

SIMPLE STRESS & STRAIN

Types Of Stress

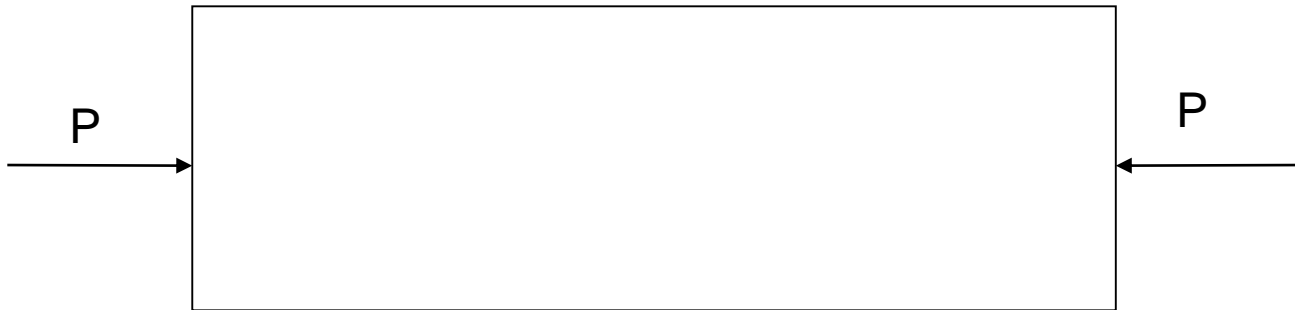
- 1. Tensile stress :- When a section is subjected to two equal and opposite pulls , as a result the body tends to lengthen , the stress induced is called tensile stress . The stress per cross sectional area of the body is known as intensity of tensile stress .This is denoted by p_t or σ_t or f_t .
- Now if the 'P' is the force and the cross sectional area is 'A'
then $f_t = P / A$



2. Compressive Stress

- When a section is subjected to two equal and opposite pushes, as a result the body tends to shorten and thus the stress induced is called compressive stress. The stress per unit cross sectional area of the body is known as intensity of compressive stress. It is opposite in sign of tensile stress.

This is denoted by ' σ_c ', now if the compressive load is 'P' kg and the cross sectional area 'A', then $\sigma_c = P / A$.



3. Shear Stress

- When a body is subjected to a vertical force, the system of internal forces develop within the body is known as shear stress and the stress per unit cross sectional area of the body is known as intensity of shear stress.

This is denoted by σ_s . If the vertical force (push) is 'P' kg and the cross sectional area is 'A' then $\sigma_s = P / A$

