or Ampere's Circuital Law
- Biot-Savart Law
- Savart Law
- Force Between two Parallel Conductors
- Magnitude of Mutual Force
- Definition of Ampere
- Magnetic Circuit
- Definitions
- Composite Series Magnetic Circuit
- How to Find Ampere-turns ?
- Comparison Between Magnetic and Electric Circuits
- Parallel Magnetic Circuits
- Series-Parallel Magnetic Circuits
- Leakage Flux and Hopkinson's Leakage Coefficient
- Magnetisation Curves
- Magnetisation curves by Ballistic Galvanometer
- Magnetisation Curves by Fluxmete

MAGNETISM AND ELECTRO-MAGNETISM

Designing high speed magnetic levitation trains is one of the many applications of electromagnetism. Electromagnetism defines the relationship between magnetism and electricity

Absolute and Relative Permeabilities of a Medium

The phenomena of magnetism and electromagnetism are dependent upon a certain property of the medium called its permeability. Every medium is supposed to possess two permeabilities :

(i) absolute permeability (μ) and *(ii)* relative permeability (μ_r).

For measuring relative permeability, vacuum or free space is chosen as the reference medium. It is allotted an absolute permeability of $\mu_0 = 4\pi \times 10^{-7}$ henry/metre. Obviously, relative permeability of vacuum with reference to itself is unity. Hence, for free space,

absolute permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m

relative permeability $\mu_r = 1$.

Now, take any medium other than vacuum. If its relative permeability, as compared to vacuum is μ_r, then its absolute permeability is $\mu = \mu_0\, \mu_r$ H/m.

Laws of Magnetic Force

Coulomb was the first to determine experimentally the quantitative expression for the magnetic force between two *isolated* point poles. It may be noted here that, in view of the fact that magnetic poles always exist in pairs, it is impossible, in practice, to get an isolated pole. The concept of an isolated pole is purely theoretical. However, poles of a thin but long magnet may be assumed to be point poles for all practical purposes (Fig. 6.1). By using a torsion balance, he found that the force between two magnetic poles placed in a medium is

(i) directly proportional to their pole strengths

(ii) inversely proportional to the square of the distance between them and

(iii) inversely proportional to the absolute permeability of the surrounding medium.

Fig. 6.1 Fig. 6.2

For example, if m_1 and m_2 represent the magnetic strength of the two poles (its unit as yet being undefined), r the distance between them (Fig. 6.2) and μ the absolute permeability of the surrounding medium, then the force F is given by

$$F \propto \frac{m_1 m_2}{\mu r^2} \quad \text{or} \quad F = k\frac{m_1 m_2}{\mu r^2} \quad \text{or} \quad \vec{F} = \frac{k\, m_1 m_2\, \hat{r}}{\mu r^2} \quad \text{in vector from}$$

where $\hat{r}$ is a unit vector to indicate direction of r.

$$\text{or} \qquad \vec{F} = k\frac{m_1 m_2}{r^3}\vec{r} \quad \text{where } \vec{F} \text{ and } \vec{r} \text{ are vectors}$$

In the S.I. system of units, the value of the constant k is $= 1/4\pi$.

$$F = \frac{m_1 m_2}{4\pi\mu r^2}\ \text{N or } F = \frac{m_1 m_2}{4\pi\mu_0\mu_r r^2}\ \text{N} \qquad \text{– in a medium}$$

In vector form,

$$\boxed{\vec{F} = \frac{m_1 m_2}{4\pi\mu r^3}\vec{r} = \frac{m_1 m_2}{4\pi\mu r^2}\hat{r}\ \text{N}}$$

If, in the above equation,

$$m_1 = m_2 = m \text{ (say)} ;\ r = 1 \text{ metre} ;\ F = \frac{1}{4\pi\,\mu_0}\ \text{N}$$

Then $m^2 = 1$ or $m = \pm 1$ weber*

Hence, a unit magnetic pole may be defined as ***that pole which when placed in vacuum at a distance of one metre*** from a similar and equal pole repels it with a force of $1/4\pi\ \mu_0$ ***newtons***.**

Magnetic Field Strength (H)

Magnetic field strength at any point within a magnetic field is numerically equally to the force experienced by a *N*-pole of one weber placed at that point. Hence, unit of *H* is N/Wb.

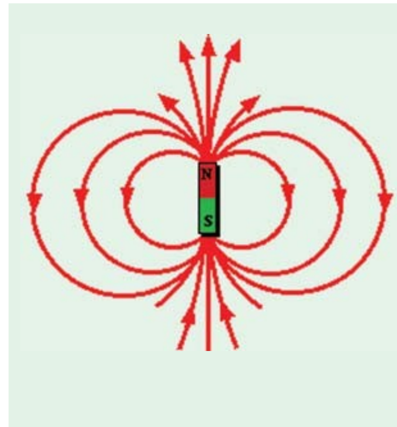
Suppose, it is required to find the field intensity at a point *A* distant *r* metres from a pole of *m* webers. Imagine a similar pole of one weber placed at point *A*. The force experienced by this pole is

$$F = \frac{m \times 1}{4\pi\mu_0 r^2}\ \text{N} \qquad \therefore H = \frac{m}{4\pi\mu_0 r^3}\ \text{N/Wb (or A/m)***}\ \text{or oersted.}$$

Also, if a pole of *m* Wb is placed in a uniform field of strength *H* N/Wb, then force experienced by the pole is = *mH* newtons.

It should be noted that field strength is a vector quantity having both magnitude and direction

$$\therefore \qquad \vec{H} = \frac{m}{4\pi\mu_0 r^2} \qquad \hat{r} = \frac{m}{4 \quad {}_0 r^3} r$$



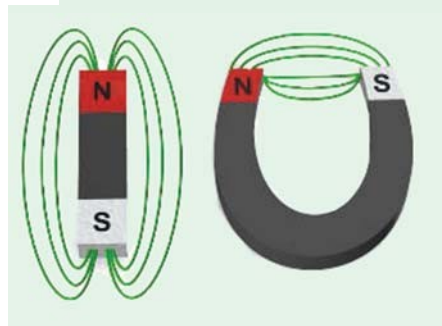
It would be helpful to remember that following terms are sometimes interchangeably used with field intensity : Magnetising force, strength of field, magnetic intensity and intensity of magnetic field.

Magnetic Potential

The magnetic potential at any point within a magnetic field is measured by the work done in shifting a *N*-pole of one weber from infinity to that point against the force of the magnetic field. It is given by

$$M = \frac{m}{4\pi\mu_0 r}\ \text{J/Wb} \qquad \text{...(Art. 4.13)}$$

It is a scalar quantity.



Magnetic lines of force

Flux per Unit Pole

A unit *N*-pole is supposed to radiate out a flux of one weber. Its symbol is Φ. Therefore, the flux coming out of a *N*-pole of *m* weber is given by

$$\Phi = m\ \text{Wb}$$

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* To commemorate the memory of German physicist Wilhelm Edward Weber (1804-1891).

** A unit magnetic pole is also defined as that magnetic pole which when placed at a distance of one metre from a very long straight conductor carrying a current of one ampere experiences a force of $1/2\pi$ newtons (Art. 6.18).

*** It should be noted that N/Wb is the same thing as ampere/metre (A/m) or just A/m cause 'turn' has no units

Flux Density (B)

It is given by the flux passing per unit area through a plane at right angles to the flux. It is usually designated by the capital letter B and is measured in weber/meter^2. It is a Vector Quantity.

It ΦWb is the total magnetic flux passing normally through an area of A m^2, then

$$B = \Phi/A \text{ Wb/m}^2 \text{ or tesla (T)}$$

Note. Let us find an expression for the flux density at a point distant r metres from a unit N-pole (*i.e.* a pole of strength 1 Wb.) Imagine a sphere of radius r metres drawn round the unit pole. The flux of 1 Wb radiated out by the unit pole falls normally on a surface of $4\pi r^2.m^2$. Hence

$$B = \frac{\Phi}{A} = \frac{1}{4\pi r^2} \text{ Wb/m}^2$$

Absolute Permeability (μ) and Relative Permeability (μ_r)

In Fig. 6.3 is shown a bar of a magnetic material, say, iron placed in a uniform field of strength H N/Wb. Suppose, a flux density of B Wb/m^2 is developed in the rod.

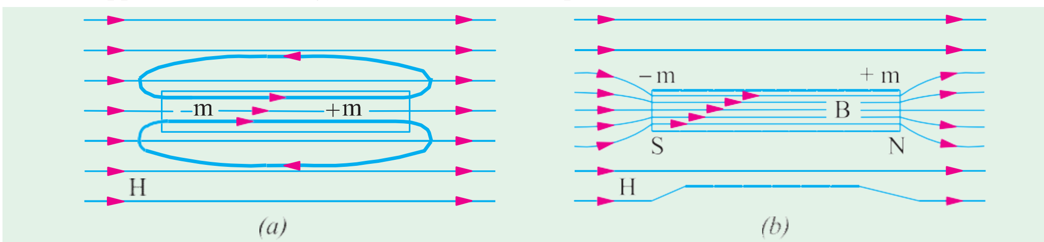


Fig. 6.3

Then, the absolute permeability of the material of the rod is defined as

$$\mu = B/H \text{ henry/metre or } B = \mu H = \mu_0 \mu_r H \text{ Wb/m}^2 \quad ...(i)$$

When H is established in air (or vacuum), then corresponding flux density developed in air is

$$B_0 = \mu_0 H$$

Now, when iron rod is placed in the field, it gets magnetised by induction. If induced pole strength in the rod is m Wb, then a flux of m Wb emanates from its N-pole, re-enters its S-pole and continues from S to N-pole within the magnet. If A is the face or pole area of the magentised iron bar, the *induction* flux density in the rod is

$$B = m/A \text{ Wb/m}^2$$

Hence, total flux density in the iron rod consists of two parts [Fig. 6.3 (*b*)].

(i) B_0 –flux density in air even when rod is not present

(ii) B_i –induction flux density in the rod

$$B = B_0 + B_i = \mu_0 H + m/A$$

Eq. (*i*) above may be written as $B = \mu_r \cdot \mu_0 H = \mu_r B_0$

$$\therefore \quad \mu_r = \frac{B}{B_0} = \frac{B \text{ (material)}}{B_0 \text{(vacuum)}} \quad \text{...for same } H$$

Hence, relative permeability of a material is equal to ***the ratio of the flux density produced in that material to the flux density produced in vacuum by the same magnetising force.***

Intensity of Magnetisation (I)

It may be defined as the induced pole strength developed per unit area of the bar. Also, it is the magnetic moment developed per unit volume of the bar.

Let $\quad m$ = pole strength induced in the bar in Wb

A = face or pole area of the bar in m^2

Then $\quad I = m/A$ Wb/m^2

Hence, it is seen that intensity of magnetisation of a substance may be defined as ***the flux density produced in it due to its own induced magnetism***.

If l is the magnetic length of the bar, then the product $(m \times l)$ is known as its magnetic moment M.

$$\therefore \quad I = \frac{m}{A} = \frac{m \times l}{A \times l} = \frac{M}{V} = \text{magnetic moment/volume}$$

Susceptibility (K)

Susceptibility is defined as ***the ratio of intensity of magnetisation I to the magnetising force H.***

$$\therefore \quad K = I/H \text{ henry/metre.}$$

Relation Between B, H, I and K

It is obvious from the above discussion in Art. 6.7 that flux density B in a material is given by

$$B = B_0 + m/A = B_0 + I \qquad \therefore B = \mu_0 H + I$$

Now absolute permeability is $\mu = \dfrac{B}{H} = \dfrac{\mu_0 H + I}{H} = \mu_0 + \dfrac{I}{H} \qquad \therefore \mu = \mu_0 + K$

Also $\quad \mu = \mu_0 \mu_r \therefore \mu_0 \mu_r = \mu_0 + K$ or $\mu_r = 1 + K/\mu_0$

For ferro-magnetic and para-magnetic substances, K is positive and for diamagnetic substances, it is negative. For ferro-magnetic substance (like iron, nickel, cobalt and alloys like nickel-iron and cobalt-iron) μ_r is much greater than unity whereas for para-magnetic substances (like aluminium), μ_r is slightly greater than unity. For diamagnetic materials (bismuth) $\mu_r < 1$.

Example 6.1. *The magnetic susceptibility of oxygen gas at 20°C is 167 × 10^{-11} H/m. Calculate its absolute and relative permeabilities.*

Solution. $\quad \mu_r = 1 + \dfrac{K}{\mu_0} = 1 + \dfrac{167 \times 10^{-11}}{4\pi \times 10^{-7}} = \mathbf{1.00133}$

Now, absolute permeability $\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1.00133 = \mathbf{12.59 \times 10^{-7}}$ ***H*/m**

Boundary Conditions

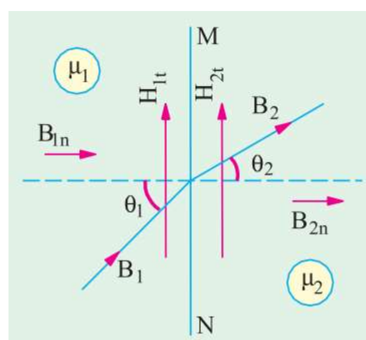


Fig. 6.4

The case of boundary conditions between two materials of different permeabilities is similar to that discussed in Art. 4.19.

As before, the two boundary conditions are

(i) the normal component of flux density is continuous across boundary. $\quad B_{1n} = B_{2n} \quad$ ...(*i*)

(ii) the tangential component of H is continuous across boundary $\quad H_{1t} = H_{2t}$

As proved in Art. 4.19, in a similar way, it can be shown that

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2}$$

This is called the law of magnetic flux refraction.

Weber and Ewing's Molecular Theory

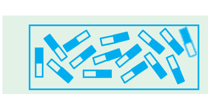


Fig. 6.5

This theory was first advanced by Weber in 1852 and was, later on, further developed by Ewing in 1890. The basic assumption of this theory is ***that molecules of all substances are inherently magnets in themselves, each having a N and S pole***. In an unmagnetised state, it is supposed that these small molecular magnets lie in all sorts of haphazard manner forming more or less closed loops (Fig. 6.5). According to the laws of attraction and repulsion, these closed magnetic circuits are satisfied internally, hence there is no resultant external magnetism exhibited by the iron bar. But when such an iron bar is placed in a magnetic field or under the influence of a magnetising force, then these molecular magnets start turning round their axes and orientate themselves more or less along straight lines parallel to the direction of the magnetising force. This linear arrangement of the molecular magnets results in *N* polarity at one end of the bar and *S* polarity at the other (Fig. 6.6). As the small magnets turn more nearly in the direction of the magnetising force, it requires more and more of this force to produce a given turning moment, thus accounting for the magnetic saturation. On this theory, the hysteresis loss is supposed to be due to molecular friction of these turning magnets.

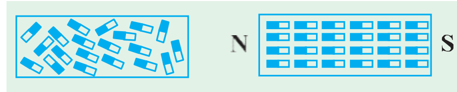
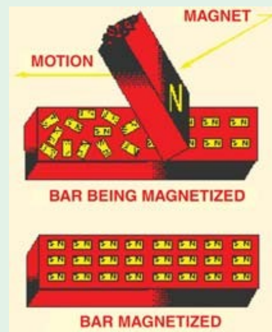
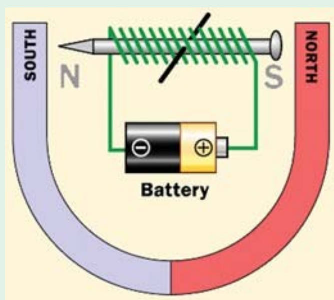


Fig. 6.6



Molecular magnets which are randomly arranged in the unmagnetised state, get oriented under the influence of an external magnetizing force

Because of the limited knowledge of molecular structure available at the time of Weber, it was not possible to explain firstly, as to why the molecules themselves are magnets and secondly, why it is impossible to magnetise certain substances like wood etc. The first objection was explained by Ampere who maintained that orbital movement of the electrons round the atom of a molecule constituted a flow of current which, due to its associated magnetic effect, made the molecule a magnet. Later on, it became difficult to explain the phenomenon of diamagnetism (shown by materials like water, quartz, silver and copper etc.) erratic behaviour of ferromagnetic (intensely magnetisable) substances like iron, steel, cobalt, nickel and some of their alloys etc. and the paramagnetic (weakly magnetisable) substances like oxygen and aluminium etc. Moreover, it was asked : if molecules of all substances are magnets, then why does not wood or air etc. become magnetised ?



An iron nail converts into a magnet as soon as the external magnetizing force starts acting on it

All this has been explained satisfactorily by the atom-domain theory which has superseded the molecular theory. It is beyond the scope of this book to go into the details of this theory. The interested reader is advised to refer to some standard book on magnetism. However, it may just be mentioned that this theory takes into account not only the planetary motion of an electron but its rotation about its own axis as well. This latter rotation is called 'electron spin'. The gyroscopic behaviour of an electron gives rise to a magnetic moment which may be either positive or negative. A substance is ferromagnetic or diamagnetic accordingly as there is an excess of unbalanced positive spins or negative spins. Substances like wood or air are non-magnetisable because in their case, the positive and negative electron spins are equal, hence they cancel each other out.

Curie Point

As a magnetic material is heated, its molecules vibrate more violently. As a consequence, individual molecular magnets get out of alignment as the temperature is increased, thereby reducing the magnetic strength of the magnetised substance. Fig. 6.7 shows the approximate decrease of magnetic strength with rise in temperature. Obviously, it is possible to partially or even completely destroy the magnetic properties of a material by heating. The temperature at which the vibrations of the molecular magnets become so random

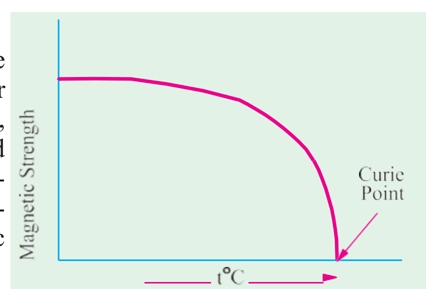


Fig. 6.7

and out of alignment as to reduce the magnetic strength to zero is called Curie point. More accurately, it is that critical temperature above which ferromagnetic material becomes paramagnetic.

