CHAPTER 12

ELEMENTS OF ELECTRO-MECHANICAL ENERGY CONVERSION

Learning Objectives

- Introduction
- Salient aspects of conversions
- Energy-balance
- Magnetic-field system: Energy and Co-energy
- Linear System
- A Simple Electro-mechanical System
- In terms of Field Energy
- In terms of field Co-energy
- Energy in terms of Electrical parameters
- Rotary Motion
- Description of Simple System
- Energy stored in the c oils
- Different Categories
- Onec oil each on Stator and on Rotor
- Vital Role of Air-gap
- Static ally induced emf and Dynamic ally induced emf.

An electric motor is a machine which converts electrical energy into mechanical (rotational or kinetic) energy

25.1. Introduction

"Energy can ***neither*** be created ***nor*** be destroyed". We can only change its forms, using appropriate energy-conversion processes.

An interesting aspect about the energy in "Electrical form" is that neither it is so available ***directly from nature*** nor it is required to be finally consumed in that form. Still, it is the most popular form of Energy, since it can be transported at remote Load-locations, for optimum utilization of resources. Further, technological progress has now made it possible to device Electrical-Power-modulation systems so flexible and controllable that modern systems tend to be energy-efficient, with increase in life-span of main equipment and the associated auxiliary components (like switches, connecting cables, contactors, etc.) since it is now possible to avoid overstrain (= over-currents or over-voltage) on the system. This means a lot for the total production process (for which electrical energy is being used) since the quality of the production improves, and plant-maintenance is minimal. Energy-conversion systems then assume still higher importance.

Generator

Energy conversion takes place between well known pairs of forms of Energy: Electrical ↔ Chemical, Electrical ↔ Thermal, Electrical ↔ Optical, Electrical ↔ Sound, and Electrical ↔ Mechanical are the common forms with numerous varieties of engineering - applications. Electrical ↔ mechanical conversion is the focus of discussion in this chapter.

The elements of electro-mechanical energy conversion shall deal with basic principles and systems dealing with this aspect. Purpose of the study is to have a general approach to understand to design, and later to modify the system with the help of modern technologies, for overall improvisation.

It is necessary to be aware about:

(a) basic conditions to be fulfilled by the conversion system.

(b) methods for innovating the conversion systems.

Electromechanical energy-conversion finds applications in following categories of systems:

(a) transducers: Devices for obtaining signals for measurement / control,

(b) force-producing devices: Solenoid-actuators, relays, electromagnets,

(c) devices for continuous-energy-conversion: Motors / Generators

These systems have different configurations. But the principles of their working are common. Understanding these principles enables us to analyze / design / improvise / innovate such systems. As a result of such development, newer types of motors and the associated modern power controllers have recently been manufactured and become popular. Controllers using powerelectronics switching devices offer energy-efficient, user-friendly, and high-performance drives. Their initial investment may be larger but two important parameters justify their use: ***(i)*** Considerable energy is saved, resulting into payback periods as short as 18-24 months, ***(ii)*** The controllers ensure to limit the currents to pre-set values under conditions of starting /overload/ unbalanced supply. Hence, the entire system enjoys longer life. Both these effects lead to better production-process and hence these are readily acceptable by industries.

Salient Aspects of Conversions

Purpose of electro-mechanical conversion device is to change the form of energy. Here, for simpler discussion, only rotary systems will be dealt with. When it is converting mechanical input to electrical output the device is "generating". With electrical input, when mechanical output is obtained, the device is motoring.

Some simple aspects of an electrical machine (motor / generator) have to be noted at this place:

(1) Electrical machine has a Stator, a Rotor, and an air-gap in between the two. For a flux path, the magnetic circuit has these three parts in series. In general, magnetic poles are established in Stator and in Rotor.

(2) Magnetic effects of following types can be categorized:

- **(a)** **Electromagnetic:** Due to currents passed through windings on Stator and / or Rotor, producing certain number of poles on these members.
- **(b)** **Permanent Magnets:** One side (Stator or Rotor) can have permanent magnets.
- **(c)** **Reluctance variation:** Surface of Rotor near the air-gap can be suitably shaped to have a particular pattern of Reluctance-variation so as to control the machine behaviour as per requirements.

Stator and Rotor

(3) Basic conditions which must be satisfied by such devices are:

- **(a)** Equal number of poles must be created on the two sides.
- **(b)** In some cases, reluctance-variation is primarily used for machine-action. The Stator side must accommodate a winding carrying current for the electromagnetic effect, when rotor surface is shaped so as to have the desired pattern of reluctance variation. Or, non-cylindrical rotor cannot have the current -carrying winding for machine action.

(4) Out of stator, rotor and air-gap, maximum energy-storage at any angular position takes place in the air-gap, since its reluctance is highest out of the three members.

(5) Stored energy must depend on rotor-position and the device tends to occupy that angular position which corresponds to maximum stored energy. If this position varies as a function of time, the device produces continuous torque.

(6) Ideal output of a motor is a constant unidirectional torque with given currents through its windings. In some cases, the output torque (as a compromise) is an average value of a cyclically varying torque.

(7) Where current-switching is done for motor-control, as in modern controllers, instantaneous effect has to be understood to conclude on any of the points mentioned above.

(8) A device can work either as a generator or as a motor, provided pertinent conditions are satisfied for the concerned mode of operation.

Energy – Balance

For an electro-mechanical system, following terms are important:

(i) Electrical port (= armature terminals): receiving / delivering electrical energy.

(ii) Mechanical port (= shaft): delivering / receiving mechanical energy.

(iii) Coupling field: Magnetic field *or* Electric field.

Even though, theoretically, both the types of fields mentioned above are able to convert the energy, the magnetic medium is most popular since the voltage levels required are not very high, and the devices of given power rating are smaller in size and are economical. Hence, only those will be dealt with.

Conversion of electrical energy into mechanical energy

It is obvious that an electrical motor receives energy at the electrical port and delivers it at the mechanical port. While an electric generator receives the energy at the mechanical port and delivers it at the electrical port. It is also known that the following losses take place in such systems and are dissipated away as heat: ***(i)*** i^2r losses in the windings of the machines, ***(ii)*** friction and windage losses, ***(iii)*** core-losses.

These can be either neglected or attached to electrical port, mechanical port and coupling magnetic field respectively, for simpler analysis. With this, the simple energy balance equation can be written as:

Change in Electrical Energy = Change in Mechanical Energy + Change in Field-Energy

$$dW_{elec} = dW_{mech} + dW_{fld} \qquad \ldots (25.1)$$

It is natural that this equation has +ve signs for electrical and mechanical-energy-terms when the device is motoring. For generating mode, however, both the terms assume –ve signs.

In case no mechanical work is done, eqn. (25.1) reduces to eqn. (25.2) below indicating that Electrical energy - input is stored in the magnetic field.