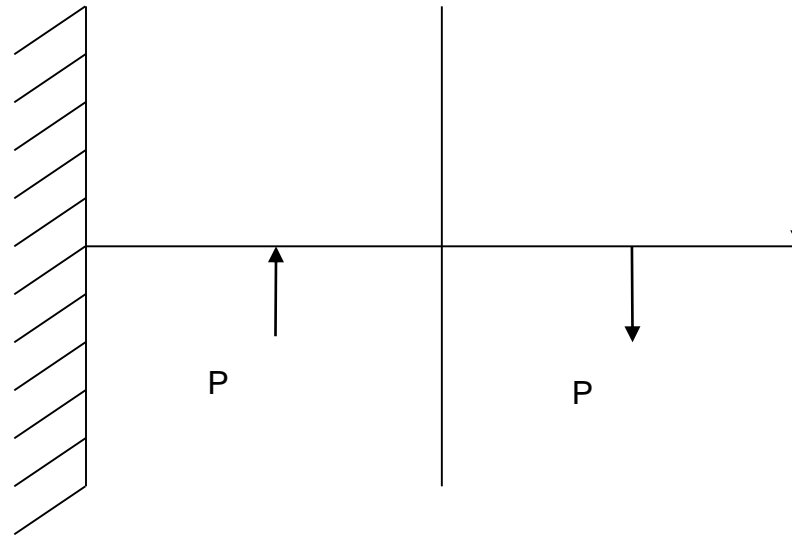
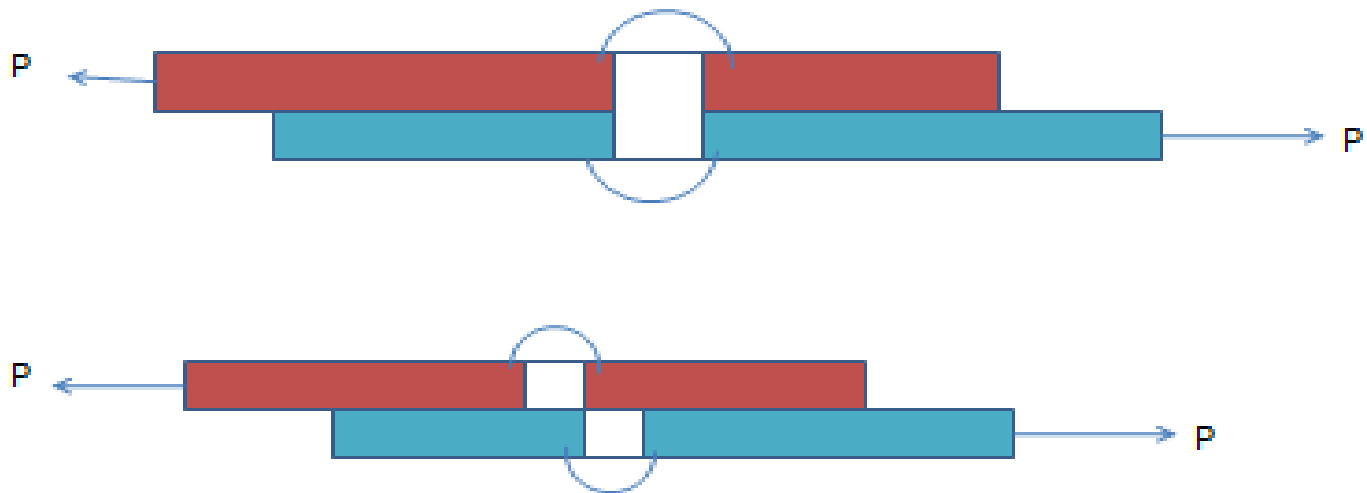


FIGURE SHOWS HOW SHEARING OCCURS IN A RIVETTED JOINT



STRAIN

- Whenever a single force or a system of force acts on a body, it undergoes some deformation. The deformation per unit length is known as strain.

Mathematically , $e = \delta l / l$ ($\delta = \text{delta}$)

Where , $e = \text{strain}$

$\delta l = \text{change of length of the body .}$

$l = \text{Original length of the body .}$

Types of strain

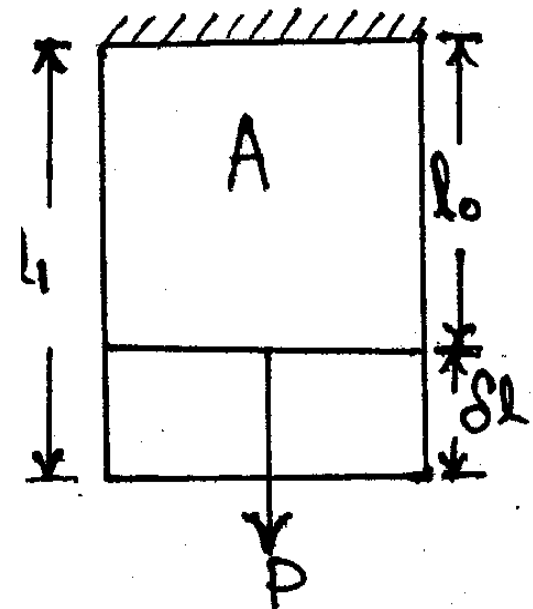
1. **Tensile Strain :-** (ϵ_t) Tensile strain is the deformation produced in a body per unit length due to some tensile force. In the figure let 'A' be a bar of length l_0 . Now an external force 'P' is applied at the bottom end of the bar keeping the top end fixed. As a result the bar elongated by δl .