ELECTROMAGNETISM

6.14 . Force on a Current-carrying Conductor Lying in a Magnetic Field

It is found that whenever a current-carrying conductor is placed in magnetic field, it experiences a force which acts in a direction perpendicular both to the direction of the current and the field. In Fig. 6.8 is shown a conductor *XY* lying at right angles to the uniform horizontal field of flux density *B* Wb/m^2 produced by two solenoids *A* and *B*. If *l* is the length of the conductor lying within this field and *I* ampere the current carried by it, then the magnitude of the force experienced by it is

$$F = BIl = \mu_0 \mu_r HIl \text{ newton}$$

Using vector notation $\vec{F} = I\,\vec{l} \times \vec{B}$ and $F = IlB \sin\theta$ where θ is the angle between $\vec{l}$ and $\vec{B}$ which is 90º in the present case

or $$F = Il\,B \sin 90^\circ = Il\,B \text{ newtons} \qquad (\because \sin 90^\circ = 1)$$

The direction of this force may be easily found by Fleming's left-hand rule.

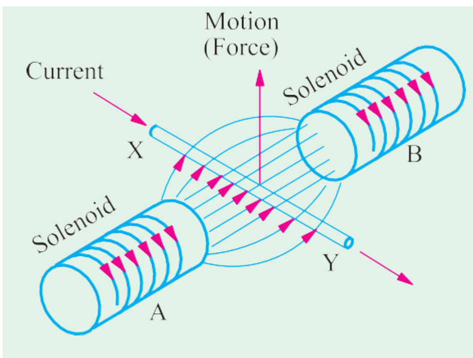


Fig. 6.8

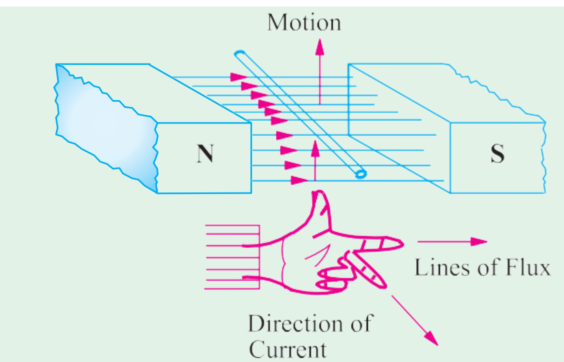


Fig. 6.9

Hold out your left hand with forefinger, second finger and thumb at right angles to one another. If the forefinger represents the direction of the field and the second finger that of the current, then thumb gives the direction of the motion. It is illustrated in Fig. 6.9.

Fig. 6.10 shows another method of finding the direction of force acting on a current carrying conductor. It is known as Flat Left Hand rule. The force acts in the direction of the thumb obviously, the direction of motor of the conductor is the same as that of the force.

It should be noted that no force is exerted on a conductor when it lies parallel to the magnetic field. In general, if the conductor lies at an angle θ with the direction of the field, then *B* can be resolved into two components, $B \cos\theta$ parallel to and $B \sin\theta$ perpendicular to the conductor. The former produces no effect whereas the latter is responsible for the motion observed. In that case,

$F = BIl \sin\theta$ newton, which has been expressed as cross product of vector above.*

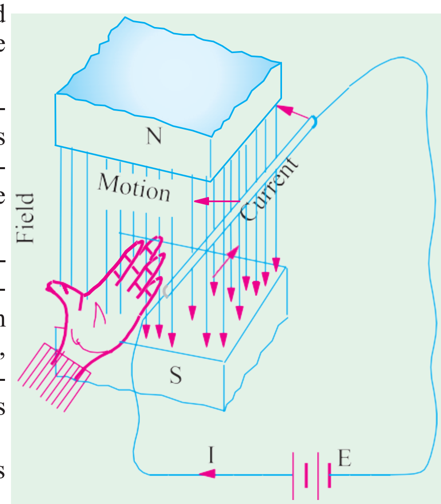


Fig. 6.10

* It is simpler to find direction of Force (Motion) through cross product of given vectors $I\,\vec{l}$ and $\vec{B}$.

Ampere's Work Law or Ampere's Circuital Law

The law states that m.m.f.* (magnetomotive force corresponding to e.m.f. *i.e*. electromotive force of electric field) around a closed path is equal to the current enclosed by the path. Mathematically, $\int \vec{H} \cdot d\vec{s} = I$ amperes where H is the vector representing magnetic field strength in dot product with vector $d\vec{s}$ of the enclosing path S around current I ampere and that is why line integral (“) of dot product $\vec{H} \cdot d\vec{s}$ is taken.

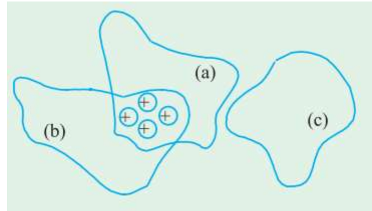


Fig. 6.11

Work law is very comprehensive and is applicable to all magnetic fields whatever the shape of enclosing path *e.g*. (*a*) and (*b*) in Fig. 6.11. Since path *c* does not enclose the conductor, the m.m.f. around it is zero.

The above work Law is used for obtaining the value of the magnetomotive force around simple idealized circuits like **(*i*)** a long straight current-carrying conductor and **(*ii*)** a long solenoid.

(i) Magnetomotive Force around a Long Straight Conductor

In Fig. 6.12 is shown a straight conductor which is assumed to extend to infinity in either direction. Let it carry a current of I amperes upwards. The magnetic field consists of circular lines of force having their plane perpendicular to the conductor and their centres at the centre of the conductor.

Suppose that the field strength at point C distant r metres from the centre of the conductor is H. Then, it means that if a unit N-pole is placed at C, it will experience a force of H newtons. The direction of this force would be tangential to the circular line of force passing through C. If this unit N-pole is moved once round the conductor *against* this force, then work done *i.e.*

m.m.f. = force × distance = I

i.e. $I = H \times 2\pi r$ joules = Amperes

or $H = \dfrac{I}{2\pi r}$

$$= \int \vec{H} \cdot d\vec{s} \text{ Joules} = \text{Amperes} = I$$

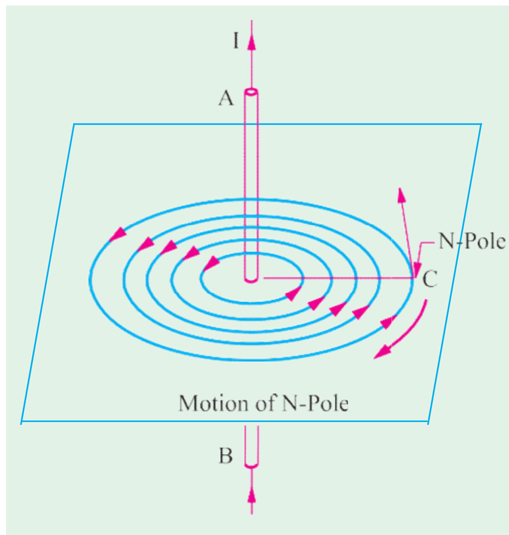


Fig. 6.12

Obviously, if there are N conductors (as shown in Fig. 6.13), then

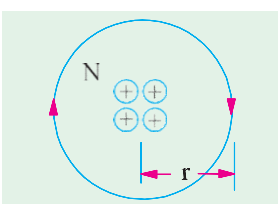


Fig. 6.13

and

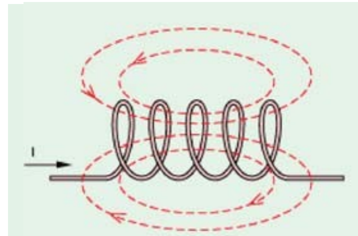
$H = \dfrac{NI}{2\pi r}$ A/m or Oersted

$B = \mu_0 \dfrac{NI}{2\pi r}$ Wb/m^2 tesla ...in air

$= \dfrac{\mu_0 \mu_r NI}{2\pi r}$ Wb/m^2 tesla ...in a medium

** M.M.F. is not a force, but is the work done.

(ii) Magnetic Field Strength of a Long Solenoid



Magnetic field around a coil carrying electric current

Let the Magnetic Field Strength along the axis of the solenoid be H. Let us assume that

(i) the value of H remains constant throughout the length l of the solenoid and

(ii) the volume of H outside the solenoid is negligible.

Suppose, a unit N-pole is placed at point A outside the solenoid and is taken once round the completed path (shown dotted in Fig. 6.14) in a direction opposite to that of H. Remembering that the force of H newtons acts on the N-pole only over the length l (it being negligible elsewhere), the work done in one round is

$= H \times l$ joules = Amperes

The 'ampere-turns' linked with this path are NI where N is the number of turns of the solenoid and I the current in amperes passing through it. According to Work Law

$$H \times l = NI \quad \text{or } H = \frac{NI}{l} \text{ A/m or Oersted}$$

$$\text{Also } B = \frac{\mu_0 NI}{l} \text{ Wb/m}^2 \quad \text{or tesla ...in air}$$

$$= \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2 \text{ or tesla ...in a medium}$$

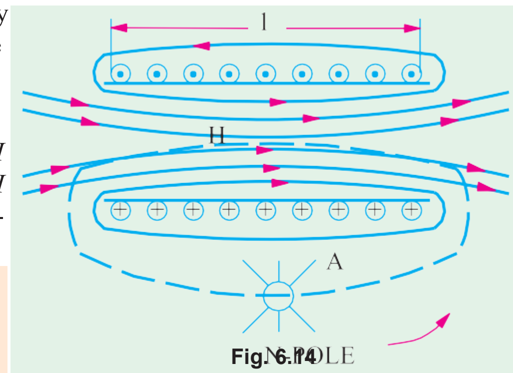


Fig. 6.14

Biot-Savart Law*

The expression for the magnetic field strength dH produced at point P by a vanishingly small length dl of a conductor carrying a current of I amperes (Fig. 6.15) is given by

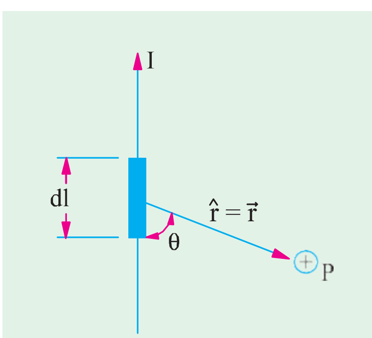


Fig. 6.15

$$dH = \frac{Idl \sin\theta}{4\pi r^2} \text{ A/m}$$

$$\text{or} \quad \vec{dH} = (Id\vec{l} \times \hat{r}) / 4\pi r^2 \text{ in vector form}$$

The direction of $\vec{dH}$ is perpendicular to the plane containing both '$\vec{dl}$' and $\vec{r}$ *i.e.* entering.

$$\text{or} \quad dB_0 = \frac{\mu_0 Idl}{4\pi r^2} \sin\theta \text{ Wb/m}^2$$

$$\text{and} \quad \vec{dB_0} = \frac{\mu_0 I \vec{dl} \times \hat{r}}{4\pi r^2} \text{ in vector form}$$

Applications of Biot-Savart Law

(i) Magnetic Field Strength Due to a Finite Length of Wire Carrying Current

Consider a straight wire of length l carrying a steady current I. We wish to find magnetic field strength (H) at a point P which is at a distance r from the wire as shown in Fig. 6.16.

* After the French mathematician and physicist Jean Baptiste Biot (1774-1862) and Felix Savart (1791-1841) a well-known French physicist.

The magnetic field strength $\vec{dH}$ due to a small element dl of the wire shown is

$$\vec{dH} = \frac{I\,\vec{dl} \times \vec{s}}{4\pi s^2}\,\hat{u} \quad \text{(By Biot-Savart Law)}$$

or $$\vec{dH} = \frac{Idl \sin\theta}{4\pi \times s^2}\,\hat{u}$$ (where $\hat{u}$ is unit vector perpendicular to plane containing $\vec{dl}$ and $\hat{s}$ and into the plane.)

or $$\vec{dH} = \frac{Idl \cos\phi}{4\pi s^2}\,\hat{u}$$ ...[$\because$ θ and ϕ are complementary angles]

The magnetic field strength due to entire length l:

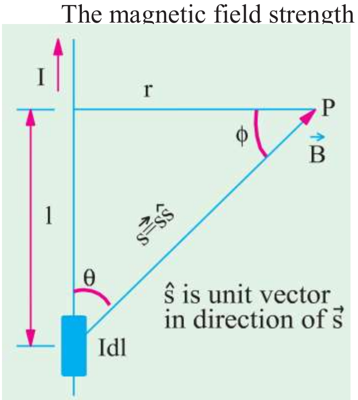


Fig. 6.16

$$\vec{H} = \frac{I}{4\pi}\int_0^l \frac{\cos\phi\,dl}{s^2}\,\hat{u} \qquad \left(\because \cos\phi = \frac{r}{s} \text{ in Fig. 6.16}\right)$$

$$= \frac{I}{4\pi}\int_0^l \frac{r/s}{s^2}\,dl\,\hat{u}$$

$$= \frac{Ir}{4\pi}\int_0^l \frac{dl}{s^3}\,\hat{u} = \frac{Ir}{4\pi}\int_0^l \frac{dl}{(r^2+l^2)^{3/2}}\,\hat{u}$$

($\because$ r is constant) ; $s = \sqrt{r^2+l^2}$ in Fig. 6.16

$$= \frac{Ir}{4\pi r^3}\int_0^l \frac{dl}{[1+(r/l)^2]^{3/2}}\,\hat{u} \quad \text{(Taking } r^3 \text{ out from denominator)}$$

To evaluate the integral most simply, make the following substitution

$$\frac{l}{r} = \tan\phi \text{ in Fig. 6.16}$$

$\therefore$ $l = r\tan\phi$ $\therefore dl = r\sec^2\phi\, d\phi$ and $1 + (r/l)^2 = 1 + \tan^2\phi = \sec^2\phi$ and limits get transformed *i.e.* become 0 to ϕ.

$$\vec{H} = \frac{Ir}{4\pi r^3}\int_0^{\phi}\frac{r\sec^2\phi}{\sec^2\phi}\,d\phi\,\hat{u} = \frac{Ir^2}{4\pi r^3}\int_0^{\phi}\cos\phi\, d\phi\,\hat{u} = \frac{1}{4\pi r}\left|\sin\phi\right|_0^{\phi}\hat{u}$$

$$= \frac{I}{4\pi r}\sin\phi\,\hat{u}$$

N.B. For wire of infinite length extending it at both ends *i.e.* $-\infty$ to $+\infty$ the limits of integration would be $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$, giving $\vec{H} = \frac{I}{4\pi r} \times 2\hat{u}$ or $\vec{H} = \frac{I}{2\pi r}\,\hat{u}$.

(ii) Magnetic Field Strength along the Axis of a Square Coil

This is similar to (*i*) above except that there are four conductors each of length say, $2a$ metres and carrying a current of I amperes as shown in Fig. 6.17. The Magnetic Field Strengths at the axial point P due to the opposite sides ab and cd are H_{ab} and H_{cd} directed at right angles to the planes containing P and ab and P and cd respectively. Now, H_{ab} and H_{cd} are numerically equal,

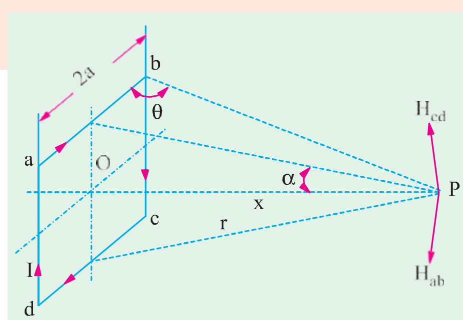


Fig. 6.17

hence their components at right angles to the axis of the coil will cancel out, but the axial components will add together. Similarly, the other two sides *da* and *bc* will also give a resultant axial component only.

As seen from Eq. (*ii*) above,

$$H_{ab} = \frac{I}{4\pi r}[\cos\theta - \cos(180^\circ - \theta)] = \frac{I \,.\, 2\cos\theta}{4\pi r} = \frac{I\cos\theta}{2\pi r}$$

Now $\quad r = \sqrt{a^2 + x^2} \qquad \therefore H_{ab} = \dfrac{I \,.\, \cos\theta}{2\pi\sqrt{a^2+x^2}}$

Its axial components is $\quad H_{ab} = H_{ab} \,.\, \sin\alpha = \dfrac{I\cos\theta}{2\pi\sqrt{a^2+x^2}} \,.\, \sin\alpha$

All the four sides of the rectangular coil will contribute an equal amount to the resultant magnetic field at *P*. Hence, resultant magnetising force at *P* is

$$H = 4 \times \frac{I\cos\theta}{2\pi\sqrt{a^2+x^2}} \,.\, \sin\alpha,$$

Now $\quad \cos\theta = \dfrac{a}{\sqrt{(2a^2+x^2)}}$ and $\sin\alpha = \dfrac{a}{\sqrt{a^2+x^2}}$

$$\therefore \quad H = \frac{2a^2 \,.\, I}{\pi(a^2+x^2) \,.\, \sqrt{x^2+2a^2}} \text{ AT/m.}$$

In case, value of *H* is required at the centre *O* of the coil, then putting $x = 0$ in the above expression,

we get $\quad H = \dfrac{2a^2 \,.\, I}{\pi a^2 \sqrt{2} \,.\, a} = \dfrac{\sqrt{2} \,.\, I}{\pi a}$ AT/m

Note. The last result can be found directly as under. As seen from Fig. 6.18, the field at point *O* due to any side is, as given by Eq. (*i*)

$$= \frac{I}{4\pi a}\int_{\pi/4}^{-\pi/4} \sin\theta \,.\, d\theta = \frac{I}{4\pi a}\Big|-\cos\theta\Big|_{45^\circ}^{-45^\circ} = \frac{I}{4\pi a} \,.\, 2\cos 45^\circ = \frac{I}{4\pi a} \,.\, \frac{2}{\sqrt{2}}$$

Resultant magnetising force due to all sides is

$$H = 4 \times \frac{1}{4\pi a} \,.\, \frac{2}{\sqrt{2}} = \frac{\sqrt{2}\, I}{\pi a} \text{ AT/m} \qquad \text{...as found above}$$

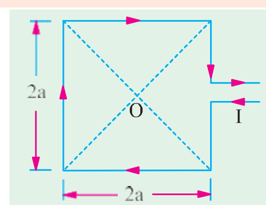


Fig. 6.18

(iii) Magnetising Force on the Axis of a Circular Coil

In Fig. 6.19 is shown a circular one-turn coil carrying a current of *I* amperes. The magnetising force at the axial point *P* due to a small element '*dl*' as given by Laplace's Law is

$$\left|\vec{dH}\right| = \frac{I dl}{4\pi(r^2 + x^2)}$$

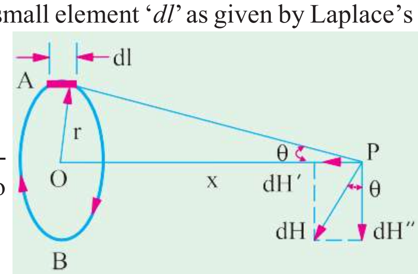


Fig. 6.19

The direction of *dH* is at right angles to the line *AP* joining point *P* to the element '*dl*'. Now, *dH* can be resolved into two components :