$$dW_{elec} = dW_{fld} \qquad \ldots (25.2)$$

Magnetic-field System: Energy and Co-energy

Linear System

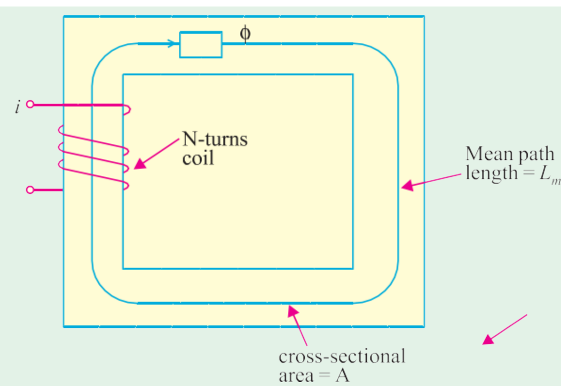


Fig. 25.1 (*a*) Magnetic circuit

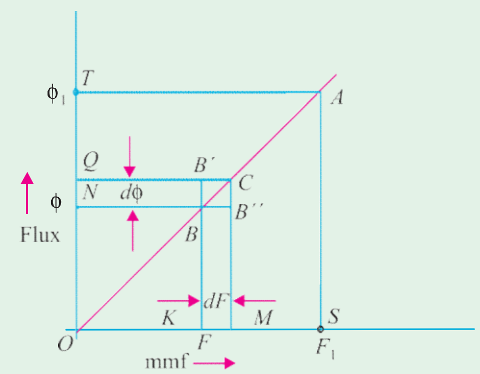


Fig. 25.1 (*b*) Characteristic of a magnetic circuit

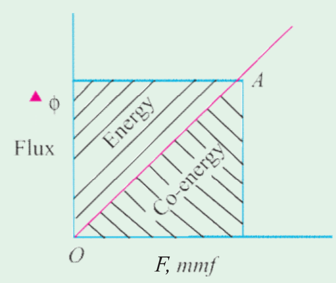


Fig. 25.1 (*c*) Energy and co-energy

A simple magnetic current is shown in Fig 25.1 (*a*), with assumptions that air-gap length at the joints is negligible, and the magnetic medium is not saturated. With A as the cross-sectional area of the core and L_m as the mean length of the path, a coil with N turns carrying a current of i amp has an mmf of F, establishing a flux of ϕ, related by

$$\phi = F \times \mathcal{P}_m$$

where $\mathcal{P}_m$ = Permeance of the Magnetic circuit

$$= \mu_o \mu_r A / L_m$$

with μ_r = relative permeability of the magnetic medium,

This corresponds to the following relationships:

Coil Inductance, $L = N^2 \mathcal{P}_m = N\phi / i = \lambda / i$

where λ = flux-linkage of the coil, in weber-turns

W_{fld} = Energy stored in the coil = $\frac{1}{2} Li^2 = \frac{1}{2} N^2 \mathcal{P}_m i^2 = \frac{1}{2} F^2 \mathcal{P}_m$

$$= \tfrac{1}{2} F(F \mathcal{P}_m) = \tfrac{1}{2} F\phi \quad \ldots \textbf{(25.3)}$$

In this eqn., $\mathcal{P}_m$ is the slope of the characteristic in Fig 25.1 (*b*). Hence, the inductance is proportional to the slope of F-ϕ plot. In Fig., 25.1 (*b*), for the operating point A, the mmf is F_1 and the flux ϕ_1. At the point A, the energy stored in the field is given by eqn. below:

$W_{fld} = \frac{1}{2} F_1 \phi_1$

F_1 is due to the current i_1. W_{fld} is given by area $OATO$ in Fig 25.1(*b*).

In Fig 25.1(*b*), the origin refers to the system without magnetization.

The system can reach the point A, starting from O as the current in the coil is increased from O to i_1.

Let us understand the intermediate events.

At point B, the flux is ϕ due to the mmf F.

An increment in coil current results into increase in mmf by dF. This increases the core-flux by $d\phi$. New operating point is C.

Eqn. (25.3) is to be suitably re-written in terms of these incremental values.

$$W_{fld} = \int dW_{fld} = \int_0^{\phi_1} F d\phi = \int_0^{\phi_1} \text{(area of the strip } NBB'Q) = \text{area of triangle } OAT$$

$$= (1/\mathcal{P}_m) \int_0^{\phi_1} \phi \cdot d\phi = \tfrac{1}{2} \cdot \phi_1 \cdot F_1 \quad \ldots \textbf{(25.4)}$$

Alternatively, we have the area of elemental strip $kBB''M = \phi \,.\, dF$

$$\text{Area of } \Delta \text{ OAS} = \int_0^{F_1} \text{(area of the strip } KBB''M)$$

$$= \int_0^{F_1} \phi \,.\, dF = \mathcal{P}m \int_0^{F_1} F \; dF = \mathcal{P}m \; F_1^2/2 = \tfrac{1}{2} \; F_1 \; \phi_1 \qquad \ldots (25.5)$$

In order to distinguish with respect to the terms in eqn. (25.4), this area is called as the "Co-energy of the field" and is represented by W'_{fld}. For a Linear system, however, for a given operating point, say A, the two energy-terms are equal. Hence,

$$W_{fld} = W'_{fld} = \tfrac{1}{2} F_1\phi_1 = \tfrac{1}{2} \; Li_1^{\,2} \qquad \ldots (25.6)$$

In order to have a simple and clear distinction between the two energy terms, it can be said that the differential variable ***"Current"*** (or mmf) is related to Co-energy and the differential variable "flux - linkage" (or flux) is related to Energy.

This energy stored in the magnetic field comes from the electrical source connected to the coil in Fig. 25.1(*a*).

A Simple Electromechanical System

A simple electro-mechanical system is shown in Fig. 25.2(*a*).

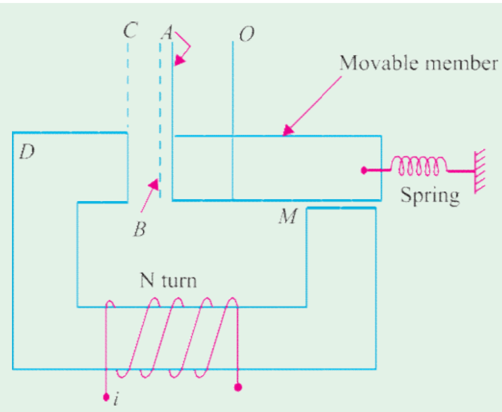


Fig. 25.2 (*a*)

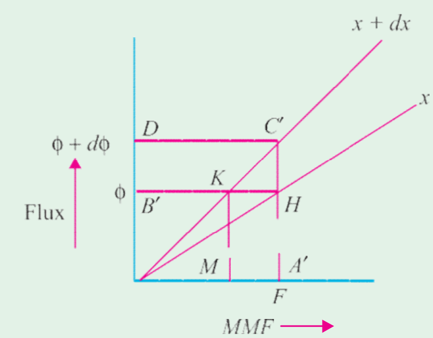


Fig. 25.2 (*b*)

Reference point O corresponds to the unstretched spring. Energy stored in the spring is then Zero. In position A of the movable member, the spring is elongated by x, and the corresponding energy stored in the spring is $\tfrac{1}{2}\, K_s\, x^2$, where K_s is "spring-constant" of the linear system in Nw / m. In Fig 25.2 (*a*), the distance OA is x. The elemental distance AB is dx, so that OB is $x + dx$. For simpler analysis, it is assumed that magnetic material is highly permeable and that the clearance at point M (for movement of the member) is negligible. So that the mmf of the coil is required to drive the flux in the region $OABC$ only. The flux-mmf

A simple electromechanical system

relationships are plotted for these two positions in Fig 25.2 (*b*). In position A, the movable member has moved a distance of x from its unstretched position or reference point. Let the operating point be H, so that the coil-mmf OA (=F) establishes a flux OB (=ϕ). In this position, the movable member experiences a force in such a direction that the energy stored in the field tends to increase. It tends to reach B, so that an additional displacement of dx shifts the characteristic upwards and final operating point in position B is C'. From H to C', the operating point can move in any one of the following ways:

(a) HC' vertically, if the mechanical movement is too slow so that change of flux is slow and induced emf in the coil is negligible. This corresponds to the coil-mmf remaining constant at F during the transition. Constant mmf means vertical travel of the operating point from H to C'.

(b) H to K horizontally and then K to C'along the characteristic corresponding to ($x + dx$) as the displacement of the movable part. This is possible when the motion is very fast, resulting into flux remaining constant till the operating point traverses from H to K. Then, from K to C', the flux increases, an emf in induced the coil and the mmf finally reaches its value of F, at the point C'.

(c) In reality, the transition from H to C' will be somewhere in between these two extremes mentioned above.

However, for simplicity, one of these extreme conditions has to be accepted. In ***(a)*** above, the mmf remains constant. In ***(b)*** above, the flux (and hence the flux-linkage) remains constant. Let us take the case of constant-mmf. If the process has taken a time of dt,

Electrical-energy input during the process = dW_{elec}

= (voltage applied to the coil) × current × $dt = eidt$

= $d\lambda/dt \times i \times dt = id\lambda = i\,N\,d\phi = Fd\phi$ = area of rectangle $B'HC'D$

In this case, coil-resistance has been neglected.