Deformation (elongation)

i.e., tensile strain (ϵ_t) = $\frac{\text{Deformation (elongation)}}{\text{Initial length}}$

$$= \frac{\delta l}{l_1 - l_0}$$

$$= \frac{\delta l}{l_0}$$



2. Compressive Strain (e_c)

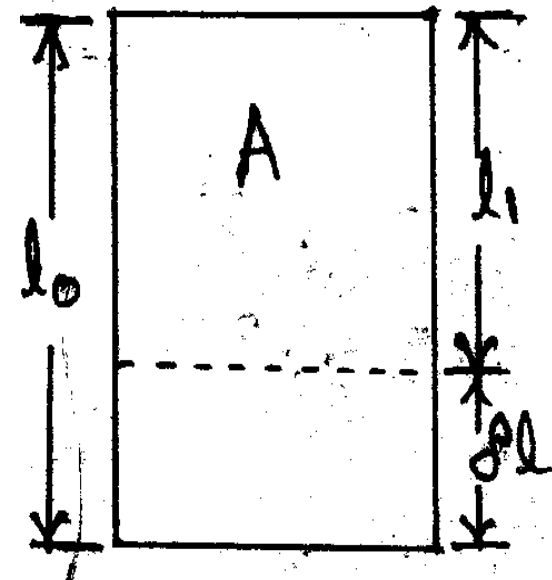
- Compressive strain is the deformation produced in a body per unit length due to some compressive force.

In the figure let 'A' be a bar of length l_0 . Now an external force 'P' is applied at the bottom end of the bar keeping the top end fixed. As a result the bar is compressed by δl .

$$\text{compressive strain } e_c = \frac{\text{Deformation (compression)}}{\text{Initial length}}$$

$$= \frac{\delta l}{l_0 - l_1}$$

$$= \frac{\delta l}{l_0}$$



3. Shear strain

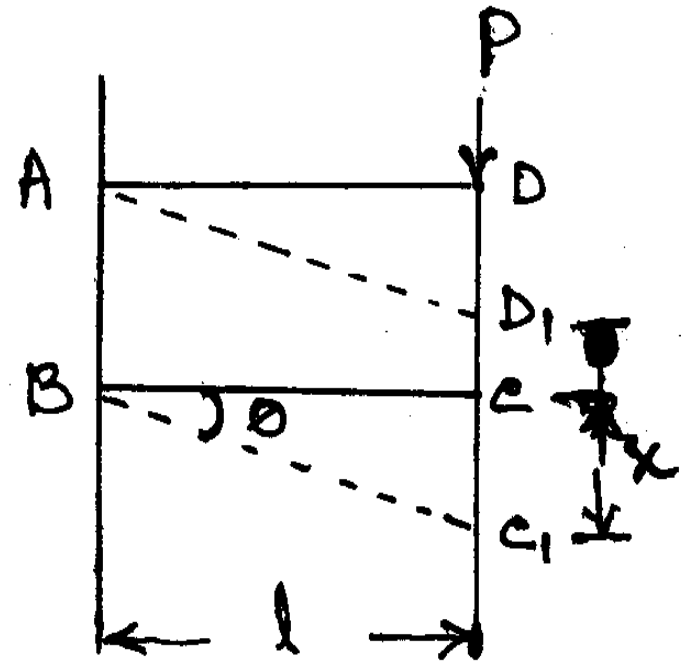
- Shear strain (e_s) is the angular deformation (distortion) in a body per unit length due to some external vertical load.

Let ABCD be a rectangular block whose AB side is fixed. Now a vertical force 'P' is applied at the bottom end of the bar keeping the top end fixed. As a result the block ABCD is distorted to ABC₁D₁ through an angle θ . Let CC₁ be X.

Then, $\tan \theta = CC_1 / BC = x / l$

i.e, $\tan \theta = \theta$, Since θ is very small, therefore $\tan \theta = \theta$ in radian,

this angular deformation is called shear strain.



4. Volumetric Strain

Volumetric Strain (e_v) is the deformation produced in a body per unit per volume due to some external load (either tensile or compressive) in the figure cube having side 'a' is shown so the volume of the cube is a^3 .