- the axial component $dH' = dH \sin\theta$
- the vertical component $dH'' = dH \cos\theta$

Now, the vertical component $dH \cos\theta$ will be cancelled by an equal and opposite vertical component of *dH* due to element '*dl*' at point *B*. The same applies to all other diametrically opposite pairs of *dl*'s taken around the coil. Hence, the resultant magnetising force at *P* will be equal to the sum of all the axial components.

$$\therefore \quad H = \sum dH' = \sum dH \sin\theta = \int \frac{I\,.\,dl\,.\,r}{4\pi(r^2+x^2)^{3/2}} \qquad \left(\because \sin\theta = \frac{r}{\sqrt{r^2+x^2}}\right)$$

$$= \frac{I\,.\,r}{4\pi(r^2+x^2)^{3/2}} \int_0^{2\pi r} dl = \frac{I\,.\,r\,.\,2\pi r}{4\pi(r^2+x^2)^{3/2}} = \frac{Ir^2}{2(r^2+x^2)^{3/2}}$$

$$= \frac{I}{2r} \cdot \frac{r^3}{(r^2+x^2)^{3/2}} \qquad \therefore \; H = \frac{I\sin^3\theta}{2r} \text{ AT/m}$$

or $$H = \frac{NI}{2r}\sin^3\theta \text{ AT/m} \qquad \text{–for an } N\text{-turn coil} \qquad ...(iii)$$

In case the value of H is required at the centre O of the coil, then putting $\theta = 90°$ and $\sin\theta = 1$ in the above expression, we get

$$H = \frac{I}{2r} \text{ – for single-turn coil} \quad \text{or} \quad H = \frac{NI}{2r} \text{ –for } N\text{-turn coil}$$

Note. The magnetising force H at the centre of a circular coil can be directly found as follows :

With reference to the coil shown in the Fig. 6.20, the magnetising force dH produced at O due to the small element dl (as given by Laplace's law) is

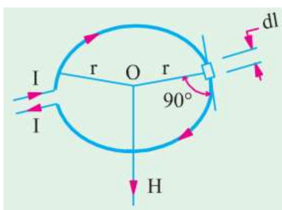


Fig. 6.20

$$dH = \frac{I\,.\,dl\sin\theta}{4\pi r^2} = \frac{I\,.dl}{4\pi r^2} \qquad (\because \sin\theta = \sin 90° = 1)$$

$$\therefore \sum dH = \sum \frac{I\,.\,dl}{4\pi r^2} = \frac{I}{4\pi r^2}\sum dl \qquad \text{or } H = \frac{I\,.\,2\pi r}{4\pi r^2} = \frac{I}{2r}$$

$$\therefore \quad H = \frac{I}{2r} \text{ AT/m –for 1-turn coil ; } \frac{NI}{2r} \text{ AT/m –for } N\text{-turn coil.}$$

(iv) Magnetising Force on the Axis of a Short Solenoid

Let a short solenoid having a length of l and radius of turns r be uniformly wound with N turns each carrying a current of I as shown in Fig. 6.21. The winding density *i.e.* number of turns per unit length of the solenoid is N/l. Hence, in a small element of length dx, there will be $N.dx/l$ turns. Obviously, a very short element of length of the solenoid can be regarded as a concentrated coil of very short axial length and having $N.dx/l$ turns. Let dH be the magnetising force contributed by the element dx at any axial point P. Then, substituting dH for H and $N.dx/l$ for N in Eq. ***(iii)***, we get

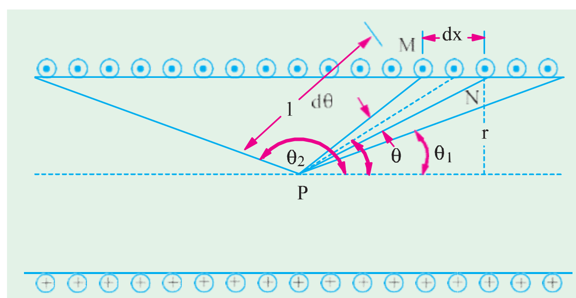


Fig. 6.21

$$dH = \frac{N\,.\,dx}{l}\,.\,\frac{I}{2r}\,.\,\sin^3\theta$$

Now $\qquad dx\,.\,\sin\theta = r\,.\,d\theta/\sin\theta$* $\;\therefore\; dx = r\,.\,d\theta/\sin^2\theta$

Substituting this value of dx in the above equation, we get

$$dH = \frac{NI}{2\,l}\sin\theta\,.\,d\theta$$

Total value of the magnetising force at P due to the whole length of the solenoid may be found by integrating the above expression between proper limits.

* Because $l\sin\theta = r \;\therefore l = r/\sin\theta$. Now, $M/N = l.d\theta = r\,d\theta/\sin\theta$. Also, $MN = dx$, $\sin\theta$, hence $dx = r\,d\theta/\sin^2\theta$.

$$\therefore \qquad H = \frac{NI}{2l}\int_{\theta_1}^{\theta_2} \sin\theta \,.\, d\theta = \frac{NI}{2l}\Big|-\cos\theta\Big|_{\theta_1}^{\theta_2}$$

$$= \frac{NI}{2l}(\cos\theta_1 - \cos\theta_2) \qquad \ldots(iv)$$

The above expression may be used to find the value of H at any point of the axis, either inside or outside the solenoid.

(i) At mid-point, $\theta_2 = (\pi - \theta_1)$, hence $\cos\theta_2 = -\cos\theta_1$

$$\therefore \qquad H = \frac{2NI}{2l}\cos\theta_1 = \frac{NI}{l}\cos\theta_1$$

Obviously, when the solenoid is very long, $\cos\theta_1$ becomes nearly unity. In that case,

$$H = \frac{NI}{l}\ \text{AT/m} \qquad \text{–Art. 6.15 } (ii)$$

(ii) At any point on the axis inside a very *long* solenoid but not too close to either end, $\theta_1 \cong 0$ and $\theta_2 \cong \pi$ so that $\cos\theta_1 \cong 1$ and $\cos\theta_2 = -1$. Then, putting these values in Eq. (*iv*) above, we have

$$H \cong \frac{NI}{2l} \times 2 = \frac{NI}{l}$$

It proves that inside a very long solenoid, H is practically constant at all axial points excepts those lying too close to either end of the solenoid.

(iii) Towards either end of the solenoid, H decreases and exactly at the ends, $\theta_1 = \pi/2$ and $\theta_2 \cong \pi$, so that $\cos\theta_1 = 0$ and $\cos\theta_2 = -1$.Hence, from Eq. (*iv*) above, we get

$$H = \frac{NI}{2l}$$

In other words, value of H is decreased to half the normal value well inside the solenoid.

Example 6.2. *Calculate the magnetising force and flux density at a distance of 5 cm from a long straight circular conductor carrying a current of 250 A and placed in air. Draw a curve showing the variation of B from the conductor surface outwards if its diameter is 2 mm.*

Solution. As seen from Art. 6.15 (*i*),

$$H = \frac{I}{2\pi r} = \frac{250}{2\pi \times 0.05} = \mathbf{795.6\ AT/m}$$

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 795.6 = \mathbf{10^{-3}\ Wb/m^2}$$

In general, $B = \dfrac{\mu_0 I}{2\pi r}$

Now, at the conductor surface, $r = 1\ \text{mm} = 10^{-3}$ m

$$\therefore \qquad B = \frac{4\pi \times 10^{-7} \times 250}{2\pi \times 10^{-3}} = \mathbf{0.05\ Wb/m^2}$$

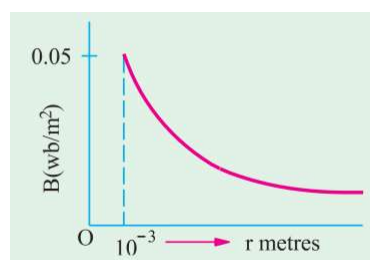


Fig. 6.22

The variation of B outside the conductor is shown in Fig. 6.22.

Example 6.3. *A wire 2.5 m long is bent (i) into a square and (ii) into a circle. If the current flowing through the wire is 100 A, find the magnetising force at the centre of the square and the centre of the circle.* **(Elec. Measurements; Nagpur Univ. 1992)**

Solution. (*i*) Each side of the square is $2a = 2.5/4 = 0.625$ m

Value of H at the centre of the square is [Art 6.17 (*ii*)]

$$= \frac{2\sqrt{2}\,I}{\pi a} = \frac{2\sqrt{2} \times 100}{\pi \times 0.3125} = \mathbf{144\ AT/m}$$

(*ii*) $2\pi r = 2.5$; $r = 0.398$ m

Value of H at the centre is $= I/2r = 100/2 \times 0.398 = \mathbf{125.6\ AT/m}$

Example 6.4. *A current of 15 A is passing along a straight wire. Calculate the force on a unit magnetic pole placed 0.15 metre from the wire. If the wire is bent to form into a loop, calculate the diameter of the loop so as to produce the same force at the centre of the coil upon a unit magnetic pole when carrying a current of 15 A.* **(Elect. Engg. Calcutta Univ.)**

Solution. By the force on a unit magnetic pole is meant the magnetising force H.

For a straight conductor [Art 6.15 (*i*)] $H = I/2\,\pi\,r = 15/2\pi \times 0.15 = 50/\pi$ AT/m

Now, the magnetising force at the centre of a loop of wire is [Art. 6.17 (***iii***)]

$$= I/\,2r = I/D = 15/D \text{ AT/m}$$

Since the two magnetising forces are equal

$\therefore \qquad 50/\pi = 15/D;\ D = 15\,\pi/50 = 0.9426 \text{ m} =$ **94.26 cm.**

Example. 6.5. *A single-turn circular coil of 50 m. diameter carries a direct current of 28×10^4 A. Assuming Laplace's expression for the magnetising force due to a current element, determine the magnetising force at a point on the axis of the coil and 100 m. from the coil. The relative permeability of the space surrounding the coil is unity.*

Solution. As seen from Art 6.17 (*iii*), $H = \dfrac{I}{2r}\,.\,\sin^3\theta$ AT/m

Here $\qquad \sin\theta = \dfrac{r}{\sqrt{r^2 + x^2}} = \dfrac{25}{\sqrt{25^2 + 100^2}} = 0.2425$

$$\sin^3\theta = (0.2425)^3 = 0.01426 \quad \therefore\ H = \frac{28\ 10^4}{2\ 25}\ 0.01426 \quad \mathbf{76.8\ AT/m}$$

Force Between Two Parallel Conductors

(i) Currents in the same direction. In Fig. 6.23 are shown two parallel conductors P and Q carrying currents I_1 and I_2 amperes in the same direction *i.e.* upwards. The field strength in the space between the two conductors is decreased due to the two fields there being in opposition to each other. Hence, the resultant field is as shown in the figure. Obviously, the two conductors are attracted towards each other.

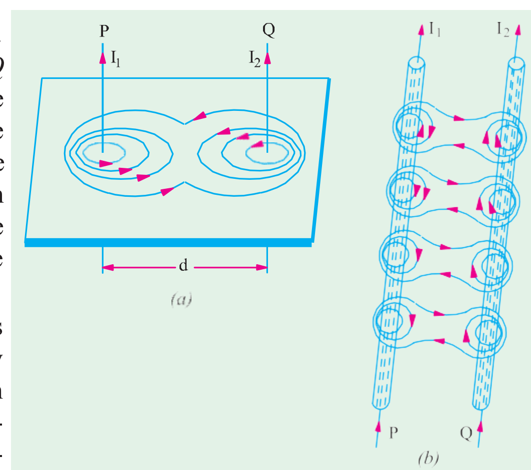


Fig. 6.23

(ii) Currents in opposite directions. If, as shown in Fig. 6.24, the parallel conductors carry currents in opposite directions, then field strength is increased in the space between the two conductors due to the two fields being in the same direction there. Because of the lateral repulsion of the lines of the force, the two conductors experience a mutual force of repulsion as shown separately in Fig. 6.24 (*b*).

Magnitude of Mutual Force

It is obvious that each of the two parallel conductors lies in the magnetic field of the other conductor. For example, conductor P lies in the magnetic field of Q and Q lies in the field of P. If 'd' metres is the distance between them, then flux density at Q due to P is [Art. 6.15 (***i***)]

$$B = \frac{\mu_0 I_1}{2\pi\,d} \text{ Wb/m}^2$$

If l is the length of conductor Q lying in this flux density, then force (either of attraction or repulsion) as given in Art. 6.14 is

$$F = BI_2 l \text{ newton} \quad \text{or} \quad F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N}$$

Obviously, conductor P will experience an equal force in the opposite direction.

The above facts are known as Laws of Parallel Currents and may be stated as follows:

(i) Two parallel conductors attract each other if currents through them flow in the *same* direction and repel each other if the currents through them flow in the *opposite* directions.

(ii) The force between two such parallel conductors is proportional to the product of current strengths and to the length of the conductors considered and varies inversely as the distance between them.

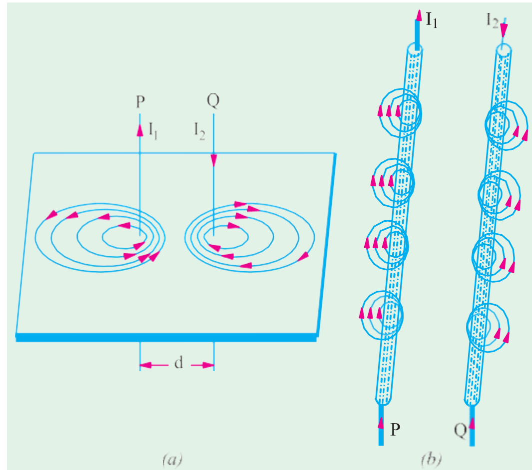


Fig. 6.24

Definition of Ampere

If has been proved in Art. 6.19 above that the force between two infinitely long parallel currently-carrying conductors is given by the expression

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N} \quad \text{or } F = \frac{4\pi \times 10^{-7} I_1 I_2 l}{2\pi d} = 2 \times 10^{-7} \frac{I_1 I_2}{d} \text{ N}$$

The force per metre run of the conductors is

$$F = 2 \times 10^{-7} \frac{I_1 I_2}{d} \text{ N/m}$$

If $I_1 = I_2 = 1$ ampere (say) and d = 1 metre, then $F = 2 \times 10^{-7}$ N

Hence, we can define one ampere current as ***that current which when flowing in each of the two infinitely long parallel conductors situated in vacuum and separated 1 metre between centres, produces on each conductor a force of 2 × 10^{-7} N per metre length***.

Example 6.6. *Two infinite parallel conductors carry parallel currents of 10 amp. each. Find the magnitude and direction of the force between the conductors per metre length if the distance between them is 20 cm.* **(Elect. Engg. Material - II Punjab Univ. May 1990)**

Solution. $$F = 2 \times 10^{-7} \frac{10 \times 10 \times 1}{0.2} \text{ N} = \mathbf{10^{-4}\ N}$$

The direction of force will depend on whether the two currents are flowing in the same direction or in the opposite direction. As per Art. 6.19, it would be a force of attraction in the first case and that or repulsion in the second case.

Example 6.7. *Two long straight parallel wires, standing in air 2 m apart, carry currents I_1 and I_2 in the same direction. The magnetic intensity at a point midway between the wires is 7.95 AT/m. If the force on each wire per unit length is 2.4 × 10^{-4} N, evaluate I_1 and I_2.*

Solution. As seen from Art. 6.17, the magnetic intensity of a long straight current-carrying conductor is

$$H = \frac{I}{2\pi r} \text{ AT/m}$$

Also, it is seen from Fig. 6.23 that when the two currents flow in the same direction, net field strength midway between the two conductors is the difference of the two field strengths.

Now, $H_1 = I_1/2\pi$ and $H_2 = I_2/2\pi$ because $r = 2/1 = 2$ metre

$$\therefore \qquad \frac{I_1}{2\pi} - \frac{I_2}{2\pi} = 7.95 \quad \therefore \; I_1 - I_2 = 50 \qquad ...(i)$$

Force per unit length of the conductors is $F = 2 \times 10^{-7} I_1 I_2/d$ newton

$$\therefore \qquad 2.4 \times 10^{-4} = 2 \times 10^{-7} I_1 I_2/2 \quad \therefore \; I_1 I_2 = 2400 \qquad ...(ii)$$

Substituting the value of I_1 from **(*i*)** in **(*ii*)**, we get

$$(50 + I_2)I_2 = 2400 \quad \text{or} \quad I_2^2 + 50I_2 - 2400 = 0$$

$$\text{or} \qquad (I_2 + 80)(I_2 - 30) = 0 \quad \therefore \; I_2 = \mathbf{30\ A} \quad \text{and} \quad I_1 = 50 + 30 = \mathbf{80\ A}$$

Tutorial Problems No. 6.1

1. The force between two long parallel conductors is 15 kg/metre. The conductor spacing is 10 cm. If one conductor carries twice the current of the other, calculate the current in each conductor. **[6,060 A; 12,120 A]**
2. A wire is bent into a plane to form a square of 30 cm side and a current of 100 A is passed through it. Calculate the field strength set up at the centre of the square. **[300 AT/m]**

(Electrotechnics - I, M.S. Univ. Baroda)

MAGNETIC CIRCUIT

Magnetic Circuit

It may be defined as the route or path which is followed by magnetic flux. The law of magnetic circuit are quite similar to (but not the same as) those of the electric circuit.

Consider a solenoid or a toroidal iron ring having a magnetic path of l metre, area of cross section A m^2 and a coil of N turns carrying I amperes wound anywhere on it as in Fig. 6.25.

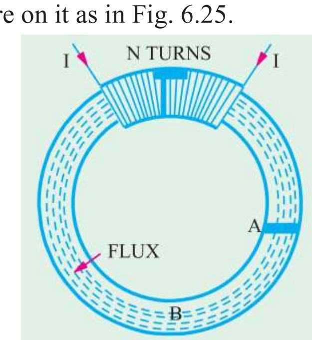


Fig. 6.25

Then, as seen from Art. 6.15, field strength inside the solenoid is

$$H = \frac{NI}{l} \text{ AT/m}$$

$$\text{Now} \qquad B = \mu_0 \mu_r H = \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2$$

Total flux produce $\Phi = B \times A = \dfrac{\mu_0 \mu_r A\, NI}{l}$ Wb

$$\therefore \qquad \boxed{\Phi = \frac{NI}{l/\mu_0 \mu_r A} \text{Wb}}$$

The numerator 'NI' which produces magnetization in the magnetic circuit is known as magnetomotive force (m.m.f.). Obviously, its unit is ampere-turn (AT)*. It is analogous to e.m.f. in an electric circuit.