In terms of Field Energy

At the previous operating point H, the energy stored in the magnetic field,

W_{fld1} = area of $\Delta\, OHB'$

At the new position corresponding to the operating point C', the field energy stored is given by W_{fld2} = area of the $\Delta\, OC'D$

The difference of these two is the change in the energy stored in the magnetic field $= dW_{fld} = \frac{1}{2}\,[OA' \times A'C' - OA' \times A'H]$

$$= \tfrac{1}{2}\, OA'\,[A'C' \times A'H] = \tfrac{1}{2}\, OA'\, HC'$$

$$= \tfrac{1}{2}\,.\,F\,.\,d\phi = \tfrac{1}{2}\, dW_{elec}$$

Out of the energy delivered by the source, half is stored in the magnetic field. Where has the remaining half been utilized? Obviously, this must have been transformed into the mechanical work done. In this case, neglecting losses, it is finally stored in the stretched spring due to its elongation by dx.

Comparing this with the equation (25.1),

$$dW_{elec} = dW_{mech} + dW_{fld}$$

$$= dW_{mech} + \tfrac{1}{2}\, dW_{elec}$$

or $\quad dW_{mech} = dW_{fld} = \tfrac{1}{2}\, dW_{elec}$

Consider that a force F is operative at the displacement of x. This force is in such a direction that x increases or the movable member is attracted towards D. In the same direction, a displacement by dx results into the increase in the energy stored by the spring. Relating the concerned terms,

$$F = k_s.x$$

dW_{mech} = mechanical work done against the force of the stretched spring

$$= -F\,dx = dW_{fld}$$

or $F = -dW_{fld} / dx$, in this case

$$= -\delta W_{fld} / \delta x, \text{ in general}$$

Alternatively, the difference in the energy stored in the spring also gives a very useful relationship.

In the position corresponding to $x + dx$, the energy stored in the spring

$= \frac{1}{2} k_s (x + dx)^2$. Similarly, at x, the energy $= \frac{1}{2} k_s x^2$

$$\text{Difference} = \tfrac{1}{2} k_s [(x + dx)^2 - (x)^2]$$

$$= \tfrac{1}{2} k_s [x^2 + 2.x.dx + (dx)^2 - x^2]$$

$$= \tfrac{1}{2} k_s [2.\,x.\,dx], \text{ neglecting } (dx)^2$$

$$= k_s.\,x.\,dx = F\,dx$$

This difference is nothing but dW_{mech}, which is equal in magnitude to dW_{fld} and confirms the relationship obtained earlier.

In terms of field Co-energy

Proceeding along lines similar to those while dealing with field-energy above, following relationships exist. For simpler discussion, the transition is assumed to be along HKC'. Neglecting area of the small triangle HKC', we have

at x, W'_{fld1} = area of $\Delta\, OA'H$

at $x + dx$, W'_{fld2} = area of $\Delta\, OA'C'$ (neglecting $\Delta\, HKC'$)

$$dW'_{fld} = W'_{fld2} - W'_{fld1} = \phi.\,dF \text{ where } dF = MA'$$

$$= \text{Co-energy in the field } F\,dx$$

Hence, $F = +\,dW'_{fld} / dx$, in this case

$$= +\,\delta W'_{fld} / \delta x, \text{ in general}$$

Energy in Terms of Electrical Parameters

In the preceding article, the energy and force were related in terms of magnetic-system parameters, namely flux and mmf, through the third parameter, the permeance.

It is at times convenient to relate these things in terms of electrical-system-parameters, namely, the inductances and currents. That is being dealt with here only for linear systems. Let $\mathscr{P}$ be the permeance of the magnetic circuit and L be the coil-inductance.

$$W_{fld} = \text{Field-energy} = \tfrac{1}{2}\, F\,\phi = \tfrac{1}{2}\, Ni\,(Ni\,\mathscr{P}) = \tfrac{1}{2}\,(N^2\,\mathscr{P})\,i^2$$

$$= \tfrac{1}{2}\, Li^2$$

$$F = -\,dW_{fld}/dx = -\tfrac{1}{2}\,.\,i^2.\,dL/dx$$

Thus, a force exists if the coil-inductance is dependent on x. Such analysis is more suitable when the system has more than one coils coupled through the magnetic circuit. If two such coils are considered, following data should be known for evaluation of the force, in case of linear displacement:

L_{11} = self inductance of coil - 1

L_{22} = self inductance of coil - 2

L_{12} = Mutual inductance between two coils, 1 and 2

i_1, i_2 = currents through the two coils.

W_{fld} = Total energy stored in the field $= ½ L_{11} i_1^2 + ½ L_{22} i_2^2 + L_{12} i_1 i_2$

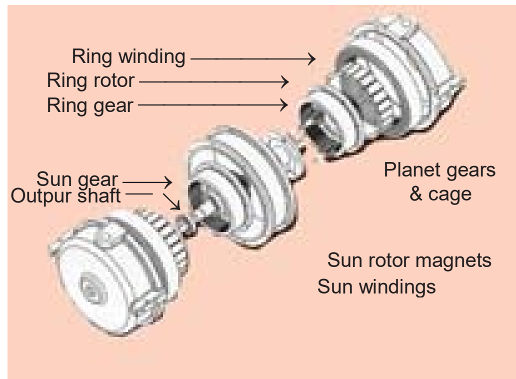
Magnitude of Force, $F = dW_{fld}/dx = ½ i_1^2 dL_{11}/dx + ½ i_2^2 dL_{22}/dx + i_1 i_2 dL_{12}/dx$

From the right-hand side of this equation, it is noted that the inductance-term which is dependent on x contributes to the force.

Rotary Motion

Most popular systems for electro-mechanical energy are Generators and Motors. The preceding discussion dealt with the Linear motions, wherein x represented the displacement parameter, and force was being calculated.

Now we shall deal with the rotary systems, wherein angular displacement parameters (such as θ) and corresponding torque developed by the system will be correlated, through a systematic procedure for a typical rotary machine.



Rotary motion

Description of Simple System

A simple rotary system has a 'stator' and a 'rotor'. Air-gap separates these two. Stator has two similar coils 'a' and 'b' located at 90^0 electrical, with respect to each other. Inner surface of the stator is cylindrical. Outer surface of the rotor is also cylindrical resulting into uniform air-gap length for the machine.

The diagram represents a two-pole machine. Axis of coil 'a' maybe taken as reference, with respect to which the rotor-coil axis makes an angle of θ, at a particular instant of time. For a continuous rotation of the rotor at ω radians / sec, θ = ωt. Coil-'b'-axis is perpendicular to the reference, as shown. Due to the uniform air-gap length, and due to the perpendi-cularity between coils 'a' and 'b', inductance-parameters exhibit the following patterns:

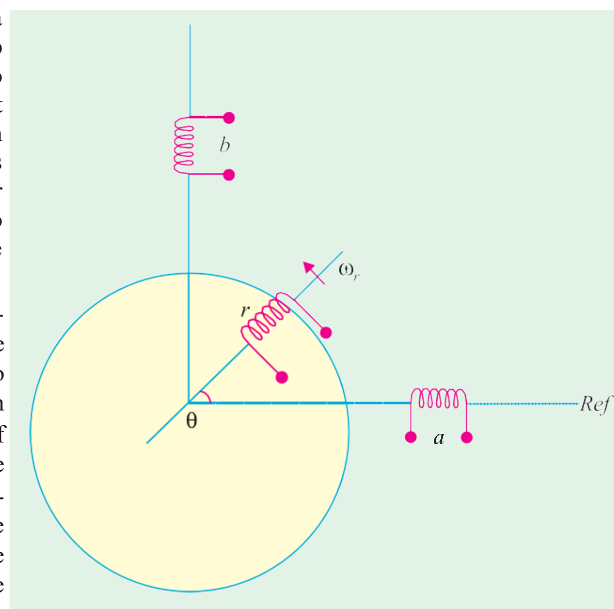


Fig. 25.3. Simple rotary system

Let x represents the inductance parameter as a function of θ. The subscripts indicate the particular parameter. x_{aa} = self-inductance of coil 'a', x_{ab} = mutual inductance between coils a and b and so on. x represents value of the particular inductance parameter, which will help in knowing the variation of inductance with θ.