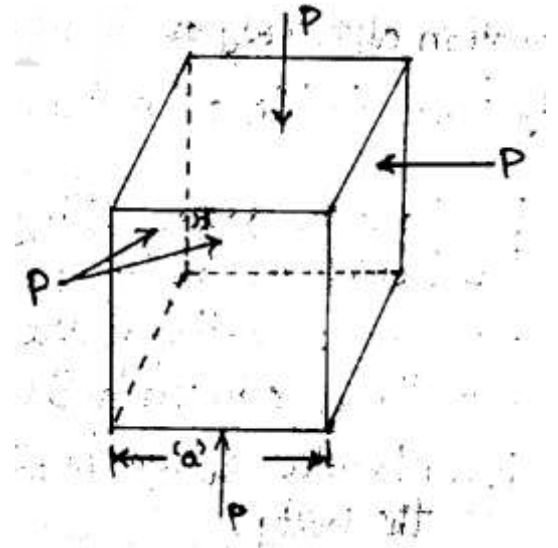
Now an external force (here compressive) P is applied on all places of the cube. As a result inside of the cube becomes $a - \delta a$, where δa is the deformation (compression) in each side.

Change in volume

$$\text{Volumetric strain } (e_v) = \frac{\text{Change in volume}}{\text{Initial volume}}$$

$$\text{Volumetric strain } e_v = 3 \delta a / a ,$$

$$e_v = 3 \cdot \text{Linear strain (Tensile or compressive)}$$



- Hookes Law :- It states when a material is loaded within its elastic limit the stress is proportional to the strain .

Mathematically, Stress \propto Strain

$$\text{Stress} / \text{Strain} = E = \text{a constant}$$

It may be noted that Hookes law equally holds good for tension as well as compression .

- Modulus Of Elasticity :- (Youngs Modulus)

Whenever a material is loaded within its elastic limit the stress is proportional to the strain .

$$P \propto e$$

$$\text{therefore } \sigma / e = E$$

where , p or f or σ = Stress

e = Strain

E = A constant of proportional known as Modulus of Elasticity or Young Modulus .

Deformation

- Deformation of a body due to force acting on it :- Consider a body subjected to a tensile stress .

Let ,

p = Load or force acting on the body .

l = Length of the body

A = Cross sectional area of the body

f = Stress induced in the body

E = Modulus of elasticity for the material of the body .

e = Strain

δl = Deformation of the body .

We know that the stress , $p = P / A$ And $p / e = E$

Therefore , strain , $e = P / AE$ [since , $f = P / A$]

Or , $\delta l / l = P / AE$ [since $e = \delta l / l$]

Therefore , $\delta l = Pl / AE$

Principle Of Superposition

- Whenever a body is subjected to a number of forces acting on its outer edges as well as some other sections along the length of the body in such the forces are split up and their effects are considered on individual sections . The principle of finding out the resultant deformation is called the principle of super position .

The relation for the resulting deformation is modified as ,

$$\delta l = \frac{P_1 l_1}{AE} + \frac{P_2 l_2}{AE} + \frac{P_3 l_3}{AE} + \dots$$

Where : $P_1, P_2, P_3 \dots$, Forces acting on the section 1 , 2 , 3 --
 $l_1, l_2, l_3 \dots$, length of the section 1 ,2 ,3 , ----