The denominator $\dfrac{l}{\mu_0 \mu_r A}$ is called the ***reluctance*** of the circuit and is analogous to resistance in electric circuits.

$$\therefore \qquad \text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}} \quad \text{or} \quad \Phi = \frac{F}{S}$$

Sometimes, the above equation is called the "Ohm's Law of Magnetic Circuit" because it resembles a similar expression in electric circuits *i.e.*

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* Strictly speaking, it should be only 'ampere' because turns have no unit.

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}} \quad \text{or} \quad I = \frac{V}{R}$$

Definitions Concerning Magnetic Circuit

1. Magnetomotive force (m.m.f.). It drives or tends to drive flux through a magnetic circuit and corresponds to electromotive force (e.m.f.) in an electric circuit.

M.M.F. is equal to the work done in joules in carrying a unit magnetic pole once through the entire magnetic circuit. It is measured in ampere-turns.

In fact, as p.d. between any two points is measured by the work done in carrying a unit charge from one points to another, similarly, m.m.f. between two points is measured by the work done in joules in carrying a unit magnetic pole from one point to another.

2. Ampere-turns (AT). It is the unit of magnetometre force (m.m.f.) and is given by the product of number of turns of a magnetic circuit and the current in amperes in those turns.

3. Reluctance. It is the name given to that property of a material which opposes the creation of magnetic flux in it. It, in fact, measures the opposition offered to the passage of magnetic flux through a material and is analogous to resistance in an electric circuit even in form. Its units is **AT/Wb.***

$$\text{reluctance} = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}; \quad \text{resistance} = \rho \frac{l}{A} = \frac{l}{\sigma A}$$

In other words, the reluctance of a magnetic circuit is the number of amp-turns required per weber of magnetic flux in the circuit. Since 1 AT/Wb = 1/henry, the unit of reluctance is "reciprocal henry."

4. Permeance. It is reciprocal of reluctance and implies the case or readiness with which magnetic flux is developed. It is analogous to conductance in electric circuits. It is measured in terms of Wb/AT or henry.

5. Reluctivity. It is specific reluctance and corresponds to resistivity which is 'specific resistance'.

Composite Series Magnetic Circuit

In Fig. 6.26 is shown a composite series magnetic circuit consisting of three different magnetic materials of different permeabilities and lengths and one air gap ($\mu_r = 1$). Each path will have its own reluctance. The total reluctance is the sum of individual reluctances as they are joined in series.

$$\therefore \quad \text{total reluctance} = \Sigma \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{l_1}{\mu_0 \mu_{r_1} A_1} + \frac{l_2}{\mu_0 \mu_{r_2} A_2} + \frac{l_3}{\mu_0 \mu_{r_3} A_3} + \frac{l_a}{\mu_0 A_g}$$

$$\therefore \quad \text{flux } \Phi = \frac{\text{m.m.f.}}{\dfrac{l}{\mu_0 \mu_r A}}$$

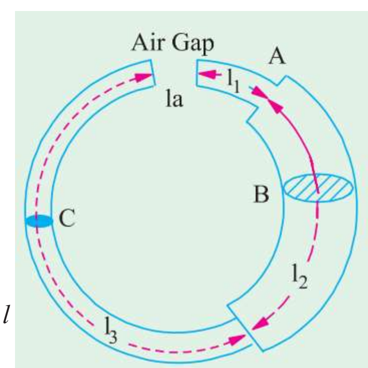


Fig. 6.26

How to Find Ampere-turns ?

It has been shown in Art. 6.15 that $H = NI/l$ AT/m or $NI = H \times l$

$\therefore$ ampere-turns AT $= H \times l$

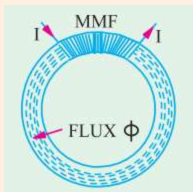
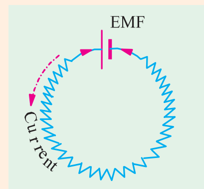
Hence, following procedure should be adopted for calculating the total ampere turns of a composite magnetic path.

* From the ratio $\Phi = \dfrac{\text{m.m.f.}}{\text{reluctance}}$, it is obvious that reluctance = m.m.f./Φ. Since m.m.f. is in ampere-turns and flux in webers, unit of reluctance is ampere-turn/weber (AT/Wb) or A/Wb.

(i) Find H for each portion of the composite circuit. For air, $H = B/\mu_0$, otherwise $H = B/\mu_0\mu_r$.
(ii) Find ampere-turns for each path separately by using the relation AT $= H \times l$.
(iii) Add up these ampere-turns to get the total ampere-turns for the entire circuit.

Comparison Between Magnetic and Electric Circuits.

SIMILARITIES

Magnetic Circuit	Electric Circuit
 Fig. 6.27	 Fig. 6.28
1. Flux $= \dfrac{\text{m.m.f.}}{\text{reluctance}}$	Current $= \dfrac{\text{e.m.f.}}{\text{resistance}}$
2. M.M.F. (ampere-turns)	E.M.F. (volts)
3. Flux Φ (webers)	Current I (amperes)
4. Flux density B (Wb/m^2)	Current density (A/m^2)
5. Reluctance $S = \dfrac{l}{\mu A} = \dfrac{l}{\mu_0 \mu_r A}$	resistance $R = \rho\dfrac{l}{A} = \dfrac{l}{\rho A}$
6. Permeance (= 1/reluctance)	Conductance (= 1/resistance)
7. Reluctivity	Resistivity
8. Permeability (= 1/reluctivity)	Conductivity (= 1/resistivity)
9. Total m.m.f. $= \Phi S_1 + \Phi S_2 + \Phi S_3 + \ldots$	9. Total e.m.f. $= IR_1 + IR_2 + IR_3 + \ldots$

DIFFERENCES

1. Strictly speaking, flux does not actually 'flow' in the sense in which an electric current flows.

2. If temperature is kept constant, then resistance of an electric circuit is constant and is independent of the current strength (or current density). On the other hand, the reluctance of a magnetic circuit does depend on flux (and hence flux density) established in it. It is so because μ (which equals the slope of B/H curve) is not constant even for a given material as it depends on the flux density B. Value of μ is large for low value of B and ***vice-versa***. Hence, reluctance is small ($S = l/\mu A$) for small values of B and large for large values of B.

3. Flow of current in an electric circuit involves continuous expenditure of energy but in a magnetic circuit, energy is needed only creating the flux initially but not for maintaining it.

Parallel Magnetic Circuits

Fig. 6.29 (a) shown a parallel magnetic circuit consisting of two parallel magnetic paths ACB and ADB acted upon by the same $m.m.f.$ Each magnetic path has an average length of $2(l_1 + l_2)$. The flux produced by the coil wound on the central core is divided equally at point A between the two outer parallel paths. The reluctance offered by the two parallel paths is = half the reluctance of each path.

Fig. 6.29 (b) shows the equivalent electrical circuit where resistance offered to the voltage source is $= R \parallel R = R/2$

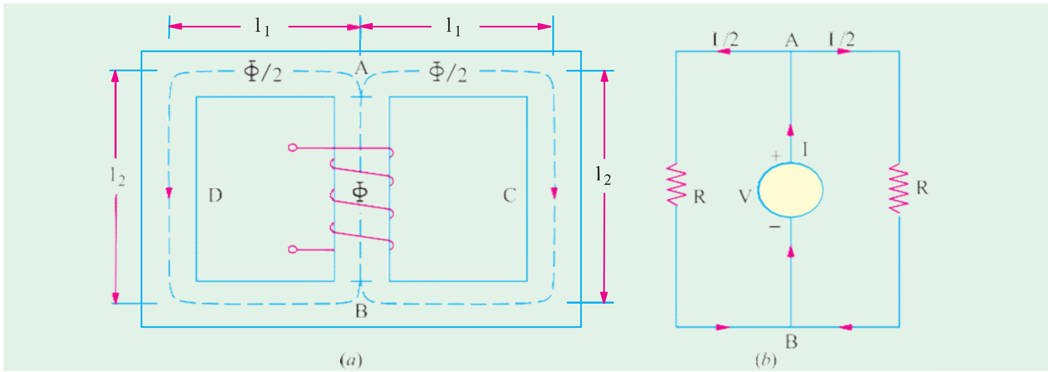


Fig. 6.29

It should be noted that reluctance offered by the central core AB has been neglected in the above treatment.

Series-Parallel Magnetic Circuits

Such a circuit is shown in Fig. 6.30 (*a*). It shows two parallel magnetic circuits *ACB* and *ACD* connected across the common magnetic path *AB* which contains an air-gap of length l_g. As usual, the flux Φ in the common core is divided equally at point *A* between the two parallel paths which have equal reluctance. The reluctance of the path *AB* consists of ***(i)*** air gap reluctance and ***(ii)*** the reluctance of the central core which comparatively negligible. Hence, the reluctance of the central core *AB* equals only the air-gap reluctance across which are connected two equal parallel reluctances. Hence, the *m.m.f.* required for this circuit would be the sum of ***(i)*** that required for the air-gap and ***(ii)*** that required for either of two paths (not both) as illustrated in Ex. 6.19, 6.20 and 6.21.

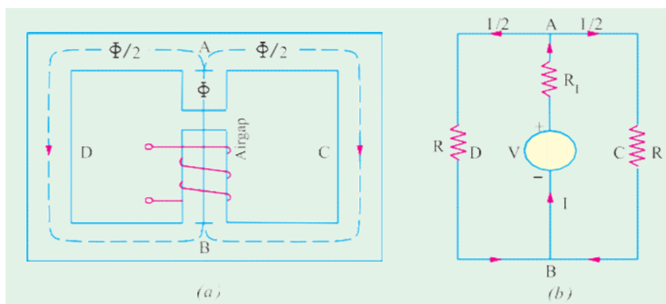


Fig. 6.30

The equivalent electrical circuit is shown in Fig. 6.30 (*b*) where the total resistance offered to the voltage source is = $R_1 + R$æ$R = R_1 + R/2$.

Leakage Flux and Hopkinson's Leakage Coefficient

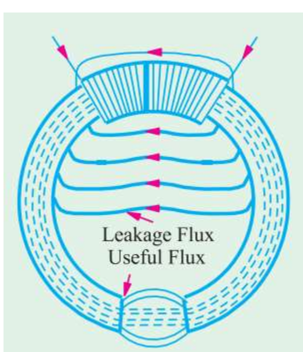


Fig. 6.31

Leakage flux is the flux which follows a path not intended for it. In Fig. 6.31 is shown an iron ring wound with a coil and having an air-gap. The flux in the air-gap is known as the useful flux because it is only this flux which can be utilized for various useful purposes.

It is found that it is impossible to confine all the flux to the iron path only, although it is usually possible to confine most of the electric current to a definite path, say a wire, by surrounding it with insulation. Unfortunately, there is no known insulator for magnetic flux. Air, which is a splendid insulator of electricity, is unluckily a fairly good magnetic conductor. Hence, as shown, some of the flux leaks through air surrounding the iron ring. The presence of leakage flux can be detected by a compass. Even in the best designed dynamos, it is found that 15 to 20% of the total flux produced leaks away without being utilised usefully.

If, Φ_t = total flux produced ; Φ = useful flux available in the air-gap, then

$$\text{leakage coefficient } \lambda = \frac{\text{total flux}}{\text{useful flux}} \quad \text{or } \lambda = \frac{\Phi_t}{\Phi}$$

In electric machines like motors and generators, magnetic leakage is undesirable, because, although it does not lower their power efficiency, yet it leads to their increased weight and cost of manufacture. Magnetic leakage can be minimised by placing the exciting coils or windings as close as possible to the air-gap or to the points in the magnetic circuit where flux is to be utilized for useful purposes.

It is also seen from Fig. 6.31 that there is fringing or spreading of lines of flux at the edges of the air-gap. This fringing increases the effective area of the air-gap.

The value of λ for modern electric machines varies between 1.1 and 1.25.

Magnetisation Curves

The approximate magnetisation curves of a few magnetic materials are shown in Fig. 6.32.

These curves can be determined by the following methods provided the materials are in the form of a ring :

(a) By means of a ballistic galvanometer and **(*b*)** By means of a fluxmeter.

Magnetisation Curves by Ballistic Galvanometer

In Fig. 6.33 shown the specimen ring of uniform cross-section wound uniformly with a coil *P* which is connected to a battery *B* through a reversing switch *RS*, a variable resistance R_1 and an ammeter. Another secondary coil *S* also wound over a small portion of the ring and is connected through a resistance *R* to a ballistic galvanometer *BG*.

The current through the primary *P* can be adjusted with the help of R_1. Suppose the primary current is *I*. When the primary current is reversed by means of *RS*, then flux is reversed through *S*, hence an induced e.m.f. is produced in it which sends a current through *BG*. This current is of very short duration. The first deflection or 'throw' of the *BG* is proportional to the quantity of electricity or charge passing through it so long as the time taken for this charge to flow is short as compared with the time of one oscillation.

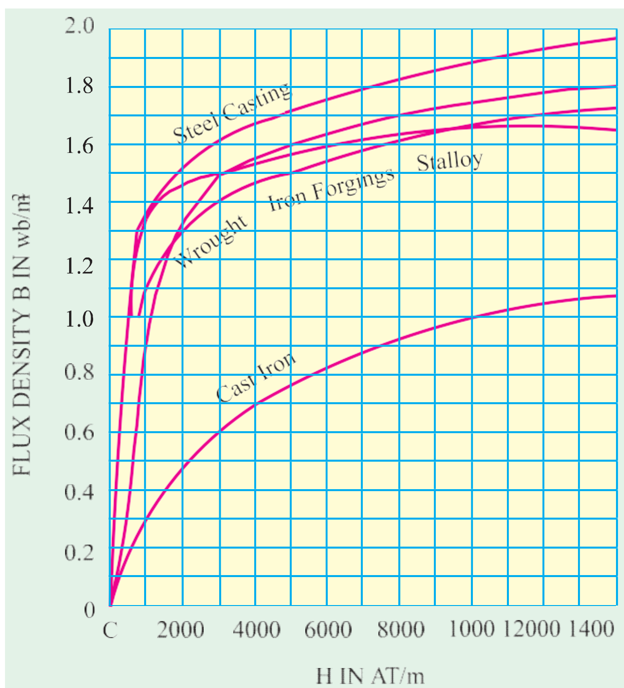


Fig. 6.32

If θ = first deflection or 'throw' of the galvanometer when primary current *I* is reversed.

k = ballistic constant of the galvanometer *i.e.* charge per unit deflection.

then, charge passing through *BG* is = $k\theta$ coulombs ...(*i*)

Let Φ= flux in Wb produced by primary current of *I* amperes ; *t* = time of reversal of flux ; then rate of change of flux = $\frac{2\Phi}{t}$ Wb/s

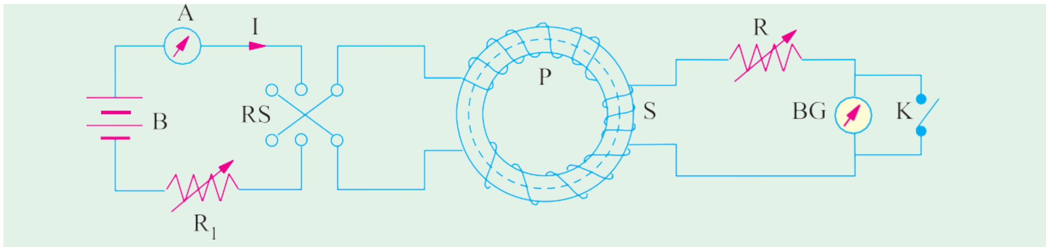


Fig. 6.33

If N_2 is the number of turns in secondary coil S, then average e.m.f. induces in it is

$$= N_2 \cdot \frac{2\Phi}{t} \text{ volt.}$$

Secondary current or current through $BG = \dfrac{2N_2\,\Phi}{R_s t}$ amperes

where R_s is the total resistance of the secondary circuit.

Charge flowing through BG = average current × time $= \dfrac{2\,N_2\,\Phi}{R_s\,t} \times t = \dfrac{2\,N_2\,\Phi}{R_s}$ coulomb ...***(ii)***

Equation ***(i)*** and ***(ii)***, we get $k\theta = \dfrac{2\,N_2\,\Phi}{R_s}$ $\quad \therefore \; \Phi = \dfrac{k\theta\, R_s}{2N_2}$ Wb

If A m^2 is the cross-sectional area of the ring, then flux density is

$$B = \frac{\Phi}{A} = \frac{k\theta\, R_s}{2N_2 A} \text{ Wb/m}^2$$

If N_1 is the number of primary turns and l metres the mean circumference of the ring, then, magnetising force $H = N_1 I/l$ AT/m.

The above experiment is repeated with different values of primary current and form the data so obtained, the B/H curves or magnetisation curves can be drawn.

Magnetisation Curves by Fluxmeter

In this method, the BG of Fig. 6.31 is replaced by a fluxmeter which is just a special type of ballistic galvanometer. When current through P is reversed, the flux is also reversed. The deflection of the fluxmeter is proportional to the change in flux-linkages of the secondary coil. If the flux is reversed from $+\Phi$ to $-\Phi$, the change in flux-linkages in secondary S in $= 2\,\Phi\,N_2$.