(a) Self inductances of coils 'a' and 'b' are not dependent on rotor position.

$x_{aa} = L_{aa}$ and $x_{bb} = L_{bb}$, at all values of θ.

(b) Mutual inductance between stator coils 'a' and 'b' is zero, due to perpendicularity.

x_{ab} = zero, for all values of θ.

(c) Self-inductance of rotor-coil is constant and not dependent on θ.

$x_{ra} = L_{rr}$ (= constant), at all values of θ.

(d) When 'r' and 'a' coils have their axes aligned, at θ = 0, the mutual inductance between them is maximum, which is denoted by x_{ar}. At θ = 90^0, their axes are perpendicular resulting into no coupling or zero mutual inductance. When θ = 180^0, r and a coils are aligned in anti-parallel way and hence maximum mutual inductance exists between them with negative sign.

Further, whatever happens to coupling between r and a at a value of θ happens to that between r and b with a delay of 90^0. All these are mathematically represented as :

$$x_{ra} = L_{ra} \cos\theta = L_{ra} \cos(\omega_r t)$$
$$x_{rb} = L_{ra} \sin\theta = L_{ra} \sin(\omega_r t) = L_{ra} \cos(\omega_r t - 90^0)$$

Since 'a' and 'b' coils are alike, the maximum mutual inductance is represented by the same term L_{ra}.

Energy stored in the coils

Energy stored in the magnetic field can *either* be expressed in terms of mmf and flux *or* be expressed in terms of inductance-terms and coil currents. If i_a, i_b and i_r are the coil-currents, stored-energy-terms are as given below:

(i) W_1 = Energy in Self-ind. of coil 'a': ½ $L_{aa} i_a^2$

(ii) W_2 = Energy in Self-ind. of coil 'b': ½ $L_{bb} i_b^2$

(iii) W_3 = Energy in Self-ind. of coil 'r': ½ $L_{rr} i_r^2$

(iv) W_4 = Energy in mutual inductance between 'a' and 'r': $x_{ra}\, i_r\, i_a = x_{ra}\, i_r\, i_a = x_{ra} .\cos\theta . i_r . \, i_a$

(v) W_5 = Energy in mutual inductance between 'b' and 'r' : $x_{rb}\, i_r .\, i_b = x_{ra} \sin\theta .\, i_r .\, i_b$

W = Total energy stored in the system = Sum of all the energy-terms cited above

$= W_1 + W_2 + W_3 + W_4 + W_5$

T = Torque produced = $\delta W / \delta\theta$

If i_a, i_b, i_r are assumed to be constant currents, for simplicity, so that their derivatives with respect to θ (and hence with respect to time t) are zero, the energy-terms which include constant inductances do not contribute to torque. W_1, W_2, and W_3 thus cannot contribute to torque. W_4 and W_5 contribute to torque related by:

$$T = \delta W / \delta\theta = \delta / \delta\theta\, [W_4 + W_5] = \delta / \delta\theta\, [L_{ra}\, i_r i_a \cos\theta + L_{ra}\, i_r i_b \sin\theta\,]$$
$$= L_{ra}\, i_r [-i_a \sin\theta + i_b \cos\theta\,]$$

If $i_a = i_b = i_s$, T = $L_{ra}\, i_r\, i_s \quad [-\sin\theta + \cos\theta\,]$

For such a system, the torque is zero at θ = + 45°, and the torque is maximum at θ = – 45°.

If one of the stator currents is reversed, the result differs. For this, let $i_a = -i_s$ and $i_b = +i_s$

$$T = L_{ra}\, i_r\, i_s [\sin\theta + \cos\theta\,]$$

And the maximum torque occurs at θ = 45^0. This is a position for rotor, which is midway between the two stator coils.

Different Categories

From the torque expressions above, it is clear that the torque exists only when stator and rotor-coils carry currents. When only stator-coils (or only rotor coil) carry current, torque cannot be produced.

(a) One coil each on Stator and on Rotor

In the above mentioned case, let us excite only one stator-coil. Let $i_a = i_s$, $i_b = 0_1$ and i_r maintained as before.

$T = -L_{ra}\, i_r i_s \sin\theta$

Following observations are made for such a case:

(i) At $\theta = 0$, $T = 0$, Mutual inductance x_{ra} is maximum ($= L_{ra}$) and hence the stored energy in mutual inductance is maximum, but torque is zero.

(ii) At $\theta = 90^0$, T is maximum. x_{ra} is zero, hence the concerned stored energy is zero.

(iii) As seen earlier, $W_4 = L_{ra}\, i_r i_a \cos\theta = L_{ra}\, i_r i_s \cos\theta$

Torque $= dW_4 / d\theta = i_r i_s\, d\,(L_{ra} \cos\theta) / d\theta$

Contributed by W_4, the power is related as follows:

Power = rate of change of energy with time

$$= dW_4 / dt = dW_4 / d\theta .\, d\theta / dt = T.\, \omega_r = i_r i_s L_{ra} (-\sin\theta)\, \omega_r$$

Magnitudes of these terms are maximum for $\theta = 90^0$. If θ can be set at 90^0, at all instants of time, torque obtained is maximum. Such a situation does exist in a d. c. machine in which rotor carries an armature winding which is a lap- or wave-connected commutator winding. The brushes are so placed on the commutator that rotor-coil-axis satisfies the above-mentioned condition of $\theta = 90^0$, irrespective of the rotor-position or rotor speed. Such an equivalence of a **rotating** armature coil with such an **effectively stationary** coil is referred to as a **quasi-stationary** coil. It means that a rotating coil is being analyzed as a stationary coil due to its typical behaviour for electro-mechanical energy conversion purposes.

Permanent magnet synchronous motor for washing machine

(b) Two stator coils carrying two-phase currents and rotor-coil carrying d. c.: When two stator coils carry two-phase alternating currents, a synchronously rotating mmf is established. If the rotor-coil carries direct current, and the rotor is run at same synchronous speed, a unidirectional constant torque is developed. Mathematically, similar picture can be visualized, with a difference that the total system is imagined to rotate at synchronous speed. Such a machine is Synchronous machine, (to be discussed in Later chapters). It can be understood through the simple system described here.

Permanent magnet

(c) Machines with Permanent Magnets.

With suitable interpretation, the field side of the simple system can be imagined to be with permanent magnets in place of coil-excited electromagnets. All the interpretations made above are valid, except for the difference that in this case there is no scope for controlling the rotor-coil-current-magnitude.

(*d*) Machines with no rotor coil, but with premeance variation.

Smooth cylindrical rotor surfaces do not exist in such cases. There are no rotor-coils. Due to geometry of the rotor surface, stator-coil- self inductances vary with rotor position. Thinking on lines of relating energy terms and their derivatives for torque-calculations, the working principles can be understood. With simple construction, Reluctance motors belong to this category.

Current in stator coils

(*e*) Switched currents in Stator Coils.

In yet another type, stator coils are distributed and properly grouped. One group carries currents during certain time interval. Then, this current is switched off. Another group carries current in the next time interval and so on. The rotor surface is so shaped that it responds to this current switching and torque is produced. Even though stator-coil-inductances are complicated functions ofrotor position, the method of analysis for such machines is same. Prominent types of machines of this type are: switched reluctance motors, stepper motors, etc.

Vital Role of Air-gap

Magnetic circuit of an electrical machine has a flux established due to coil-mmfs. This flux is associated with stator core, rotor core and air-gap. An important point for understanding is to know which out of these three stores major portion of the field energy. Through an illustrative case, it will be clear below, in example 25.1.