STRESSES IN COMPOSITE BARS

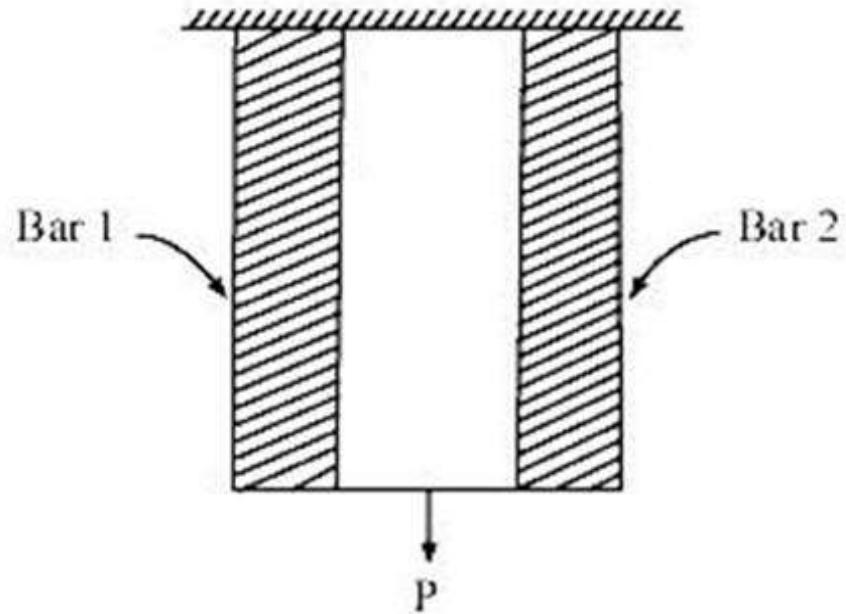
A composite bar may be defined as a bar made up of two or more different materials joined together in such a manner that the system extends or contracts as one unit, equally, when subjected to tension or compression. Following two points should be considered while solving numerical problems.

1. Extension or contraction of the bar being equal the strain i.e deformation per unit length is also equal.

2. The total external load on the bar is equal to the sum of the loads carried by the different materials.

Composite bar

COMPOSITE BAR



Composite bar

Let

P = total load on the bar.

A_1 = area of the bar 1

E_1 = modulus of elasticity of bar 1

P_1 = load shared by bar 1

A_2, E_2, P_2 = corresponding values for bar 2

l = length of the composite bar.

δl = elongation of composite bar.

We know that total load on the bar-

$$P = P_1 + P_2 \text{----- (i)}$$

Stress in bar1, $\sigma_1 = P_1/A_1$

Strain in bar 1 $e_1 = \sigma_1/E_1$

$$\Rightarrow e_1 = P_1/A_1E_1$$

Elongation in the bar $\delta l = e.l = P_1l/A_1E_1$ ----- (ii)

Similarly elongation of bar 2 = P_2l/A_2E_2 -----(iii)

Composite bar

Since both the elongations are equal so equating (ii) and (iii),

$$P_1/A_1E_1 = P_2/A_2E_2 \quad \text{or} \quad P_1/A_1E_1 = p_2/A_2E_2 \quad \text{-----(iv)}$$

$$\text{or } P_2 = P_1 \times A_2E_2/A_1E_1$$

$$\text{But } P = P_1 + P_2 = P_1 + P_1 \times A_2E_2/A_1E_1$$

$$= P_1(1 + A_2E_2/A_1E_1) = P_1 \frac{(A_1E_1 + A_2E_2)}{A_1E_1}$$

$$\text{Or, } P_1 = P \times \frac{A_1E_1}{A_1E_1 + A_2E_2} \quad \text{----- (v)}$$

$$P_2 = P \times \frac{A_2E_2}{A_1E_1 + A_2E_2} \quad \text{----- (vi)}$$

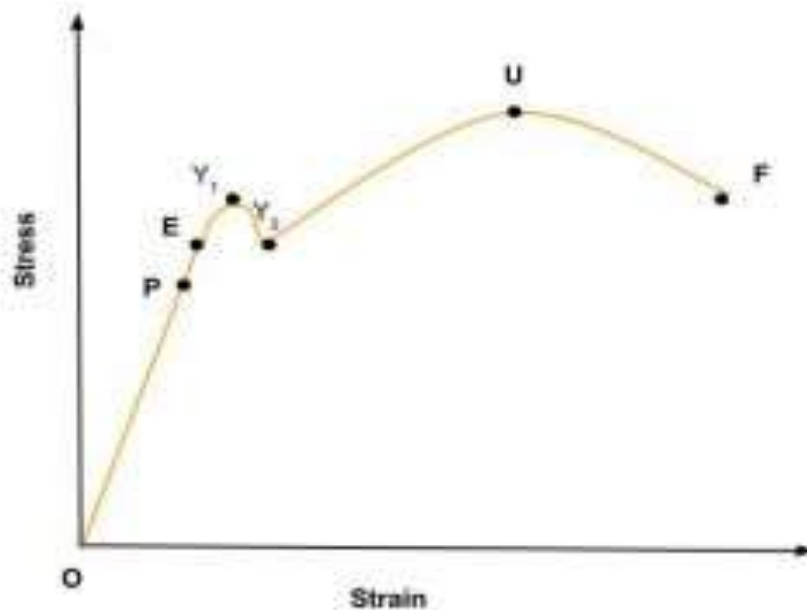
From these equations we can find out the loads shared by the different materials.