If $\quad \theta$ = corresponding deflection of the fluxmeter

$\quad C$ = fluxmeter constant *i.e.* weber-turns per unit deflection.

then, $\quad$ change of flux-linkages in $S = C\,\theta$

$$\therefore \quad 2\Phi\,N_2 = C\theta \quad \text{or } \Phi = \frac{C\theta}{N_2} \text{ Wb}; \quad B = \frac{\Phi}{A} = \frac{C\theta}{2N_2 A} \text{ Wb/m}^2$$

Example 6.8. *A fluxmeter is connected to a search-coil having 600 turns and mean area of 4 cm*^{2}*. The search coil is placed at the centre of an air-cored solenoid 1 metre long and wound with 1000 turns. When a current of 4 A is reversed, there is a deflection of 20 scale divisions on the fluxmeter. Calculate the calibration in Wb-turns per scale division.*

(Measurements-I, Nagpur Univ. 1991)

Solution. Magnetising force of the solenoid is $H = NI/l$ AT/m

$B = \mu_0 H = \mu_0 NI/l = 4\pi \times 10^{-7} \times 1000 \times 4/1 = 16\pi \times 10^{-4}$ Wb/m^2

Flux linked with the search coil is $\Phi = BA = 64\pi \times 10^{-8}$ Wb

Total change of flux-linkages on reversal

$= 2 \times 64\pi \times 10^{-8} \times 600$ Wb-turns –Art. 6.29

$= 7.68\pi \times 10^{-4}$ Wb-turns

Fluxmeter constant C is given by $= \dfrac{\text{Change in flux-linkages}}{\text{deflection produced}}$

$= 7.68\pi \times 10^{-4}/20 =$ **1.206×10^{-4} Wb-turns/division**

Example 6.9. *A ballistic galvanometer, connected to a search coil for measuring flux density in a core, gives a throw of 100 scale divisions on reversal of flux. The galvanometer coil has a resistance of 180 ohm. The galvanometer constant is 100 μC per scale division. The search coil has an area of 50 cm², wound with 1000 turns having a resistance of 20 ohm. Calculate the flux density in the core.* **(Elect. Instru & Measu. Nagpur Univ. 1992)**

Solution. As seen from Art. 6.28.

$$k\theta = 2N_2\Phi/R_s \quad \text{or } \Phi = k\theta R_s/2N_2 \text{ Wb}$$

$\therefore$ $$BA = k\theta R_s/2N_2 \quad \text{or } B = k\theta R_s/2N_2A$$

Here $k = 100\ \mu\text{C/division} = 100 \times 10^{-6} = 10^{-4}$ C/division

$\theta = 100;\ A = 50\text{ cm}^2 = 5 \times 10^{-3}\text{ m}^2$

$R_s = 180 + 20 = 200\ \wedge$

$\therefore$ $B = 10^{-4} \times 100 \times 200/2 \times 1000 \times 5 \times 10^{-3} =$ **0.2 Wb/m²**

Example 6.10. *A ring sample of iron, fitted with a primary and a secondary winding is to be tested by the method of reversals to obtain its B/H curve. Give a diagram of connections explain briefly how the test could be carried out.*

In such a test, the primary winding of 400 turns carries a current of 1.8 A. On reversal, a change of 8 × 10⁻³ Wb-turns is recorded in the secondary winding of 10 turns. The ring is made up of 50 laminations, each 0.5 mm thick with outer and inner diameters of 25 and 23 cm respectively. Assuming uniform flux distribution, determine the values of B, H and the permeability.

Solution. Here, change of flux-linkages $= 2\Phi N_2 = 8 \times 10^{-3}$ Wb-turns

$\therefore$ $2\Phi \times 10 = 8 \times 10^{-3}$ or $\Phi = 4 \times 10^{-4}$ Wb and $A = 2.5 \times 10^{-4}\text{m}^2$

$\therefore$ $$B = \frac{4 \times 10^{-4}}{2.5 \times 10^{-4}} = 1.6\text{ Wb/m}^2;\ H = \frac{NI}{l} = \frac{400 \times 1.8}{0.24\pi} = 955\text{ AT/m}$$

Now $$\mu_0\mu_r = \frac{B}{H};\ \mu_r = \frac{B}{\mu_0 H} = \frac{1.6}{4\pi \times 10^{-7} \times 955} = \mathbf{1333}$$

Example 6.11. *An iron ring of 3.5 cm² cross-sectional area with a mean length of 100 cm is wound with a magnetising winding of 100 turns. A secondary coil of 200 turns of wire is connected to a ballistic galvanometer having a constant of 1 micro-coulomb per scale division, the total resistance of the secondary circuit being 2000 ∧. On reversing a current of 10 A in the magnetising coil, the galvanometer gave a throw of 200 scale divisions. Calculate the flux density in the specimen and the value of the permeability at this flux density.* **(Elect. Measure, A.M.I.E Sec.B. 1992)**

Solution. Reference may please be made to Art. 6.28.

Here $N_1 = 100;\ N_2 = 200 : A = 3.5 \times 10^{-4}\text{ m}^2;\ l = 100\text{ cm} = 1\text{m}$

$k = 10^{-6}$ C/division, $\theta = 100$ divisions; $R_s = 2000\ \wedge;\ I = 10$ A

$$B = \frac{k\theta R_s}{2N_2A} = \frac{10^{-6} \times 100 \times 2000}{2 \times 200 \times 3.5 \times 10^{-4}} = 1.43\text{ Wb/m}^2$$

Magnetising force $H = N_1 I/l = 100 \times 10/1 = 1000$ AT/m

$$\mu = \frac{B}{H} = \frac{1.43}{1000} = \mathbf{1.43 \times 10^{-3}\ H/m}$$

Note. The relative permeability is given by $\mu_r = \mu / \mu_0 = 1.43 \times 10^{-3}/4\pi \times 10^{-7}$ = **1137.**

Example 6.12. *An iron ring has a mean diameter of 0.1 m and a cross-section of 33.5 × 10^{-6} m². It is wound with a magnetising winding of 320 turns and the secondary winding of 220 turns. On reversing a current of 10 A in the magnetising winding, a ballistic galvanometer gives a throw of 272 scale divisions, while a Hilbert Magnetic standard with 10 turns and a flux of 2.5 × 10^{-4} gives a reading of 102 scale divisions, other conditions remaining the same. Find the relative permeability of the specimen.* **(Elect. Measu. A.M.I.E. Sec B, 1991)**

Solution. Length of the magnetic path $l = \pi D = 0.1\ \pi$ m

Magnetising Force, $H = NI/l = 320 \times 10/0.1\ \pi = 10{,}186$ AT/m

Flux density $B = \mu_0 \mu_r \mathbf{H} = 4\pi \times 10^{-7} \times \mu_r \times 10{,}186 = 0.0128\ \mu_r$ ...(*i*)

Now, from Hilbert's Magnetic standard, we have

$$2.5 \times 10^{-4} \times 10 = K \times 102,\ K = 2.45 \times 10^{-5}$$

On reversing a current of 10 A in the magnetising winding, total change in Weber-turns is

$2\Phi N_s = 2.45 \times 10^{-5} \times 272$ or $2 \times 220 \times \Phi = 2.45 \times 10^{-5} \times 272$ or $\Phi = 1.51 \times 10^{-5}$ Wb

$\therefore\ B = \Phi/A = 1.51 \times 10^{-5}/33.5 \times 10^{-6} = 0.45$ Wb/m²

Substituting this value in Eq. (*i*), we have $0.0128\ \mu_r = 0.45$, $\therefore\ \mu_r =$ **35.1**

Example 6.13. *A laminated soft iron ring of relative permeability 1000 has a mean circumference of 800 mm and a cross-sectional area 500 mm². A radial air-gap of 1 mm width is cut in the ring which is wound with 1000 turns. Calculate the current required to produce an air-gap flux of 0.5 mWb if leakage factor is 1.2 and stacking factor 0.9. Neglect fringing.*

Solution. Total AT reqd. $= \Phi_g S_g + \Phi_i S_i = \dfrac{\Phi_g l_g}{\mu_0 A_g} + \dfrac{\Phi_i l_i}{\mu_0 \mu_r A_i B}$

Now, air-gap flux $\Phi_s = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb, $l_g = 1$ mm $= 1 \times 10^{-3}$ m; $A_g = 500$ mm² $= 500 \times 10^{-6}$ m²

Flux in the iron ring, $\Phi_i = 1.2 \times 0.5 \times 10^{-3}$ Wb

Net cross-sectional area $= A_i \times$ stacking factor $= 500 \times 10^{-6} \times 0.9$ m²

$$\therefore \text{ total AT reqd.} = \frac{0.5 \times 10^{-3} \times 1 \times 10^{-3}}{4\pi \times 10^{-7} \times 500 \times 10^{-6}} + \frac{1.2 \times 0.5 \times 10^{-3} \times 800 \times 10^{-3}}{4\pi \times 10^{-7} \times 1000 \times (0.9 \times 500 \times 10^{-6})} = 1644$$

$\therefore\ I = 1644/1000 =$ **1.64 A**

Example 6.14. *A ring has a diameter of 21 cm and a cross-sectional area of 10 cm². The ring is made up of semicircular sections of cast iron and cast steel, with each joint having a reluctance equal to an air-gap of 0.2 mm. Find the ampere-turns required to produce a flux of 8 × 10^{-4} Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect fringing and leakage effects.* **(Elect. Circuits, South Gujarat Univ.)**

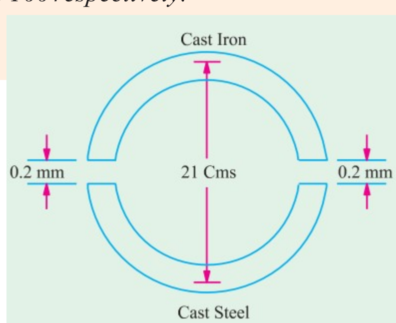


Fig. 6.34

Solution. $\Phi = 8 \times 10^{-4}$ Wb ; $A = 10$ cm² $= 10^{-3}$ m²; $B = 8 \times 10^{-4}/10^{-3} = 0.8$ Wb/m²

Air gap

$H = B/\mu_0 = 0.8/4\pi \times 10^{-7} = 6.366 \times 10^5$ AT/m

Total air-gap length $= 2 \times 0.2 = 0.4$ mm

$= 4 \times 10^{-4}$ m

$\therefore$ AT required $= H \times l = 6.366 \times 10^5 \times 4 \times 10^{-4} = 255$

Cast Steel Path (Fig. 6.34)

$H = B/\mu_0\,\mu_r = 0.8/4\pi \times 10^{-7} \times 800 = 796$ AT/m

path $= \pi\,D/2 = 21\;\pi/2 = 33$ cm $= 0.33$ m

AT required $= H \times l = 796 \times 0.33 = 263$

Cast Iron Path

$H = 0.8/\pi \times 10^{-7} \times 166 = 3{,}835$ AT/m ; path $= 0.33$ m

AT required $= 3{,}835 \times 0.33 = 1265$

Total AT required $= 255 + 263 + 1265 =$ **1783.**

Example 6.15. *A mild steel ring of 30 cm mean circumference has a cross-sectional area of 6 cm² and has a winding of 500 turns on it. The ring is cut through at a point so as to provide an air-gap of 1 mm in the magnetic circuit. It is found that a current of 4 A in the winding, produces a flux density of 1 T in the air-gap. Find* **(i)** *the relative permeability of the mild steel and* **(ii)** *inductance of the winding.* **(F.E. Engg. Pune Univ.)**

Solution. (*a*) Steel ring

$$H = B/\mu_0\,\mu_r = 1/4\pi \times 10^{-7} \times \mu_r \text{ AT/m} = 0.7957 \times 10^7/\mu_r \text{ AT/m}$$

$$\text{m.m.f.} = H \times l = (0.7957 \times 10^7/\mu_r) \times 29.9 \times 10^{-2} = 0.2379 \times 10^6/\mu_r \text{ AT}$$

(b) Air-gap

$$H = B/\mu_0 = 1/4\pi \times 10^{-7} = 0.7957 \times 10^6 \text{ AT/m}$$

$$\text{m.m.f. reqd.} = H \times l = 0.7957 \times 10^6 \times (1 \times 10^{-3}) = 795.7 \text{ AT}$$

$$\text{Total m.m.f.} = (0.2379 \times 10^6/\mu_r) + 795.7$$

Total m.m.f. available $= NI = 500 \times 4 = 2000$ AT

(*i*) $\therefore\quad 2000 = (0.2379 \times 10^6/\mu_r) + 795.7 \quad \therefore\quad \mu_r =$ **197.5**

(*ii*) Inductance of the winding $= \dfrac{N\Phi}{I} = \dfrac{NBA}{I} = \dfrac{500 \times 1 \times 6 \times 10^{-4}}{4} =$ **0.075 H**

Example 6.16. *An iron ring has a X-section of 3 cm² and a mean diameter of 25 cm. An air-gap of 0.4 mm has been cut across the section of the ring. The ring is wound with a coil of 200 turns through which a current of 2 A is passed. If the total magnetic flux is 0.24 mWb, find the relative permeability of iron, assuming no magnetic leakage.* **(Elect. Engg. A.M.Ae.S.I., June 1992)**

Solution. $\Phi = 0.24$ mWb; $A = 3$ cm² $= 3 \times 10^{-4}$ m²;

$B = \Phi/A = 0.24 \times 10^{-3}/3 \times 10^{-4} = 0.8$ Wb/m²

AT for iron ring $= H \times l = (B/\mu_0\mu_r) \times l = (0.8/4\pi \times 10^{-7} \times \mu_r) \times 0.25 = 1.59 \times 10^{-5}/\mu_r$

AT for air-gap $= H \times l = (B/\mu_0) \times l = (0.8/4\pi \times 10^{-7}) \times 0.4 \times 10^{-3} = 255$

Total AT reqd. $= (1.59 \times 10^5/\mu_r) + 255$; total AT provided $= 200 \times 2 = 400$

$\therefore\ (1.59 \times 10^5/\mu_r) + 255 = 400$ or $\mu_r =$ **1096.**

Example 6.17. *A rectangular iron core is shown in Fig. 6.35. It has a mean length of magnetic path of 100 cm, cross-section of (2 cm × 2 cm), relative permeability of 1400 and an air-gap of 5 mm cut in the core. The three coils carried by the core have number of turns $N_a = 335$, $N_b = 600$ and $N_c = 600$; and the respective currents are 1.6 A, 4 A and 3 A. The directions of the currents are as shown. Find the flux in the air-gap.* **(F.Y. Engg. Pune Univ.)**

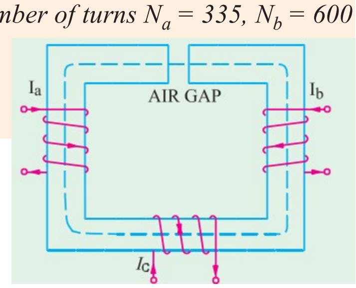


Fig. 6.35

Solution. By applying the Right-Hand Thumb rule, it is found that fluxes produced by the current I_a and I_b are directed in the clockwise direction through the iron core whereas that produced by current I_c is directed in the anticlockwise direction through the core.

$\therefore$ total m.m.f. $= N_a I_a + N_b I_b - N_c I_c = 335 \times 1.6 + 600 \times 4 - 600 \times 3 = 1136$ AT

Reluctance of the air-gap $= \dfrac{l}{\mu_0 A} = \dfrac{5 \times 10^{3}}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 9.946 \times 10^{6}$ AT/Wb

Reluctance of the iron path $= \dfrac{l}{\mu_0 \mu_r A} = \dfrac{100 - (0.5) \times 10^{-2}}{4\pi \times 10^{-7} \times 1400 \times 4 \times 10^{-4}} = 1.414 \times 10^{6}$ AT/Wb

Total reluctance $= (9.946 + 1.414) \times 10^{6} = 11.36 \times 10^{6}$ AT/Wb

The flux in the air-gap is the same as in the iron core.

Air-gap flux $= \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{1136}{11.36 \times 10^{6}} = 100 \times 10^{-6}$ Wb $=$ **100 μWb**

Example 6.18. *A series magnetic circuit comprises of three sections **(i)** length of 80 mm with cross-sectional area 60 mm², **(ii)** length of 70 mm with cross-sectional area 80 mm² and **(iii)** and air-gap of length 0.5 mm with cross-sectional area of 60 mm². Sections **(i)** and **(ii)** are if a material having magnetic characteristics given by the following table.*

H (AT/m)	*100*	*210*	*340*	*500*	*800*	*1500*
B (Tesla)	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*	*1.2*

Determine the current necessary in a coil of 4000 turns wound on section (ii) to produce a flux density of 0.7 Tesla in the air-gap. Neglect magnetic leakage. **(F.E. Pune Univ. May 1990)**

Solution. Section (*i*) It has the same cross-sectional area as the air-gap. Hence, it has the same flux density *i.e.* 0.7 Tsela as in the air-gap. The value of the magnetising force H corresponding to this flux density of 0.7 T as read from the B-H plot is 415 AT/m.

m.m.f. reqd $= H \times l = 415 \times (80 \times 10^{-3}) = 33.2$ AT

Section (*ii*) Since its cross-sectional area is different from that of the air-gap, its flux density would also be different even though, being a series circuit, its flux would be the same.

Air-gap flux $= B \times L = 0 \times (60 \times 10^{-6}) = 42 \times 10^{-6}$ Wb

Flux density in this section $= 42 \times 10^{-6}/80 \times 10^{-6} = 0.525$ T

The corresponding value of the H from the given garph is 285 AT/m

m.m.f. reqd, for this section $= 285 \times (70 \times 10^{-3}) = 19.95$ AT.

Air-gap

$H = B/\mu_0 = 0.7/4\pi \times 10^{-7} = 0.557 \times 10^{6}$ AT/m

$\therefore$ m.m.f. reqd. $= 0.557 \times 10^{-6} \times (0.5 \times 10^{-3}) = 278.5$ AT

Total m.m.f. reqd. $= 33.2 + 19.95 + 278.5 = 331.6$

$\therefore NI = 331.6$ or $I = 331.6/4000 =$ **0.083 A**

Example 6.19. *A magnetic circuit made of mild steel is arranged as shown in Fig. 6.36. The central limb is wound with 500 turns and has a cross-sectional area of 800 mm². Each of the outer limbs has a cross-sectional area of 500 mm². The air-gap has a length of 1 mm. Calculate the current rquired to set up a flux of 1.3 mWb in the central limb assuming no magnetic leakage and fringing. Mild steel required 3800 AT/m to produce flux density of 1.625 T and 850 AT/m to produce flux density of 1.3 T.* **(F.Y. Engg. Pune Univ.)**

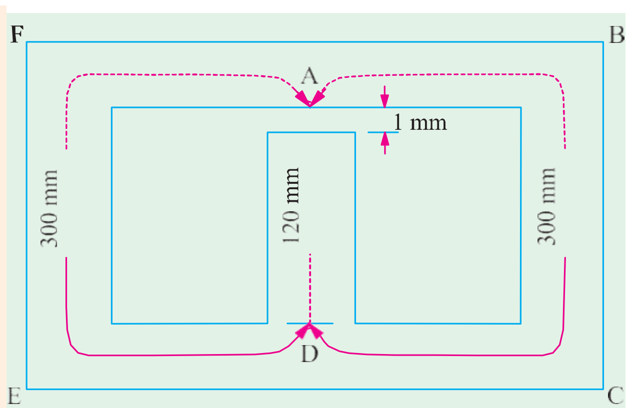


Fig. 6.36

Solution. Flux in the central limb is = 1.3 mWb = 1.3×10^{-3} Wb

Cross section $A = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{m}^2$

$\therefore$ $B = \Phi/A = 1.3 \times 10^{-6}/800 \times 10^{-6}$

$= 1.625$ T

Corresponding value of H for this flux density is given as 3800 AT/m.