Example 25.1. *Let a machine with following data be considered.*

Calculate the energy stored in the air-gap and compare the same with that stored in the cores.

Stator-core outer diameter = 15 cm

Stator-core inner diameter = 10.05 cm

Rotor-core outer diameter = 10.00 cm

Rotor-core inner diameter = 5 cm

Axial length of the machine = 8 cm

Effect of slotting is neglected. The core volumes and air-gap volume for the machine shown in Fig 25.4 have to be calculated.

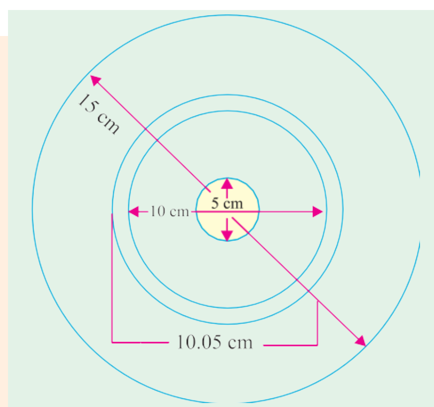


Fig. 25.4

Solution.

Volume of Stator-core = $(\pi / 4) \times (15^2 - 10.05^2) \times 8 \text{ cm}^3 = 779 \text{ cm}^3$

Volume of Rotor-core = $(\pi / 4) \times (10^2 - 5^2) \times 8 \text{ cm}^3 = 471 \text{ cm}^3$

Volume of air-gap in the machine = $(\pi / 4) \times (10.05^2 - 10^2) \times 8 \text{ cm}^3$

$= 6.3 \text{ cm}^3$

Let the relative permeability of the core material be 1000. If the flux density is B Wb / m^2, and μ is the permeability, the energy-density is ½ x B^2 / μ Joules / m^3. Let the flux density be 1.20 Wb / m^2. Energy density in air-gap = ½ × 1.20^2 / (4 π × 10^{-7}) = 572350 Joules / m^3

= 0.573 Joules / cm^3

Energy stored in air-gap = 0.573 × 6.3 = 3.6 Joules

Energy-density in Magnetic medium = ½ × 1.20^2 / (4π × 10^{-7} × 1000) = 573 J / m^3

It is assumed only for simplicity that the flux density is same for the entire core of stator and of rotor.

Energy stored in stator-core = 573 × 779 × 10^{-6} = 0.45 Joule

Energy stored in rotor-core = 573 × 471 × 10^{-6} = 0.27 Joule

It is worth noting that even though the ratio of volumes is 198, the ratio of energies is 0.2, since, for the present case,

$$K_v = \frac{\text{volume of (Stator - core + Rotor - core)}}{\text{Volume of air - gap}} = \frac{779+471}{6.3} = 198$$

k_E = Energy stored in cores / Energy stored in air-gap = (0.45 + 0.27) / 3.6 = 0.2

The ratio are like this due to μ_r being 1000, and k_v / k_E = 198 /0.2 = 1000

Alternatively, an air-gap of volume 6.3 cm^3, [surrounded by the magnetic medium of μ_r = 1000] is equivalent to the magnetic medium of volume 6.3 × 1000 cm^3.

$$\frac{\text{Converted equivalent volume of air-gap}}{\text{Volume of (Stator + Rotor)}} = \frac{6300}{779+471}$$

$$\frac{\text{Energy stored in air- gap}}{\text{Energy stored in (Stator + Rotor)}} = \frac{3.6}{(0.45 + 0.27)} = 5$$

This correlates the various parameters and confirms that the stored energy is maximum in the air-gap.

Or, one can now say that in the process of electro-mechanical energy-conversion, the air-gap plays a very vital role.

However, the stator-core and rotor-core help in completing the flux-path in a well defined manner for effective and efficient working of a rotary machine.

Example 25.2. *An electromagnetic relay has an exciting coil of 800 turns. The coil has a cross-sectional area of 5 cm × 5 cm. Neglect reluctance of the magnetic circuit and fringing.*

(a) (i) *Find the coil inductance if the air-gap length is 0.5 cm.*

(ii) *Find the field energy stored for a coil current of 1.25 amp.*

(b) *Coil-current remaining constant at 1.25 A, find the mechanical energy output based on field-energy changes when the armature moves to a position for which x = 0.25 cm. Assume slow movement of armature.*

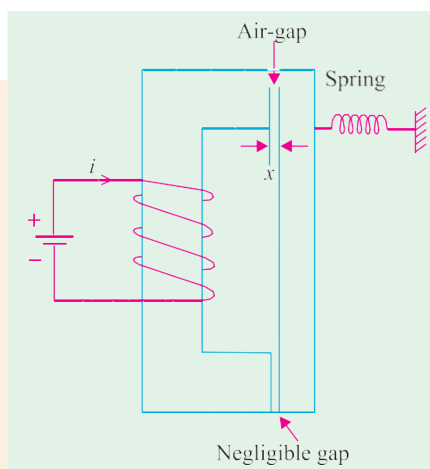


Fig. 25.5 Electro-magnetic relay

(c) *Repeat (b) above based on force-calculations and mechanical displacement.*

(d) *What will be change in above results of mechanical work done, if the mechanical movement is fast, keeping the flux initially constant ?*

Solution.

(a) ***(i) Permeance at air-gap*** $= \dfrac{\mu_0\, 5\times5\times10^{-4}}{0.5\times10^{-2}} = 4\pi\times10^{-7}\times10^{-2} = 6.28\times10^{-7}$

Coil Inductance = $N^2 \mathscr{P}_m = 800 \times 800 \times 6.28 \times 10^{-7} = 0.402$ H

(ii) Energy stored in magnetic field = $1/2\, Li^2 = 1/2 \times 0.402 \times 1.25^2$

$= 0.314$ joule

(iii) $W'_{fd} = 1/2\, L(x) i^2 = 1/2 \left[\dfrac{H^2 \mu_0 A}{l_x}\right] = \dfrac{1/2 \times 800 \times 800 \times 4\pi \times 10^{-7} \times 5 \times 5 \times 10^{-4}}{x} \times i^2$

$$= \frac{1.005\times10^{-3}}{x}\times i^2$$

$$F_f = \frac{\delta}{\delta_x}\frac{[i^2 \times (1.005\times10^{-3})]}{x} = [1.005\times10^{-3}]\times i^2 \times \frac{-1}{x^2}$$

This is to be evaluated at $x = 0.5 \times 10^{-2}$

$$= \frac{-1.005\times10^{-3}\times1.25\times1.25}{(0.5\times10^{-2})^2} = -\ 62.8 \text{ NW}$$

This force has to be balanced by the spring-tension.

(b) ***Energy-computations :*** Inductance for $x = 0.25$ cm is first calculated. $L(x_2) = N^2 \mathscr{P}_{m2}$

$= 800 \times 800 \times 2 \times 6.28 \times 10^{-7} = 0.804$ Henry

If the mechanical movement is slow, net mmf remains unchanged and the operating point moves along *HC* vertically upwards and settles at *C*. Added Electrical Energy input during change-over of the operating point from *H* to *C*,

= area of rectangle $BDCH = (\phi_2 - \phi_1)\, F_1$

$$\left[\frac{L(x_2).i - L(x_1)i}{N}\right] Ni$$

$= i^2 [L(x_2) - L(x_1)] = 1.25^2 \times [0.804 - 0.402]$

$= 0.628$ joule

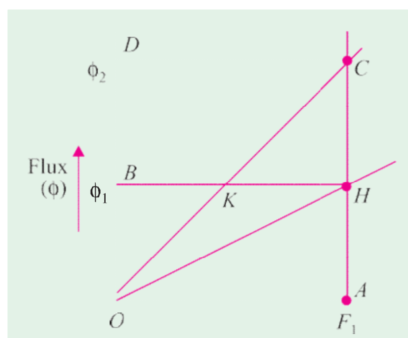


Fig. 25.6 Graphical correlation of energy-terms for the relay

Out of this, the additional stored energy in field, $dW_{fld} = ½\, [L(x_2) - L(x_1)]$

$= 0.314$ joule

The remaining 0.314 joule is transformed into mechanical form and is related to the work-done. This is obtained when the force on moving member is multiplied by the displacement.

$$\therefore \int_{x_1}^{x_2} dW_{mech} = 0.314 \text{ joule}$$

(c) As in *a* (*iii*) above,

$F(x) = [1.005 \times 10^{-3} \times 1.25^2]\, [-1/x^2]$

$$dW_{mech} = F(x).dx$$

$$W_{\text{mech}} = \int_{x_1}^{x_2} F(x)dx = k.\left[\int_{x_1}^{x_2} -1/x^2 d_x\right] = -k.\int_{x_1}^{x_2} x^{-2}dx$$

$$= 1.005\times 10^{-3} \times 1.25^2 \times \left[\frac{1}{0.25} \quad \frac{1}{0.5}\right] \times 10^2 = \frac{1.005\times 1.25^2}{10} \times 2 = 0.314 \text{ joule}$$

This agrees with answer obtained in (*b*) above.