we have also seen in equation (iv) that $P_1/A_1E_1 = P_2/A_2E_2$

$$\text{or } \sigma_1/E_1 = \sigma_2/E_2 \quad (P/A = \sigma)$$

Stress -Strain Diagram

- The diagram which shows the relation between stress & strain showing the different values of stress along y axis, is called stress strain diagram .



Stress strain diagram for mild steel.

The various terms related with the stress-strain diagram

- **Limit of Proportionality(point P) :-** It is the greatest stress up to which the strain is proportional to the stress producing it. This limit does not exist for brittle material.
- **Elastic limit (point E) :-** It is the greatest stress up to which the material recovers its original length or dimension as soon as the stress causing the strain is removed. For many materials the numerical values of the elastic limit and the proportional limit are almost identical and the terms are sometimes used synonymously. If distinction prevails between these two limits then elastic limit is always possesses greater value than proportionality limit.

- **Permanent Set** :- If a material body is loaded beyond elastic limit , it does not fully recover its original length or dimension when the load is removed . This permanent deformation of the material is known as permanent set.
- **Yield point(point y_1 and y_2)** :- The stress which the deformation of the material body grows without further increase in the load is called yield point of the material. At this point material attains permanent set. In mild steel generally two distinct yield point is seen as shown in figure. One point is called upper yield point and another is called lower yield point.
- **Ultimate Stress** :- The maximum stress up to which there is no deformation in the cross sectional area of a material body (Just before starting the formation of waist) is called ultimate stress .

SOME ADDITIONAL TERMS RELATED WITH STRESS- STRAIN

- **1. Working Stress :-** The greatest stress to which a structure subjected in actual practice and design is known as working stress . It is always well below the elastic limit.
- **2. Proof Stress :-** The largest stress repeatedly applied , which the material body withstand without taking a permanent set is known as proof stress and corresponding load is known as proof load .
- **3. Factor Of Safety :-** The ratio of ultimate stress and the working stress is called factor of safety . Now a days the general practice followed that for structural steel work subjected to gradually increasing load the factor of safety is taken as the ratio of elastic limit to the working stress whose value is taken as 2 to 2.5.

But in case of structural steel work subjected to sudden load the factor of safety is taken as the ratio of ultimate stress to the working stress. Its value varies from 4 to 6.

SOME ADDITIONAL TERMS RELATED WITH STRESS- STRAIN

Percentage elongation- Let L_0 = Initial length of the specimen ,
 L = length at fracture. Then

$$\frac{L - L_0}{L_0} \times 100 \text{ is called percentage elongation.}$$

Percentage reduction of area : - Let
 A_0 = Initial cross section of the specimen
 A = Area at neck at fracture.
Then

$$\frac{A_0 - A}{A_0} \times 100 \text{ is called percentage reduction of area}$$

SOME ADDITIONAL TERMS RELATED WITH STRESS- STRAIN

Fluctuating stress : -When a material stressed (tensile or compressive) within a range then the stress is termed as fluctuating.

Repeated stress : - When a material stressed (tensile or compressive) between zero and some other specified value then it is known as repeated stress.

Alternating or reversed stress :- When a material is stressed alternately tensile and compressive, it is called an alternating or reversed stress.

Fatigue :- Experimentally , it has been established that a material will fail at a stress considerably below its ultimate strength if that stress is repeated at sufficiently large number of times and the number of cycles necessary to cause failure would be considerably less if the same stress be reversed stress. This phenomenon is called fatigue.

Endurance limit :- If a material stressed within some range doesnot fail even when the cycle is repeated indefinite times, then this range of stress is known as endurance limit.