Since the length of the central limb is 120 mm. m.m.f. required is = $H \times l = 3800 \times (120 \times 10^{-3})$

= 456 AT/m.

Air-gap

Flux density in the air-gap is the same as that in the central limb.

$H = B/\mu_0 = 1.625/4\pi \times 10^{-7} = 0.1293 \times 10^{-7}$ AT/m

Length of the air-gap = 1 mm = 10^{-3} m

m.m.f. reqd. for the air-gap = $H \times l = 0.1293 \times 10^7 \times 10^{-3} = 1293$ AT.

The flux of the central limb divides equally at point A in figure along the two parallel path *ABCD* and *AFED*. We may consider either path, say *ABCD* and calculate the m.m.f. required for it. The same m.m.f. will also send the flux through the other parallel path *AFED*.

Flux through $ABCD = 1.3 \times 10^{-3}/2 = 0.65 \times 10^{-3}$ Wb

Flux density $B = 0.65 \times 10^{-3}/500 \times 10^{-6} = 1.3$ T

The corresponding value of H for this value of B is given at 850 AT/m.

$\therefore$ *m.m.f.* reqd. for path $ABCD = H \times l = 850 \times (300 \times 10^{-3}) = 255$ AT

As said above, this, m.m.f. will also send the flux in the parallel path *AFED*.

Total m.m.f. reqd. = 456 + 1293 + 255 = 2004 AT

Since the number of turns is 500, $I = 2004/500 =$ **4A**.

Example 6.20. *A cast steel d.c. electromagnet shown in Fig. 6.37 has a coil of 1000 turns on its central limb. Determine the current that the coil should carry to produce a flux of 2.5 mWb in the air-gap. Neglect leakage. Dimensions are given in cm. The magnetisation curve for cast steel is as under :*

Flux density (Wb/m²) :	*0.2*	*0.5*	*0.7*	*1.0*	*1.2*
Amp-turns/metre :	*300*	*540*	*650*	*900*	*1150*

(Electrotechnics-I, ; M.S. Univ. Baroda)

Solution. Two points should be noted

(i) there are two (equal) parallel paths *ACDE* and *AGE* across the central path *AE*.

(ii) flux density in either parallle path is half of that in the central path because flux divides into two equal parts at point *A*.

Total m.m.f. required for the whole electromagnet is equal to the sum of the following three m.m.fs.

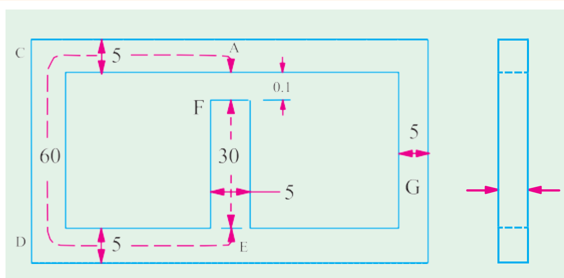


Fig. 6.37

(i) that required for path *EF*

(ii) that required for air-gap

(iii) that required for either of the two parallel paths ; say, path $ACDE_2$

Flux density in the central limb and air gap is

$= 2.5 \times 10^{-3}/(5 \times 5) \times 10^{-4} = 1 \text{ Wb/m}^2$

Corresponding value of H as found from the given data is 900 AT/m.

$\therefore$ AT for central limb $= 900 \times 0.3 = 270$

H in air-gap $= B/\mu_0 = 1/4\pi \times 10^{-7} = 79.56 \times 10^4$ AT/m

AT required $= 79.56 \times 10^4 \times 10^{-3} = 795.6$

Flux density in path *ACDE* is 0.5 Wb/m² for which corresponding value of H is 540 AT/m.

$\therefore$ AT required for path $ACDE = 540 \times 0.6 = 324$

Total AT required = 270 + 795.6 + 324 = 1390 ;Current required = 1390/1000 = **1.39 A**

Example 6.21. *A cast steel magnetic structure made for a bar of section 8 cm × 2 cm is shown in Fig. 6.35. Determine the current that the 500 turn-magnetising coil on the left limb should carry so that a flux of 2 mWb is produced in the right limb. Take μ_r = 600 and neglect leakage.*

(Elect. Technology Allahabad Univ. 1993)

Solution. Since path C and D are in parallel with each other w.r.t. path E (Fig. 6.38), the m.m.f. across the two is the same.

$$\Phi_1 S_1 = \Phi_2 S_2$$

$$\therefore \quad \Phi_1 \times \frac{15}{\mu A} = 2 \times \frac{25}{\mu A}$$

$$\therefore \quad \Phi_1 = 10/3 \text{ mWb}$$

$$\therefore \quad \Phi = \Phi_1 + \Phi_2 = 16/3 \text{ mWb}$$

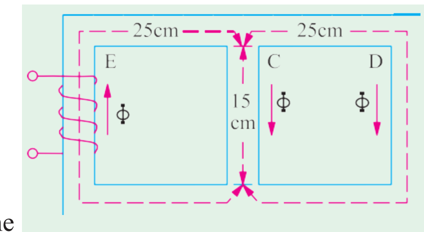


Fig. 6.38

Total AT required for the whole circuit is equal to the sum of

(i) that required for path E and ***(ii)*** that required for either of the two paths C or D.

$$\text{Flux density in path } E = \frac{16 \times 10^{-3}}{3 \times 4 \times 10^{-4}} = \frac{40}{3} \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{40 \times 0.25}{3 \times 4\pi \times 10^{-7} \times 600} = 4,420$$

$$\text{Flux density in path } D = \frac{2 \times 10^{-3}}{4 \times 10^{-4}} = 5 \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{5}{4\pi \times 10^{-7} \times 600} \times 0.25 = 1658$$

Total AT = 4,420 + 1,658 = 6,078 ;

Current needed = 6078/500 = **12.16 A**

Example 6.22. *A ring of cast steel has an external diameter of 24 cm and a square cross-section of 3 cm side. Inside and cross the ring, an ordinary steel bar 18 cm × 3 cm × 0.4 cm is fitted with negligible gap. Calculating the number of ampere-turns required to be applied to one half of the ring to produce a flux density of 1.0 weber per metre^2 in the other half. Neglect leakge. The B-H characteristics are as below :*

	For Cast Steel			*For Ordinary Plate*			
B in Wb/m^2	*1.0*	*1.1*	*1.2*	*B in Wb/m^2*	*1.2*	*1.4*	*1.45*
Amp-turn/m	*900*	*1020*	*1220*	*Amp-turn/m*	*590*	*1200*	*1650*

(Elect. Technology, Indore Univ.)

Solution. The magnetic circuit is shown in Fig. 6.39.

The m.m.f. (or AT) produced on the half A acts across the parallel magnetic circuit C and D. First, total AT across C is calculated and since these amp-turns are also applied across D, the flux density B in D can be estimated. Next, flux density in A is calculated and therefore, the AT required for this flux density. In fact, the total AT (or m.m.f.) required is the sum of that required for A and that of either for the two parallel paths C or D.

Value of flux density in C = 1.0 Wb/m^2

Mean diameter of the ring = (24 + 18)/2 = 21 cm

Mean circumference = $\pi \times 21$ = 66 cm

Length of path A or C = 66/2 = 33 cm = 0.33 m

Value of AT/m for a flux density of 1.0 Wb/m^2 as seen from the given $B.H$ characteristics = 900 AT/m

$\therefore$ Total AT for path C = 900 × 0.33 = 297. The same ATs. are applied across path D.

Length of path D = 18 cm = 0.18 m $\therefore$ AT/m for path D = 297/0.18 = 1650

Value of B corresponding to this AT/m from given table is = 1.45 Wb/m^2

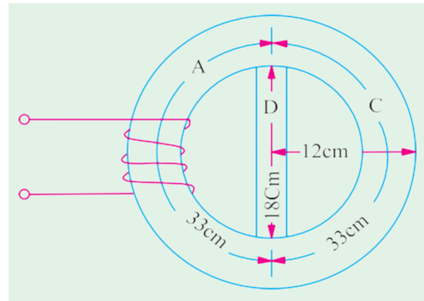


Fig. 6.39

Flux through $C = B \times A = 1.0 \times 9 \times 10^{-4} = 9 \times 10^{-4}$ Wb

Flux through $D = 1.45 \times (3 \times 0.4 \times 10^{-4}) = 1.74 \times 10^{-4}$ Wb

$\therefore$ Total flux through $A = 9 \times 10^{-4} + 1.74 \times 10^{-4} = 10.74 \times 10^{-4}$ Wb.

Flux density through $A = 10.74 \times 10^{-4}/9 \times 10^{-4} = 1.193$ Wb/m^2

No. of AT/m reqd. to produce this flux density as read from the given table = 1200 (approx.)

$\therefore$ Amp-turns required for limb A = 1200 × 0.33 = 396

Total AT required = 396 + 297 = **693**

Example 6.23. *Show how the ampere-turns per pole required to produce a given flux in d.c. generator are calculated.*

Find the amp-turns per pole required to produce a flux of 40 mWb per pole in a machine with a smooth core armature and having the following dimensions :

Length of air gap = 5 mm *Area of air-gap = 500 cm^2*

Length of pole = 12 cm *Sectional area of pole core = 325 cm^2*

Relative permeability of pole core = 1,500

Length of magnetic path in yoke betwen pole = 65 cm

Cross-sectional area of yoke = 450 cm^2 ; Relative permeability of yoke = 1,200

Leakage coefficient = 1.2

The ampere-turns for the armature core may be neglected.

Solution. Air-gap Φ = 40 mWb = 4×10^{-2} Wb ; $A = 500 \times 10^{-4} = 5 \times 10^{-3}$ m^2

$\therefore B = 4 \times 10^{-2}/5 \times 10^{-2} = 0.8$ Wb/m^2 ; $H = B/\mu_0 = 0.8/4\pi \times 10^{-7} = 63.63 \times 10^{-4}$ AT/m

Air-gap length = 5×10^{-3} m ; AT reqd. = $63.63 \times 10^{4} \times 5 \times 10^{-3}$ = 3181.5

Pole Core

$$\Phi = 1.2 \times 4 \times 10^{-2} = 4.8 \times 10^{-2} \text{ Wb} ; A = 325 \times 10^{-4} \text{ m}^2$$

$$B = 4.8 \times 10^{-2}/325 \times 10^{-4} = 1.477 \text{ Wb/m}^2$$

$$H = B/\mu_0 \mu_r = 1.477/4\pi \times 10^{-7} \times 1,500 = 783 \text{ AT/m}$$

Pole length = 0.12 m ; AT reqd. = 783 × 0.12 = 94

Yoke Path

flux = half the pole flux = $0.5 \times 4 \times 10^{-2} = 2 \times 10^{-2}$ Wb

$A = 450 \text{ cm}^2 = 45 \times 10^{-3} \text{ m}^2$; $B = 2 \times 10^{-2}/45 \times 10^{-3} = 4/9$ Wb/m^2

$$H = \frac{4/9}{4\pi \times 10^{-7} \times 1,200} = 294.5 \text{ AT/m} \quad \text{Yoke length} = 0.65 \text{ m}$$

At reqd = 294.5 × 0.65, Total AT/pole = 3181.5 + 94 + 191.4 = **3,467**

Example 6.24. *A shunt field coil is required to develop 1,500 AT with an applied voltage of 60 V. The rectangular coil is having a mean length of turn of 50 cm. Calculate the wire size. Resistivity of copper may be assumed to be 2 × 10^{-6} μ∧-cm at the operating temperature of the coil. Estimate also the number of turns if the coil is to be worked at a current density of 3 A/mm^2.*

(Basis Elect. Machines Nagpur Univ. 1992)

Solution. $NI = 1{,}500$ (given) or $N \cdot \dfrac{V}{R} = N \cdot \dfrac{60}{R} = 1{,}500$

$$\therefore \quad R = \frac{N}{25}\ \text{ohm} \qquad \text{Also } R = \rho \cdot \frac{l}{A} = \frac{2 \times 10^{-6} \times 50\, N}{A}$$

$$\therefore \quad \frac{N}{25} = \frac{\cancel{10^{-4} n}}{A} \qquad \text{or } A = 25 \times 10^{-4}\ \text{cm}^2 \text{ or } A = 0.25\ \text{mm}^2$$

$$\therefore \quad \frac{\cancel{\pi D^2}}{4} = 0/25 \qquad \text{or } D = \mathbf{0.568\ mm}$$

$$\text{Current in the coil} = 3 \times 0.25 = 0.75\ \text{A}$$

Now, $NI = 1{,}500$; $\therefore N = 1{,}500/0.75 = \mathbf{2{,}000}$

Example 6.25. *A wooden ring has a circular cross-section of 300 sq. mm and a mean diameter of the ring is 200 mm. It is uniformly wound with 800 turns.*

Calculate :

(i) *the field strength produced in the coil by a current of 2 amperes :(assume = 1)*

(ii) *the magnetic flux density produced by this current and*

(iii) *the current required to produce a flux density of 0.02 wb/m^2.*

[Nagpur University (Summer 2000)]

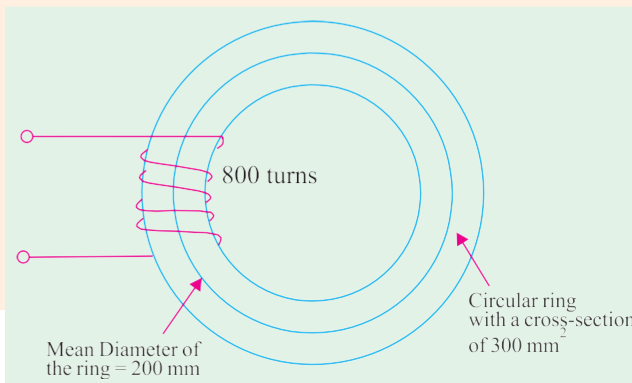


Fig. 6.40

Solution. The question assumes that the flux-path is through the ring, as shown by the dashed line, in figure, at the mean diameter, in Fig. 6.40.

With a current of 2 amp,

$$\text{Coil m.m.f.} = 800 \times 2 = 1600\ \text{AT}$$

$$\text{Mean length of path} = \pi \times 0.2 = 0.628\ \text{m}$$

(i) $$H = \frac{1600}{0.628} = 2548\ \text{amp-turns/meter}$$

(ii) $$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 1 \times 2548 = 3.20 \times 10^{-3}\ \text{Wb/m}^2$$

This Flux density is produced by a coil current of 2-amp

(iii) For producing a flux of 0.02 Wb/m^2, the coil current required is

$$2 \times \frac{0.02}{0.0032} = \mathbf{12.5\ amp}$$

Example 6.26. *A magnetic core in the form of a closed circular ring has a mean length of 30 cm and a cross-sectional area of 1 cm^2. The relative permeability of iron is 2400. What direct-current will be needed in the coil of 2000 turns uniformly wound around the ring to create a flux of 0.20 mWb in iorn ? If an air-gap of 1 mm is cut through the core perpendicualr to the direction of this flux, what current will now be needed to maintain the same flux in the air gap ?*

[Nagpur University Nov. 1999]

Solution.

$$\text{Reluctance of core} = \frac{1}{\mu_0 \mu_r} \cdot \frac{L}{a} = \frac{1}{10\pi \times 10^{-7} \times 2400} \times \frac{30 \times 10^2}{1 \times 10^{-4}}$$

$$= \frac{30 \times 10^{-9}}{4\pi \times 2400 \times 1} = 995223 \text{ MKS units}$$

$$\phi = 0.2 \times 10^{-3} \text{ Wb}$$

$$\text{MMF required} = \phi \times \text{Rel}$$

$$= 0.2 \times 10^{-3} \times 995223 = 199 \text{ amp-tunrs}$$

Direct current required through the 2000 turn coil

$$= \frac{199}{2000} = 0.0995 \text{ amp}$$

Reluctance of 1 mm air gap

$$= \frac{1}{4\pi \times 10^{-7}} \times \frac{1 \times 10^{-3}}{1 \times 10^{-4}} = \frac{10^8}{4\pi} = 7961783 \text{ MKS units}$$

Addition of two reluctances

$$= 995223 + 7961783 = 8957006 \text{ MKS units}$$

MMF required to establish the given flux

$$= 0.2 \times 10^{-3} \times 8957006 = \textbf{1791 amp turns}$$

Current required through the coil

$$= \frac{1791}{2000} \quad \textbf{0.8955 amp}$$

Note : Due to the high permeability of iron, which is given here as 2400, 1 mm of air-gap length is equivalent magnetically to 2400 mm of length through the core, for comparison of mmf required.

Example 6.27. *An iron-ring of mean length 30 cm is made up of 3 pieces of cast-iron. Each piece has the same length, but their respective diameters are 4, 3 and 2.5 cm. An air-gap of length 0.5 mm is cut in the 2.5 – cm. Piece. If a coil of 1000 turns is wound on the ring, find the value of the current has to carry to produce a flux density of 0.5 Wb/m² in the air gap. B-H curve data of cast-iron is as follows :*

B	*(Wb/m²) :*	*0.10*	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*
H	*(AT/m) :*	*280*	*680*	*990*	*1400*	*2000*	*2800*

Permeability of free space $= 4\pi \times 10^{-7}$

Neglect Leakage and fringing effects. **[Nagpur University, November 1998]**

Solution. From the data given, plot the B-H curve for cast-iron

The magnetic circuit has four parts connected in series

Part 1. Air-gap 0.5 mm length, $B = 0.5$ wb/m², and

Permeability of free sapce is known

$$H_1 = B/\mu_0 = 0.5 \times 10^7/(4\pi) = 398100$$

$$\text{AT for gap} = (0.5 \times 10^{-3}) \times H_1 = 199$$

Part 2. 2.5 cm diameter, 10 – cm long, cast-iron ring portion *B* and *H* are to be related with the help of given data. In this, out of 10 cms. 0.5 mm air-gap is cut, and this portion of ring will have cast-iron length of 99.5 mm.