With fast movement of armature, the operating point will move from H to K first, then follow the path *KC*.

This means that the energy represented by the area of the triangle KHC corresponds to the reduced consumption of energy.

OC has a slope of $\mathcal{P}_{m\,2} = 12.56 \times 10^{-7}$

OH has a slope of $\mathcal{P}_{m1} = 6.28 \times 10^{-7}$

BK = mmf required for establishing a flux of ϕ_1 with an air-gap of 0.25 cm

BK = ½ *OA* = ½ × (800 × 1.25) = 500 amp-turns = *HK* in the present case.

Area of the triangle *KHC* = ½ × *KH* × *HC*

= ½ × 500 × [$\phi_2 - \phi_1$]

= ¼th of area of rectangle *BDCH*, in this case

= ¼ × 0.628 = 0.157 joule.

Hence, Electrical energy fed during this process = area *BKCD*

= area *BDCH* – area *KHC* = 0.628 – 0.157 = 0.471 joule

Increase in field energy stored $\Delta W'_{fld}$ = area *OKH*

= area OHC – area *KHC* = 0.314 – 0.157 = 0.157

Mechanical Energy output = 0.471 – 0.157 = 0.314 Joule

It indicates that with fast movement, the electrical energy-input and the field-stored energy have decreased by 0.157 J each but the mechanical-energy-term remains unaffected by fast or slow movements of armature.

Dynamic Equations and System-model of a Simple System

It is quite necessary to analyze electro-mechanical conversion system for predicting the performance and/or for monitoring the system. A simple system is being taken up here to deal with dynamic equations and a simple model with its components is being related to the system. The details will vary from system to system, and accordingly the equations will vary.

Fig. 25.7 shows different components of such a system meant for electrical to mechanical conversion. On one side, an electrical source feeds the device at the 'electrical port'. On the other side, a force f_e is developed at the 'mechanical port'. Mechanical load is connected to this port.

(a) ***At Electrical Port*** **:** A voltage source is shown to feed the device. r is its effective internal resistance. At the electrical port, the inputs are λ (= flux linkage with the coil) and *i*. From λ , the voltage induced in the coil can always be evaluated.

(b) ***Role of the Conversion device*****:** With these inputs, the device converts the energy into mechanical form, and is available as a force f_e (in case of linear motions), and, displacement *x* measured from a suitable reference.

(c) ***At the Mechanical Port*:** The possible items are: spring, damper, mass and an applied mechanical force. Their natural and simple dependence on displacement x and its derivatives are indicated below:

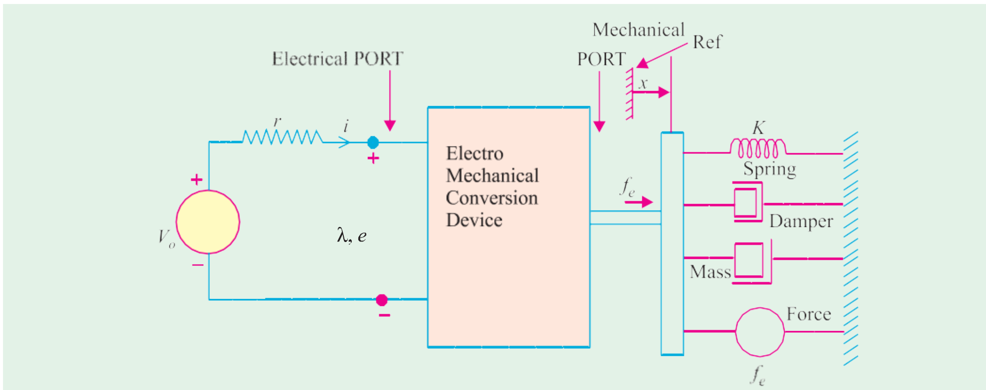


Fig. 25.7 Linear motion: MODEL

(i) **Spring:** Force required to overcome spring elongation is proportional to the displacement x.

(ii) **Damper:** Force required to overcome damping action in the system is proportional to derivative of x.

(iii) **Mass:** Force required to overcome acceleration of mass is proportional to second derivative of x.

(iv) **Applied force, f_0:** This has to be overcome by f_e . In terms of an equation, these terms are related as follows:

$$f_e = k_s (x - x_o) + B\dot{x} + M\ddot{x} + f_o$$

where

k_s = spring constant

x_o = value of x for unstretched spring

B = damping constant

M = Mass to be accelerated

f_o = External mechanical force applied to the system.

Static ally induced emf and Dynamically Induced emf :

In Fig. 25.7 source voltage is v_o. Let L (x) be the coil inductance as a function of displacement x. In a very general case,

$$\begin{aligned} v_o(t) &= ri + d\lambda/dt \\ &= ri + d/dt\,[L(x) \times i] \\ &= ri + L(x).\, di/dt + i.\, dL(x)/dx.\, dx/dt \end{aligned}$$

The second term on the right-hand side is statically induced emf (or transformer-emf), since change of current with time is responsible for it. This cannot produce any force (or torque) and hence cannot convert energy from electrical to mechanical form (or vice-versa).

The third term on the right hand side includes the speed (= dx/dt) and dependence of L (x) on x. Any of these, if non-existent, will mean that third term reduces to zero. This term relates dynamically induced emf (= speed emf) and is the main indicator of the process of electro-mechanical energy conversion. So, for conversion, there must be an inductance which varies with the system position, and a motion must be there. In addition, coil must carry a current.

Having understood the linear-motion-system, it is easier to understand the system with rotary motion, with due modifications.

Example 25.3. *A doubly excited rotating machine has the following self and mutual inductances.*

$r_s = 40\,\Omega$, $L_s = 0.16\ H$

$r_r = 2\ \Omega$, $L_r = 0.04 + 0.02\cos 2\theta$

$M_{sr} = 0.08\cos\theta$

where θ is the space-angle between axes of rotor-coil and of stator-coil. The rotor is revolving at a speed of 100 radians/sec. For i_s = 10 Amp d. c., and i_r = 2 Amp d. c., obtain an expression for torque and corresponding electrical power.

[Rajiv Gandhi Technical University, Bhopal, Summer 2001]

Solution. W_{fld} = Total energy stored

$$= \tfrac{1}{2}\, L_s\, i^2_s + 1/2\, L_r\, i^2_r + M_{sr}\, i_s i_r$$

$$= \tfrac{1}{2}(0.16)\, i^2_s + \tfrac{1}{2}\,[0.04 + 0.02\cos 2\theta]\, i^2_r + [0.08\cos\theta]\, i_s i_r$$

since i_s and i_r are direct currents of constant magnitudes, there is no variation with φ or with t.

Relating torque with W_{fld} and substituting current-magnitudes,

$$\text{Torque, } T = \frac{-dW_{fld}}{d\theta} = -\,[0 + \tfrac{1}{2}\times 0.02 \times 2^2\,(-\,2\sin 2\theta) + 0.08\,(-\sin\theta)\,(10\times 2)]$$

$$= 0.08\sin 2\theta + 1.6\sin\theta$$

$$= 1.6\sin\theta + 0.08\sin 2\theta \text{ Nw-m}$$

On the right hand side, the first term is electromagnetic Torque which is dependent on both the currents. Second term is dependent only on one current, and is of the type categorized as Reluctance-torque which depends on non-cylindrical shape,in this case, on the stator side, as shown in Fig. 25.8

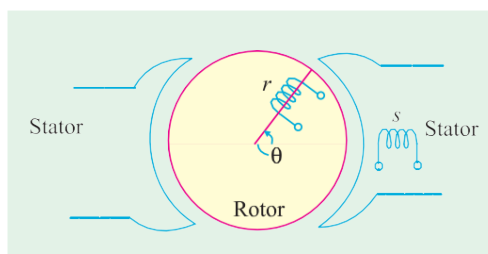


Fig. 25.8

Starting from W_{fld}, electrical power can be expressed, since it is well -known that

Power = time rate of change of energy

$$\text{Electrical power,} \quad p = \frac{dW_{fld}}{dt}$$

$$= \frac{d}{dt}\left[\tfrac{1}{2} L_s i^2_s + \tfrac{1}{2} L_r i^2_r + M_{ST}\, i_s i_r\right]$$

$$= \tfrac{1}{2} i^2_s \frac{d}{dt}(L_s) + \tfrac{1}{2} i^2_r \frac{d}{dt}(L_r) + i_s i_r \frac{d}{dt}(M_{sr})$$

$$= \frac{1}{2} i_s^2 \left[\frac{d}{d\theta}(L_s)\right]\frac{d\theta}{dt} + \frac{1}{2} i_r^2 \left[\frac{d}{d\theta}(L_r)\right]\frac{d\theta}{dt} + i_s i_r \left[\frac{d}{d\theta}(M_{sr})\right]\frac{d\theta}{dt}$$

$$= \frac{1}{2} i_s^2 \,[\text{zero}]\times 100 + \frac{1}{2} i_r^2 \,[-\,0.02\times 2\times \sin 2\theta]\times 100 + i_s i_r \,[-\,0.08\sin\theta]\times 100$$

Substituting numerical values of currents,the electrical power is expressed as a function of θ, as below :

$$p = [\text{zero} + (-\,8)\sin 2\theta + (-160)\sin\theta] \text{ watts}$$

$$= [-\,160 \sin\theta - 8 \sin 2\theta\,] \text{ watts}$$

Proper interpretation of sign of power (as dependent on θ) is important. Positive power is received by the coils, while negative power is received by the source.

Example 25.4. *An inductor has an inductance which varies with displacement x as*

$$L = 2\,L_o / [1 + (x/x_o)]$$

Where L_o = 50 mh, x_o = 0.05cm, x = displacement in cm,

The coil –resistance is 0.5 ohm.

(a) *The displacement x is held constant at 0.075 cm, and the current is increased from 0 to 3 amp. Find the resultant magnetic stored energy in the inductor.*

(b) *The current is then held constant at 3 amp and the displacement is increased to 0.15 cm. Find the corresponding change in the magnetic stored energy.*

Note. Assume that all electrical transients are negligible.

Solution. (*a*) Inductance at x = 0.075 cm is calculated first.

$$L_1 = \frac{2L_0}{1+(0.075/0.05)} = 40 \text{ mH}$$

$\lambda_1 = L_1 \times$ current $= 120 \times 10^{-3}$, corresponding to point A in Fig. 25.9

$W_{fld} = ½\, L_1 (3)^2 = ½\,(40 \times 10^{-3} \times 9) = 0.18$ joule

(*b*) Inductance for x = 0.15 cm is to be calculated now.

$$L_2 = \frac{2L_0}{1+(0.075/0.05)} = 25 \text{ mH}$$

With current held constant at 3amp, the flux-Linkage is now $\lambda_2 = (25 \times 10^{-3}) \times 3 = 75 \times 10^{-3}$

Since the current is constant at 3amp, magnetic stored energy is reduced by the area of triangle *OAB*, in Fig. 25.9

Area of triangle *OAB* = ½ × 3 × (120 –75) 10^{-3} = 0.0675 joule

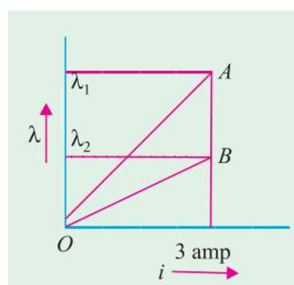


Fig. 25.9

Check : Stored-energy at *B* in terms of L_2 and *i*, is given by

$W_{fld} = ½\,(25 \times 10^{-3}) \times 3^2 = 0.1125$ joule

Alternatively, $W_{fld2} = W_{fld1}$ – area of Δ OAB

$= 0.18 - 0.0675 = 0.1125$ Joule

Example 25.5. *If the inductor in the previous case is connected to a voltage source which increases from 0 to 3 V [part (a)] and then is held constant at 3 V [part (b)], repeat the problem, assuming that electrical transients are negligible.*

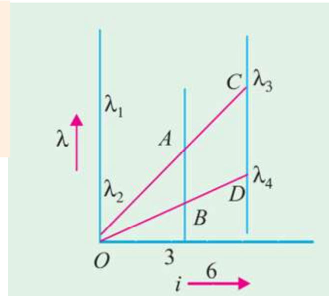


Fig. 25.10

Solution. Coil resistance is 0.5 ohm. When the voltage reaches 3 V, the coil current is 6 amp. In part (*a*), L_1 = 40 mH. Hence, W_{fld3} = energy stored = $\frac{1}{2} L_1 i_1^2$ = 0.72 joule, at point *C* in Fig. 25.10. In part (*b*), $L_2 = 25$ *mH*. The current is held constant at 6 amp. Working on similar lines,

ΔW_{fld} = change in the field energy stored = area of triangle *ODC* or $\Delta W_{fld} = W_{fld3} - W_{fld4}$

$W_{fld} = \frac{1}{2} \times 25 \times 10^{-3} \times 36 = 0.45$ Joule, at point *D*

Change in energy stored in the field $= W_{fld3} - W_{fld4} = 0.72 - 0.45 = 0.27$ joule

Or ΔW_{fld} = area of $\Delta ODC = \frac{1}{2} \times 6 \times (\lambda_3 - \lambda_4)$

Here $\lambda_3 = 40 \times 10^{-3} \times 6$, and $\lambda_4 = 25 \times 10^{-3} \times 6$

$\Delta W_{fld} = \frac{1}{2} \times 6 \times 6 \times 10^{-3} (40 - 25) = 0.27$ joule

Example 25.6. *A coil of an electromagnetic relay is associated with a magnetic circuit whose reluctance is given by*

$= a + bx$

where a and b are positive constants decided by the details of the magnetic circuit, in which x is the length of the air-gap between fixed and movable members. If the coil is connected to an A.C. source where voltage is described by

$v = V_m \sin \omega t,$

find the expression for the average force on armature, with air-gap held constant at x.

Solution. If ϕ = flux established, in Webers

N = number of turns on the coil, λ = flux-linkage in Weber-turns

$W_{fld} = \frac{1}{2} \mathcal{R} \phi^2$

And force $$F = \frac{\delta W_{fld}}{\delta x} = \frac{1}{2}\phi^2 \frac{\delta \mathcal{R}}{\delta x} = \frac{1}{2} b \phi^2$$

The current in the coil is given by

$$v = R + L\frac{di}{dt}$$

for which, the steady-state solution for current with an a.c. voltage applied to the coil is given by

$$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \angle -\theta \quad \text{where} \quad \theta = \tan^{-1}\frac{(\omega L)}{R}$$

RMS voltage, $V = (V_m) / \mathcal{R}\sqrt{2}$

Instantaneous current i is expressed as

$$i = \frac{\sqrt{2}\,V}{\sqrt{R^2+\omega^2 L^2}}\sin(\omega t-\theta)$$

Further, L = N²/ $\mathcal{R}$

$$\phi = Ni/\mathcal{R} = \frac{\sqrt{2}\,NV}{\sqrt{(R\mathcal{R})^2+(N^2\omega)^2}}\sin(\omega t-\theta)$$

Force, $$F_f = \frac{\mathcal{R}-bN^2V^2}{(R\mathcal{R})^2+(N^2\omega)^2}\sin^2(\omega t-\theta)$$

The last term $\sin^2(\omega t-\theta)$ has a time average (over a cycle) of ½.