For $B = 0.5 \text{ wb/m}^2, \; H_2 = 2000 \text{ AT/m}$

$$AT_2 = 2000 \times 9.95 \times 10^{-2} = 199$$

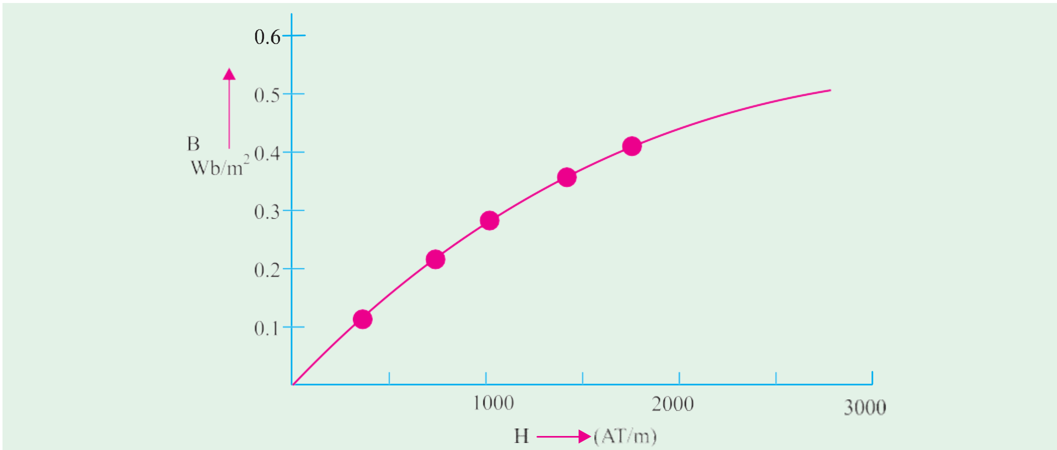


Fig. 6.41

Part 3. 3-cm diameter, 10-cm long, cast-iron ring-portion.

Here $B = 0.50 \times (2.5/3)^2 = 0.347$ Wb/m^2

For this B, H is read from B-H curve.

$H_3 = 1183$ AT/m

$AT_3 = 1183 \times 10 \times 10^{-2} = 118.3$

Part 4. 4 cm. Diameter, 10 cm long, cast-iron ring portion.

Here, $B = 0.50 \times (2.5 \times 4)^2 \times 0.195$ Wb/m^2

From, B–H curve, corresponding H is 661

$AT_4 = 661 \times 10 \times 10^{-2} = 66\ AT$

Since all these four parts in series, the total m.m.f. required is obtained by adding the above terms.

AT = 199 + 199 + 118 + 66 = 582

Coil Current = 582/1000 = 0.582 amp

Additional observations.

(a) The 2.5-cm diameter portion of the ring has $H = 2000$ for $B = 0.5$ Wb/m^2. From this, the relative permeability of cast-iron can be foud out.

$\mu_0\ \mu_r = 0.5/2000$, giving $\mu_r = 199$

An air-gap of 0.5 mm is equivalent of 99.5 mm of cast-iron length. Hence, the two m.m.fs. are equal to 199 each.

(b) The common flux for this circuit is obtained from flux-density and the concerned area.

Hence $\phi = 0.5 \times (\pi/4) \times (2.5 \times 10^{-2})^2 = 0.02453 \times 10^{-2}$

$= 0.2453$ mWb

Reluctance of total magnetic circuit

$= \text{m.m.f./flux} = 582/(2.453 \times 10^{-4})$

$= 2372650$ MKS units

Example 6.28. *A steel-ring of 25 cm mean diameter and of circular section 3 cm in diameter has an air-gap of 1.5 mm length. It is wound uniformly with 700 turns of wire carrying a current of 2 amp. Calculate : **(i)** Magneto motive force **(ii)** Flux density **(iii)** Magnetic flux **(iv)** Relative permeability. Neglect magnetic leakage and assume that iron path takes 35 % of total magneto motive force.* **[Nagpur University, April 1996]**

Solution. From the given data, length of mean path in the ring (= L_m) is to be calculated. For a mean diameter of 25 cm, with 1.5 mm of air-gap length.

$L_m = (\pi \times 0.25) - (1.5 \times 10^{-3}) = 0.7835$ m

Cross-sectional area of a 3 cm diameter ring = 7.065×10^{-4} sq.m.

Total m.m.f. due to coil = 700 × 2 = 1400 amp-turns

Since iron-path takes 355 of the total mmf, it is 490.

Remaining mmf of 910 is consumed by the air-gap.

Corresponding H for air-gap = $910/(1.5 \times 10^{-3})$ = 606666 amp-turns/m.

If Flux density is B_g, we have $B_g = \mu_0 H_g = 4\pi \times 10^{-7} \times 606666 = 0.762$ Wb/m^2

Iron-ring and air-gap are in series hence their flux is same. Since the two have some cross-sectional area, the flux density is also same. The ring has a mean length of 0.7835 m and needs an mmf of 490. For the ring.

$$H = 490/0.7845 = 625.4 \text{ amp-turns / m}$$

$$\mu_0 \mu_r = B/H = 0.752/625.4 = 1.218 \times 10^{-3}$$

$$\mu_r = (1.218 \times 10^{-3}) / (4\pi \times 10^{-7}) = 970$$

Flux = Flux density × Cross-sectional area = $0.762 \times 7.065 \times 10^{-4}$ = 0.538 milli-webers

Check. μ_r of 970 means that 1.5 mm of air-gap length is equivalent to $(1.5 \times 10^{-3} \times 970) = 1.455$ m of length through iron as a medium. With this equivalent.

$$\frac{\text{mmf of ring}}{\text{mmf for (ring + air-gap)}} = \frac{0.785}{0.785 + 1.455} = 0.35$$

which means that 35 % of total mmf is required for the ring

Example 6.29. ***(a)*** *Determine the amp-turns required to produce a flux of 0.38 mWb in an iron-ring of mean diameter 58 cm and cross-sectional area of 3 sq. cm. Use the following data for the ring :*

B Wb/m^2	*0.5*	*1.0*	*1.2*	*1.4*
μ_r	*2500*	*2000*	*1500*	*1000*

(b) *If a saw-cut of 1mm width is made in the ring, calculate the flux density in the ring, with the mmf remaining same as in (a) above.* **[Nagpur University, Nov. 1996]**

Solution. Plot the B- μ_r curve as in Fig. 6.42

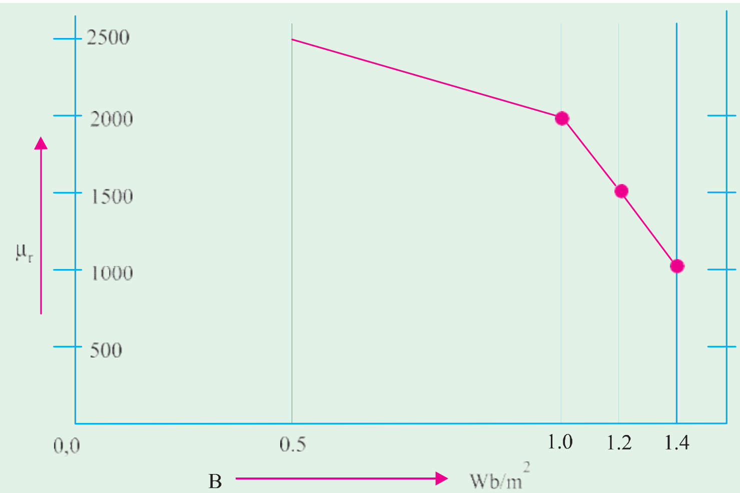


Fig. 6.42

(*a*) Cross-sectional area = 3 sq. cm = 3×10^{-4} sq. m.

Flux = 38 mWb = 0.38×10^{-3} Wb

Flux density, B = flux/area = $(0.38 \times 10^{-3})/(3 \times 10^{-4}) = 1.267$ Wb/m^2

Looking into the table relating B and μ_r, interpolation is required for evaluating μ_r for $B = 1.267$ Wb/m^2. After 1.2 Wb/m^2, μ_r decreases by 500 for a rise of 0.2 in B. Interpolation results into:

$$\mu_r = 1500 - \frac{0.067}{0.20} \times 500 = 1332$$

For mean diameter of path in the ring as 0.58 m, the length of the magnetic path in the ring is

$$l_m = p \times 0.58 = 1.8212 \text{ m}$$

Since $B = \mu_0 \mu_r H$,

$$H = 1.267/(4\pi \times 10^{-7} \times 1332) = 757$$

Hence, the required m.m.f. is

$$757 \times 1.8212 = 1378 \text{ amp-turns}$$

(*b*) If a saw-cut of 1 mm is cut in the ring, B is to be calculated with a m.m.f. of 1378. Now the magnetic circuit has two components in series : the ring with its B-μ_r curve in Fig. 6.42 and the air-gap. Since B is not known, μ_r cannot be accurately known right in the initial steps. The procedure to solve the case should be as follows :

Let B the flux density as a result of 1378 amp-turns due to the coil.

For air-gap. $H_g = B_g / (4\pi \times 10^{-7}) = 0.796 \times 10^6$ AT/m

$$AT_g = H_g \times l_g = 0.796 \times 10^6 \times 1 \times 10^{-3} \times B_g = 796\, B_g$$

Due to the air-gap, the flux-density is expected to be between 0.5 and 1 Wb/m^2, because, in (*a*) above, μ_r(for $B = 1.267$ Wb/m^2) is 1332. One mm air-gap is equivalent to 1332 mm of path added in iron-medium. This proportional increase in the reluctance of the magnetic circuit indicates that flux density should fall to a value in between 0.5 and 1 Wb/m^2.

For Iron-ring. With flux density expected to be as mentioned above, interpolation formula for μ_r can be written as :

$$\mu_r = 2500 - 500\,[(B_g - 0.50) / 0.50] = 3000 - 1000\, B_g$$

$$H_i = B_g / (\mu_0\, \mu_r) = B_g / [\mu_0\,(3000 - 1000\, B_g)]$$

Total m.m.f. = $AT_g + AT_i$ = 1378, as previously calculated

Hence,
$$1378 = \frac{1.8212 \times B_g}{\mu_0\,(3000 - 1000\, B_g)} + 796\, B_g$$

This is a quadratic equation in B_g and the solution, which gives B_g in between 0.5 & 1.0 Wb/m^2 is acceptable.

This results into $B_g = 0.925$ Wb/m^2

Corresponding $\mu_r = 3000 - 1000 \times 0.925 = 2075$

Example 6.30. *An iron-ring of mean diameter 19.1 cm and having a cross-sectional area of 4 sq. cm is required to produce a flux of 0.44 mWb. Find the coil-mmf required.*

If a saw-cut 1 mm wide is made in the ring, how many extra amp-turns are required to maintain the same flux ?

B - μ_r curve is as follows :

B (Wb/m^2)	*0.8*	*1.0*	*1.2*	*1.4*
μ_r	*2300*	*2000*	*1600*	*1100*

[Nagpur University, April 1998]

Solution. For a mean-diameter of 19.1 cm, Length of mean path, $l_m = \pi \times 0.191 = 0.6$ m

Cross-sectional area = 4 sq.cm = 4×10^{-4} m^2

Flux, ϕ = 0.44 mWb = 0.44×10^{-3} Wb

Flux density, $B = 0.44 \times 10^{-3}/(4 \times 10^{-4}) = 1.1$ Wb/m^2

For this flux density, μ_r = 1800, by simple interpolation.

$H = B/(\mu_o \mu_r) = 1.1 \times 10^7/(4\pi \times 1800) = 486.5$ amp-turns/m.

m.m.f required = $H \times l_m = 486.5 \times 0.60 = 292$

A saw-cut of 1 mm, will need extra mmf.

$H_g = B_g/\mu_o = 1.1 \times 10^7/(4\pi) = 875796$

$AT_g = H_g \times l_g = 875796 \times 1.0 \times 10^{-3} = 876$

Thus, additional mmf required due to air-gap = 876 amp-turns

Example 6.31. *A 680-turn coil is wound on the central limb of a cast steel frame as shown in Fig. 6.43 (a) with all dimensions in cms. A total flux of 1.6 mWb is required in the air-gap. Find the current in the magnetizing coil. Assume uniform flux distribution and no leakage. Data for B-H curve for cast steel is given.* **[Nagpur University, Novemeber 1997]**

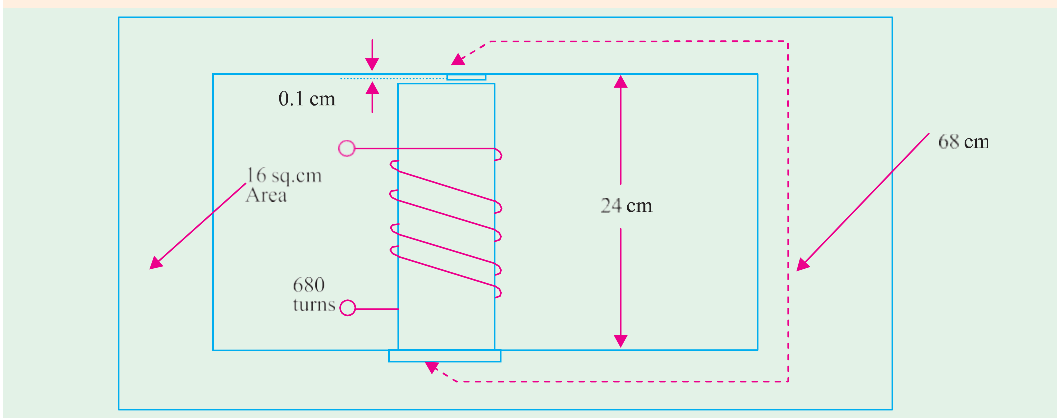
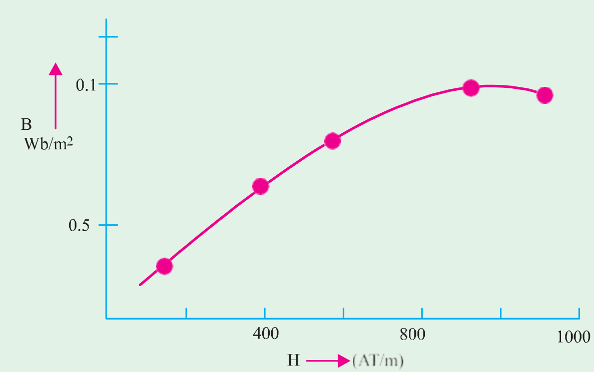


Fig. 6.43 (*a*)



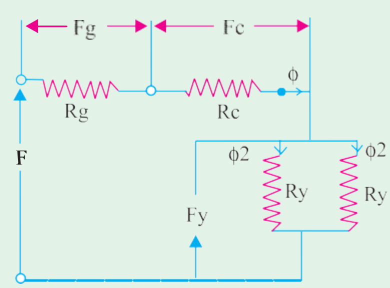


Fig. 6.43 (*b*) **Fig. 6.43 (*c*)**

Solution. ϕ = 1.6 mWb through air-gap and central limb

$\phi/2$ = 0.8 mWb through yokes

Corresponding flux densities are :

$$B_g = B_c = 1.6 \text{ mWb}/(16 \times 10^{-4}) = 1.0 \text{ Wb/m}^2$$

$$B_y = 0.8 \text{ m Wb}/(16 \times 10^{-4}) = 0.50 \text{ Wb/m}^2$$

MMF-Calculations :

(a) For Air gap : For B_g of 1 Wb/m^2, $H_g = 1.0/\mu_o$

$$AT_g = H_g \times l_g = [1/(4\pi \times 10^{-7})] \times (0.1 \times 10^{-2})$$
$$= 796 \text{ amp turns}$$

(b) For Central limb : $AT_c = H_c \times l_c = 900 \times 0.24 = 216$

$\therefore$ For $B_c = 1.00$, H_c from data = 900 AT/m

The yokes are working at a flux-density of 0.50 Wb/m^2. From the given data and the corresponding plot, interpolation can be done for accuracy.

$$H_y = 500 + [(0.5 - 0.45)/(0.775 - 0.45)] \times 200$$
$$= 530 \text{ AT/m}$$
$$F_y = 530 \times 0.68 = 360$$

Total mmf required = 796 + 216 + 360 = 1372

Hence, coil-current = 1372/680 = 2.018 A

Example 6.32. *For the magnetic circuit shown in fig. 6.44 the flux in the right limb is 0.24 mWb and the number of turns wound on the central-limb is 1000. Calculate* ***(i)*** *flux in the central limb* ***(ii)*** *the current required.*

The magnetization curve for the core is given as below :

H (AT/m) :	*200*	*400*	*500*	*600*	*800*	*1060*	*1400*
B (Nb/m$_2$) :	*0.4*	*0.8*	*1.0*	*1.1*	*1.2*	*1.3*	*1.4*

Neglect Leakage and fringing. **[Rajiv Gandhi Technical University, Bhopal, Summer 2001]**

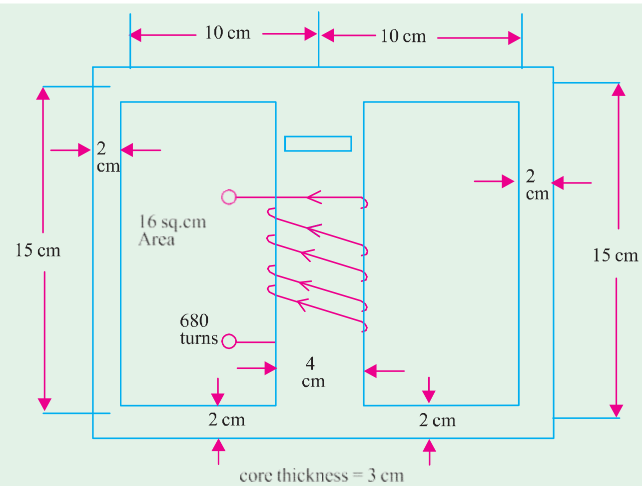


Fig. 6.44

Solution. Area of cross-section of side-limbs = 2 × 3 = 6 sq.cm

Area of cross-section of core = 3 × 4 = 12 sq.cm

Flux in side Limbs = 0.24 mWb

Flux density in side Limbs = $(0.24 \times 10^{-3})/(6 \times 10^{-4}) = 0.4$ Wb/m^2

Since the coil is wound on the central limb and the magnetic circuit is symmetrical, the flux in the central limb = 0.48 mWb. Flux density in the central limb = $(0.48 \times 10^{-3})/(12 \times 10^{-4}) = 0.4$ Wb/m^2

For the flux density of 0.40 Wb/m^2, H = 200 AT/m

Central Limb has a path length of 15 cm.

Other part carrying 0.24 mWb has a total path length of 35 cm.

Total mmf required = (200 × 0.15) + (200 × 0.35) = 100 AT

Hence, coil current = 100/1000 = 0.1 Amp.