Hence, average force, $$F_f(\text{av}) = -\frac{1}{2}\frac{\mathcal{R}\,bN^2V^2}{(R\mathcal{R})^2+(N^2\omega)^2}$$

Force is in such a direction that x will be reduced, or that the energy stored tends to increase.

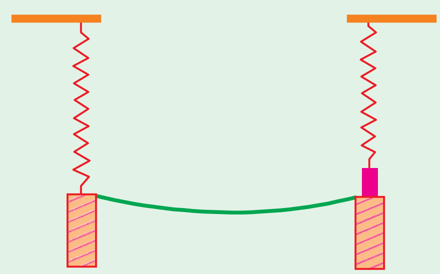
Example 25.7. *Two coupled coils have self- and mutual-inductances as expressed below:*

$$L_{11} = 1 + (1/x),$$
$$L_{22} = 0.5 + (1/x),$$
$$L_{12} = L_{21} = 1/x$$

These expressions are valid over a certain range of linear displacement x, in cms. The first coil is excited by a constant current of 20 A and the second one, by a constant current of – 10 A. Find

(a) *mechanical work done if x changes from 0.5 to 1.0 cm*

(b) *energy supplied by the two electrical sources in (a) above.*



Two coupled coils

Solution. With data given, substituting the values of currents,

$$W_{fld} = \tfrac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \tfrac{1}{2}L_{22}i_2^2$$
$$= 225 + (50/x)$$

$$F_f = -\delta W_{fld}/\delta x = 50/x^2$$

(a) $$\Delta W_{mech} = \int_{0.5}^{1.0}(50/x^2)\,dx$$

$$= 50\left[x^{-1}/-1\right]_{0.5}^{1.0} = +50 \text{ joules}$$

At $x = 0.5$, $W_{fld} = 325$ joules, and

at $x = 1.0$, $W_{fld} = 275$ joules

Thus, increase in x from 0.50 to 1.0 cm decreases the stored energy in the field from 325 to 275 joules. The field-system, thus, releases an energy of 50 joules.

(*b*) Calculations of Energy input from electrical sources –

$$\lambda_1 = L_{11}\, i_1 + L_{12}\, i_2$$

$$= 20\,[1 + (1/x)] - 10\,(1/x) = 20 + (10/x)$$

At $x = 0.5$, $\lambda_1 = 20 + 20 = 40$ Wb-turns

$x = 1.0$, $\lambda_1 = 20 + 10 = 30$ Wb-turns

$\Delta_{\text{Welec1}} = i_1$ [change in λ_1 due to displacement]

$= 20 \times (-10) = -200$ Joules

Similarly, $\lambda_2 = L_{12}\, i_1 + L_{22}\, i_2 = -5 + (10/x)$

At $x = 0.5$, $\lambda_2 = -5 + 20 = +15$ Wb-turns

$x = 1.0$, $\lambda_2 = -5 + 10 = +5$ Wb-turns

$\Delta W_{elec2} = (-10)\,(-10) = +100$ Joules.

Seeing the signs and numerical values, it can be seen that Source 1 receives an energy of 200 Joules, which comes from three constituents:

100 J from source 2,

50 J from field energy stored,

and 50 J from mechanical system.

Tutorial Problems 25.1

(a) A magnetic circuit has a coil with 1000 turns. Its reluctance is expressed as

$\mathcal{R} = [\,8.5 + 40\text{ g}] \times 10^{+3}$ MKS units

where g = air- gap length in mm, between fixed and movable parts. For a coil current of 2.0 amp held constant and with slow movement, calculate the change in the field energy stored, if the length of the air-gap changes from 0.20 to 0.15 cm. Calculate the mechanical force experienced by the system.

Hint: $\Delta\, W_e = i\,(\lambda_2 - \lambda_1)$, $\Delta\, W_{fld} = ½\, \Delta\, W_e$

Force, $F = -\Delta\, W_{\text{fld}} / \Delta\, x$

[**Ans.** $\Delta W_{fld} = 6.60$ J, Force = 13200 Nw]

(b) An electro-magnetic relay with an air-gap of x cm has the current and flux-linkage relationship as

$i = \lambda^2 + \lambda\;(\text{o}.5 - x)^2$ amp, for $x < 0.5$ cm

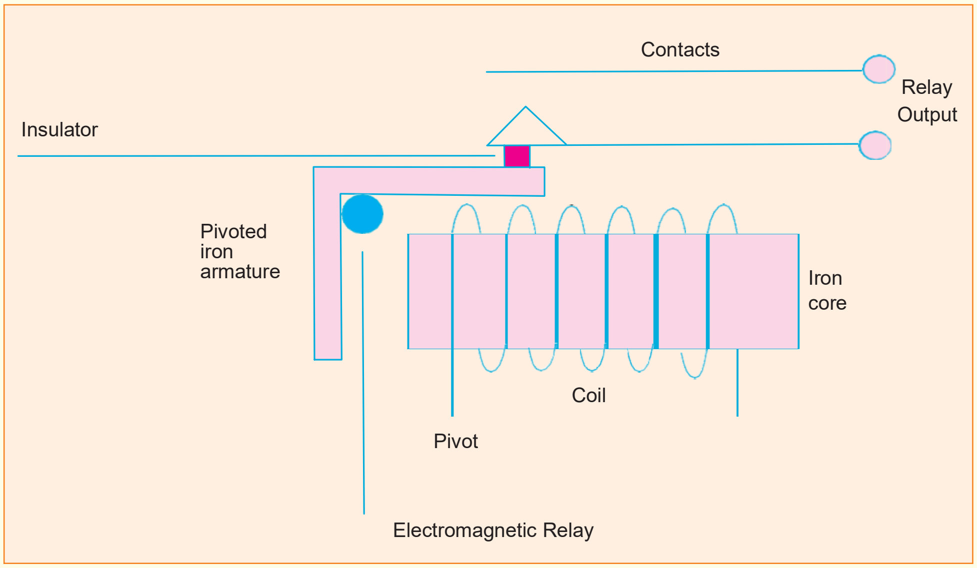
Find the force on armature as a function of λ and x.

Hint: $W_f\,(\lambda_1\, x) = \int_{0.5}^{1.0} i d\,\lambda$

And $F_f = -\,\delta W_f / \,\delta x$

[**Ans.** $W_f\;(\lambda\,, x) = (\lambda^3/3) + (\lambda^2/2)\,(0.5 - x)^2$

$F_f = \lambda^2\;(0.5 - x)$



(*c*) For a rotary system, the stator-coil and the rotor-coil have self and mutual-inductances as described below, with suffix 1 for stator and 2 for rotor:

$$L_{11} = L_{22} = 4 - (6\theta/\pi) \quad \text{for} \quad 0 < \theta < \pi/2$$

$$= 1 + (6/\pi)(\theta - 0.5\pi) \quad \text{for} \quad \pi/2 < \theta < \pi$$

(Note: Self inductances cannot be negative.)

$L_{12} = L_{21} = 6(1 - 2\theta/\pi)$ for $0 < \theta < \pi$

Evaluate the inductances and the torque for $\theta = \pi/4$ and the two coil currents of 5 amp constant in magnitude.

Hint: $\delta L / \delta\theta$ contributes to torque.

[Ans. $L_{11} = L_{22} = 2.5\ H$
$L_{12} = +3\ H$
$T = 450/\pi$ Nw**]**