Example 6.33. *A ring composed of three sections. The cross-sectional area is 0.001 m^2 for each section. The mean arc length are l_a = 0.3 m, l_b = 0.2 m, l_c = 0.1 m. An air-gap length of 0.1 mm is cut in the ring. Mr for sections a, b, c are 5000, 1000, and 10,000 respectively. Flux in the air gap is 7.5 × 10^{-4} Wb. Find **(i)** mmf **(ii)** exciting current if the coil has 100 turns, **(iii)** reluctances of the sections.* **[Nagpur University April 1999]**

Solution. Area = 0.001 sq.m

$$l_a = 0.3 \text{ m}, \; l_b = 0.2 \text{ m}, \; l_c = 0.1 \text{ m}, \; l_g = 0.1 \times 10^{-3} \text{ m}$$

$$\mu_{ra} = 5000, \; \mu_{rb} = 1000, \; \mu_{rc} = 10{,}000 \; \mu_o = 4\pi \times 10^{-7}$$

$$\phi = 7.5 \times 10^{-4} \text{ Wb}$$

(***iii***) Calculations of Reluctances of four parts of the magnetic circuit :

(***a***) Reluctance of air gap, $R_{eg} = \frac{1}{\mu_o} \times \frac{0.1 \times 10^{-3}}{0.001} = \frac{1000}{4\pi \times 0.001} = 79618$

(**b**) Reluctance of section '*a*' of ring

$$= R_{ea} = \frac{1}{\mu_o \mu_{ra}} \times \frac{0.3}{0.001} = \frac{10^7 \times 0.3}{4\pi \times 47770 \times 5000 \times 0.001} = 47770$$

(**c**) Reluctance of section '*b*' of the ring

$$= R_{eb} = \frac{1}{\mu_o \mu_{rb}} \times \frac{0.20}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 15923.6$$

(***d***) Reluctance of section '*c*' of the ring

$$= R_{ec} = \frac{1}{\mu_o \mu_{rc}} \times \frac{0.10}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 7961$$

Total Reluctance = $R_{eg} + R_{ea} + R_{eb} + R_{ec}$ = 294585

(**i**) Total mmf required = Flux × Reluctance

= 7.5 × 10^{-4} × 294585 = 221 amp-turns

(**ii**) Current required = 221/100 = 2.21 amp

Tutorial Problems No. 62

1. An iron specimen in the form of a closed ring has a 350-turn magnetizing winding through which is passed a current of 4A. The mean length of the magnetic path is 75 cm and its cross-sectional area is 1.5 cm^2. Wound closely over the specimen is a secondary winding of 50 turns. This is connected to a ballistic glavanometer in series with the secondary coil of 9-mH mutual inductance and a limiting resistor. When the magnetising current is suddenly reversed, the galvanometer deflection is equal to that produced by the reversal of a current of 1.2 A in the primary coil of the mutual inductance. Calculate the *B* and *H* values for the iron under these conditions, deriving any formula used.

   [1.44 Wb/m^2 ; 1865 AT/m] ***(London Univ.)***

2. A moving-coil ballistic galvanometer of 150 ∧ gives a throw of 75 divisions when the flux through a search coil, to which it is connected, is reversed.

   Find the flux density in which the reversal of the coil takes place, given that the galvanometer constant is 110 μ*C* per scale division and the search coil has 1400 turns, a mean are of 50 cm^2 and a resistance of 20 ∧. **[0.1 Wb/m^2]** ***(Elect. Meas. & Measuring Inst. Gujarat Univ.)***

3. A fluxmeter is connected to a search coil having 500 turns and mean area of 5 cm^2. The search coil is placed at the centre of a solenoid one metre long wound with 800 turns. When a current of 5 A is reversed, there is a deflection of 25 scale divisions on the fluxmeter. Calculate the flux-meter constant. **[10^{-4} Wb-turn/division] (*Elect. Meas. & Measuring Inst., M.S. Univ. Baroda*)**

4. An iron ring of mean length 50 cms has an air gap of 1 mm and a winding of 200 turns. If the permeability of iron is 300 when a current of 1 A flows through the coil, find the flux density. **[94.2 mWb/m^3] *(Elect. Engg. A.M.Ae.S.I.)***

5. An iron ring of mean length 100 cm with an air gap of 2 mm has a winding of 500 turns. The relative permeability of iron is 600. When a current of 3 A flows in the winding, determine the flux density. Neglect fringing. **[0.523 Wb/m^2] *(Elect. Engg. & Electronic Bangalore Univ. 1990)***

6. A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900, having a mean circumference of 40 mm and cross-sectional area of 50 mm^2. If a current of 25 amps is passed through the coil, find (*i*) m.m.f. (*ii*) reluctance of the ring and (*iii*) flux.
[(*i*) 7500 AT (*ii*) 0.7 × 10^6 AT/Wb (*iii*) 10.7 mWb]
(Elect. Engg. & Electronics Bangalore Univ.)

7. A specimen ring of transformer stampings has a mean circumference of 40 cm and is wound with a coil of 1,000 turns. When the currents through the coil are 0.25 A, 1 A and 4 A the flux densities in the stampings are 1.08, 1.36 and 1.64 Wb/m^2 respectively. Calcualte the relative permeability for each current and explain the differences in the values obtained. **[1,375,434,131]**

8. A magnetic circuit consists of an iron ring of mean circumference 80 cm with cross-sectional area 12 cm^2 throughout. A current of 2A in the magnetising coil of 200 turns produces a total flux of 1.2 mWb in the iron. Calculate :
(a) the flux density in the iron
(b) the absolute and relative permeabilities of iron
(c) the reluctance of the circuit
[1 Wb/m^2 ; 0.002, 1,590 ; 3.33 × 10^5 AT/Wb]

9. A coil of 500 turns and resistance 20 ∧ is wound uniformly on an iron ring of mean circumference 50 cm and cross-sectional area 4 cm^2. It is connected to a 24-V d.c. supply. Under these conditions, the relative permeability of iron is 800. Calculate the values of :
(a) the magnetomotive force of the coil (*b*) the magnetizing force
(*c*) the total flux in the iron (*d*) the reluctance of the ring
[(*a*) 600 AT (*b*) 1,200 AT/m (*c*) 0.483 mWb (*d*) 1.24 × 10^6 AT/Wb]

10. A series magnetic circuit has an iron path of length 50 cm and an air-gap of length 1 mm. The cross-sectional area of the iron is 6 cm^2 and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the circuit. The following points are taken from the magnetisation characteristic :

Flux density (Wb/m^2) :	1.2	1.35	1.45	1.55	
Magnetizing force (AT/m) :	500	1,000	2,000	4,500	**[6.35 A]**

11. An iron-ring of mean length 30 cm is made of three pieces of cast iron, each has the same length but their respective diameters are 4, 3 and 2.5 cm. An air-gap of length 0.5 mm is cut in the 2.5 cm piece. If a coil of 1,000 turns is wound on the ring, find the value of the current it has to carry to produce a flux density of 0.5 Wb/m^2 in the air gap. *B*/*H* characteristic of cast-iron may be drawn from the following :

B (Wb/m^2) :	0.1	0.2	0.3	0.4	0.5	0.6	
(AT/m) :	280	620	990	1,400	2,000	2,8000	**[0.58 A]**

Permeability of free space = 4π × 10^7 H/m. Neglect leakage and fringing.

12. The length of the magnetic circuit of a relay is 25 cm and the cross-sectional area is 6.25 cm^2. The length of the air-gap in the operated position of the relay is 0.2 mm. Calculate the magnetomotive force required to produce a flux of 1.25 mWb in the air gap. The relative permeability of magnetic material at this flux density is 200. Calculate also the reluctance of the magnetic circuit when the

relay is in the unoperated position, the air-gap then being 8 mm long (assume μ_r, remains constant). **[2307 AT, 1.18 × 10^7 AT/Wb]**

13. For the magnetic circuit shown in Fig. 6.45, all dimensions are in cm and all the air-gaps are 0.5 mm wide. Net thickness of the core is 3.75 cm throughout. The turns are arranged on the centre limb as shown.

Calculate the m.m.f. required to produce a flux of 1.7 mWb in the centre limb. Neglect the leakage and fringing. The magnetisation data for the material is as follows :

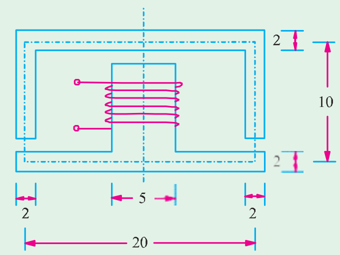


Fig. 6.45

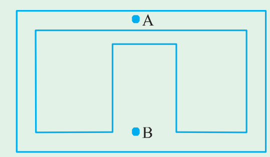


Fig. 6.46

H (AT/m) :	400	440	500	600	800
B (Wb/m^2) :	0.8	0.9	1.0	1.1	1.2

[1,052 AT]

14. In the magnetic circuit shown in Fig. 6.46 a coil of 500 turns is wound on the centre limb. The magnetic paths *A* to *B* by way of the outer limbs have a mean length of 100 cm each and an effective cross-sectional area of 2.5 cm^2. The centre limb is 25 cm long and 5cm^2 cross-sectional area. The air-gap is 0.8 cm long. A current of 9.2 A through the coil is found to produce a flux of 0.3 mWb.

15. The magnetic circuit of a choke is shown in Fig. 6.47. It is designed so that the flux in the central core is 0.003 Wb. The cross-section is square and a coil of 500 turns is wound on the central core. Calculate the exciting current. Neglect leakage and assume the flux to be uniformly distributed along the mean path shown dotted. Dimensions are in cm.

The characteristics of magnetic circuit are as given below :

B (Wb/m^2) :	0.38	0.67	1.07	1.2	1.26
H (AT/m) :	100	200	600	1000	1400

(Elect. Technology I. Gwalior Univ.)

16. A 680-turn coil is wound on the central limb of the cast steel sheet frame as shown in Fig. 6.48 where dimensions are in cm. A total flux of 1.6 mWb is required to be in the gap. Find the current required in the magnetising coil. Assume gap density is uniform and all lines pass straight across the gap. Following data is given :

H (AT/m) :	300	500	700	900	1100
B (Wb/m^2) :	0.2	0.45	0.775	1.0	1.13

(Elect. Technology ; Indore Univ.)

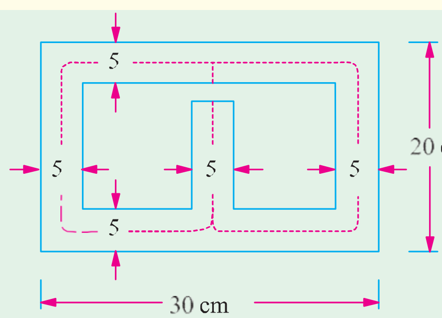


Fig. 6.47

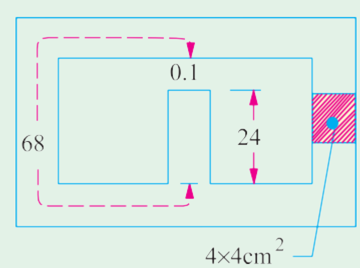


Fig. 6.48

17. In the magnetic circuit of Fig. 6.49, the core is composed of annealed sheet steel for which a stacking factor of 0.9 should be assumed. The core is 5 cm thick. When $\Phi_A = 0.002$ Wb, $\Phi_B = 0.0008$ Wb and $\Phi_C = 0.0012$ Wb. How many amperes much each coil carry and in what direction ? Use of the

following magnetisation curves can be made for solving the problem.

B (Wb/m^2) :	0.2	0.4	0.6	0.8	1.0	1.4	1.6	1.8
H (AT/m^2) :	50	100	130	200	320	1200	3800	10,000

(Elect. Technology, Vikram Univ.)

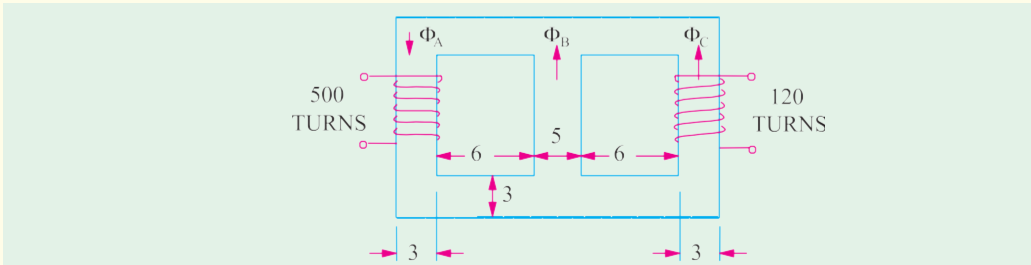


Fig. 6.49

18. A magnetic circuit with a uniform cross-sectional area of 6 cm^2 consists of a steel ring with a mean magnetic length of 80 cm and an air gap of 2 mm. The magnetising winding has 540 ampere-turns. Estimate the magnetic flux produced in the gap. The relevant points on the magnetization curve of cast steel are :

B (Wb/m^2) :	0.12	0.14	0.16	0.18	0.20
H (AT/m) :	200	230	260	290	320

[0.1128 m Wb] *(City & Guilds, London)*

19. Explain the terms related to magnetic circuits:
(*i*) reluctance (*ii*) flux density (*iii*) magnetomotive force *(Nagpur University, Summer 2002)*

20. A metal ring of mean diameter of 80 cm is made out of two semi-circular pieces of cast iron and cast steel separated at junctions by pieces of copper each of 1 mm thickness. If the ring is uniformly wound with 1000 turns, calculate the value of current required to produce a flux density of 0.85 wb/nV in the ring.
Given that relative permeability of cast iron as 200, that of cast steel is 1200 and for copper, $\mu_r = 1$. *(Nagpur University, Summer 2002)*

21. A 1154 turns coil is wound on the central limb of the case steel frame shown in Fig. 6.50. A total flux of 1.6 mwb is required in the air gap. Find the current required through the gap. Assume that the gap density is uniform and there is no leakage. Frame dimensions are given in cm. Take permeability of cast steel as 1,200. *(Nagpur University, Winter 2002)*

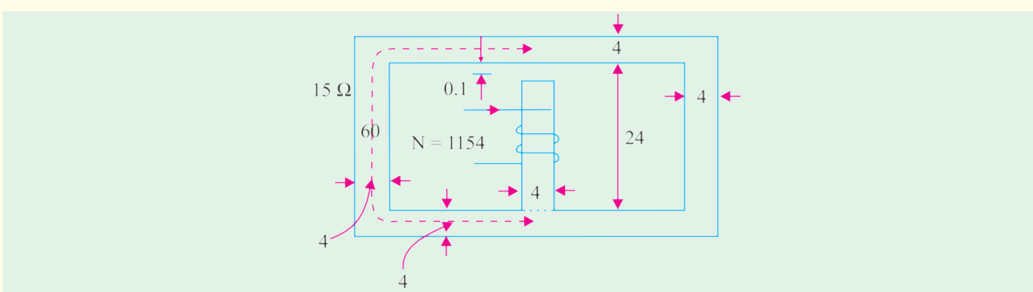


Fig. 6.50

22. Explain the terms related to magnetic circuits :
(*i*) Reluctance (*ii*) Flux density (*iii*) Coercive force (*iv*) Magnetomotive force (*v*) Residual flux. *(Nagpur University, Summer 2003)*

23. Compare electric and magnetic circuit by their similarities and dissimilarities. *(Nagpur University, Winter 2003)*

24. Compare electric and magnetic circuits with respect to their similarities and dissimilarities. *(Nagpur University, Summer 2004)*

25. A steel wire of 25 cm mean diameter and circular cross section 3 cm in diameter has an airgap of 1 mm wide. It is wound with a coil of 700 turns carrying a current of 2 A.
Calculate : (*i*) m.m.f. (*ii*) Flux density (*iii*) Reluctance (*iv*) Relative permeability.
Assume that iron path take 30% of total m.m.f. ***(Gujrat University,Summer 2003)***
26. What is a search coil in magnetic measurements? ***(Anna University, April 2002)***
27. Name the magnestic squares used to find iron loss. ***(Anna University, April 2002)***
28. What is a magnetic circuit? A magnetic circuit is made up of 3 limbs A, B and C in prallel. The reluctances of the magnetic paths of A, B and C in AT/mWb are 312, 632.6 and 520 respectively. An exciting coil of 680 turns is wound on limb B. Find the exciting current to produce of flux of lmwb in the limb A. ***(V.T.U., Belgaum Karnataka University, February 2002)***
29. An iron ring of 300cm mean circumference with a cross section of $5 cm^2$ is wound uniformly with 350 turns of wire. Find the current required to produce a flux of 0.5 Mwb in iron. Take relative permeability of iron as 400. ***(V.T.U. Belgaum Karnataka University, July/August 2002)***
30. What is Biot-Savart law? Explain briefly. Find the magnetic field due to a small circular loop carrying current I at distances from loop that are large compared with its dimensions.
(Agra Univ. 1978 Supp.)
31. Magnetic potential ***(Mumbai University, 2002) (RGPV, Bhopal 2001)***
32. Flux density ***(Pune University,2002) (RGPV, Bhopal 2001)***
33. Susceptibility ***(Mumbai University, 2002) (RGPV, Bhopal 2001)***
34. Define mm *f*, flux, reluctance, absolute and relative permeabilities with reference to magnetic circuits.
(U.P. Technical University 2003) (RGPV, Bhopal 2002)
35. Discuss B-H curve of a ferro-magnetic material and explain the following.
(*i*) Magnetic saturation (*ii*) Hysteresis (*iii*) Residual magnetism (*iv*) Coercive force
(RGPV, Bhopal 2002)
36. What is meant by leakage and fringing? Define leakage coefficient.
(RGPV, Bhopal 2002)
37. Define the following terms (any five) :
(*i*) MMF (*ii*) Reluctance (*iii*) Permeance (*iv*) Magnetisation curve (*v*) flux density
(*vi*) Magnetizing force (*vii*) Susceptibility (*viii*) Relative permeability (*ix*) Magnetic potential
(RGPV, Bhopal 2002)
38. Distinguish between leakage and fringing of flux. ***(RGPV, Bhopal 2002)***
39. Explain fringing of magnetic flux, magnetic leakage, staturation of ferowegnetic materials, B-H Curve, hysteresis and eddy current losses. ***(RGPV, Bhopal 2003)***

OBJECTIVE TESTS – 6

1. Relative permeability of vacuum is
(*a*) $4\pi \times 10^{-7}$ H/m (*b*) 1 H/m
(*c*) 1 (*d*) $1/4\,\pi$
2. Unit of magnetic flux is
(a) weber (*b*) ampere-turn
(*c*) tesla (*d*) coulomb
3. Point out the WRONG statement.
The magnetising force at the centre of a circular coil varies.
(a) directly as the number of its turns
(b) directly as the current
(c) directly as its radius
(d) inversely as its radius
4. A pole of driving point admittance function implies
(a) zero current for a finite value of driving voltage
(b) zero voltage for a finite value of driving current
(c) an open circuit condition
(*d*) None of (*a*), (*b*) and (*c*) mentioned in the question ***(ESE 2001)***

ANSWERS

1. *c* **2.** *a* **3.** *a*