Basic Fundamentals of Gear Drives

Chapter: 04

BASIC FUNDAMENTALS OF GEAR DRIVES

A gear is a toothed wheel that engages another toothed mechanism to change speed or the direction of transmitted motion. Gears are generally used for one of four different reasons:

1. To increase or decrease the speed of rotation;
2. To change the amount of force or torque;
3. To move rotational motion to a different axis (i.e. parallel, right angles, rotating, linear etc.); and
4. To reverse the direction of rotation.

Gears are compact, positive-engagement, power transmission elements capable of changing the amount of force or torque. Sports cars go fast (have speed) but cannot pull any weight. Big trucks can pull heavy loads (have power) but cannot go fast. Gears cause this.

Gears are generally selected and manufactured using standards established by American Gear Manufacturers Association (AGMA) and American National Standards Institute (ANSI).

This course provides an outline of gear fundamentals and is beneficial to readers who want to acquire knowledge about mechanics of gears. The course is divided into 6 sections:

Section -1 Gear Types, Characteristics and Applications

Section -2 Gears Fundamentals

Section -3 Power Transmission Fundamentals

Section -4 Gear Trains

SECTION -1 GEAR TYPES, CHARACTERISTICS & APPLICATIONS

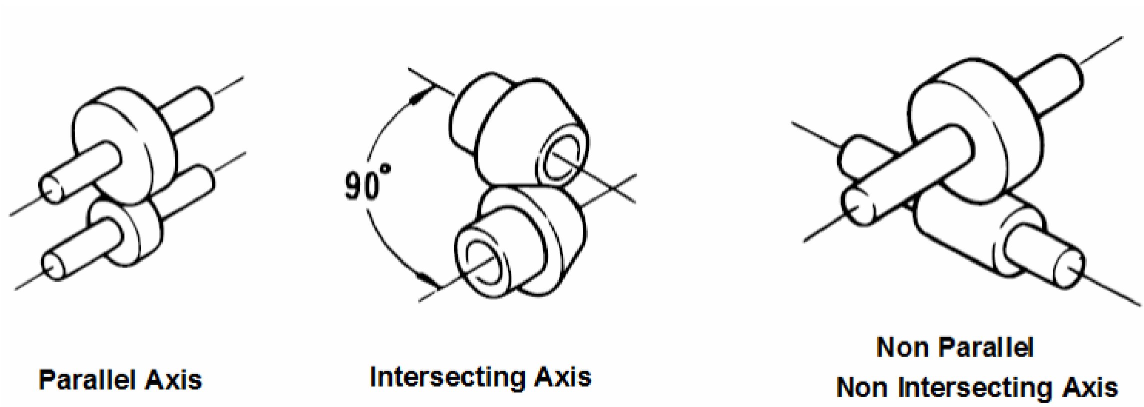
The gears can be classified according to:

1. the position of shaft axes
2. the peripheral velocity
3. the type of gears
4. the teeth position

According to the position of shaft axes:

Gears may be classified according to the relative position of the axes of revolution. The axes may be:

1. **Parallel** shafts where the angle between driving and driven shaft is 0 degree. Examples include spur gears, single and double helical gears.
2. **Intersecting** shafts where there is some angle between driving and driven shaft. Examples include bevel and miter gear.
3. **Non-intersecting and non-parallel** shafts where the shafts are not coplanar. Examples include the hypoid and worm gear.



According to peripheral velocity:

Gears can be classified as:

1. **Low** velocity type, if their peripheral velocity lies in the range of 1 to 3 m/sec.
2. **Medium** velocity type, if their peripheral velocity lies in the range of 3 to 15 m/sec.
3. **High** velocity type, if their peripheral velocity exceeds 15 m/sec.

According to type of gears:

Gears can be classified as external gears, internal gears, and rack and pinion.

1. **External gears** mesh externally - the bigger one is called “gear” and the smaller one is called “pinion”.
2. **Internal gears** mesh internally - the larger one is called “annular” gear and the smaller one is called “pinion”.
3. **Rack and pinion type –** converts rotary to linear motion or vice versa. There is a straight line gear called “rack” on which a small rotary gear called “pinion” moves.

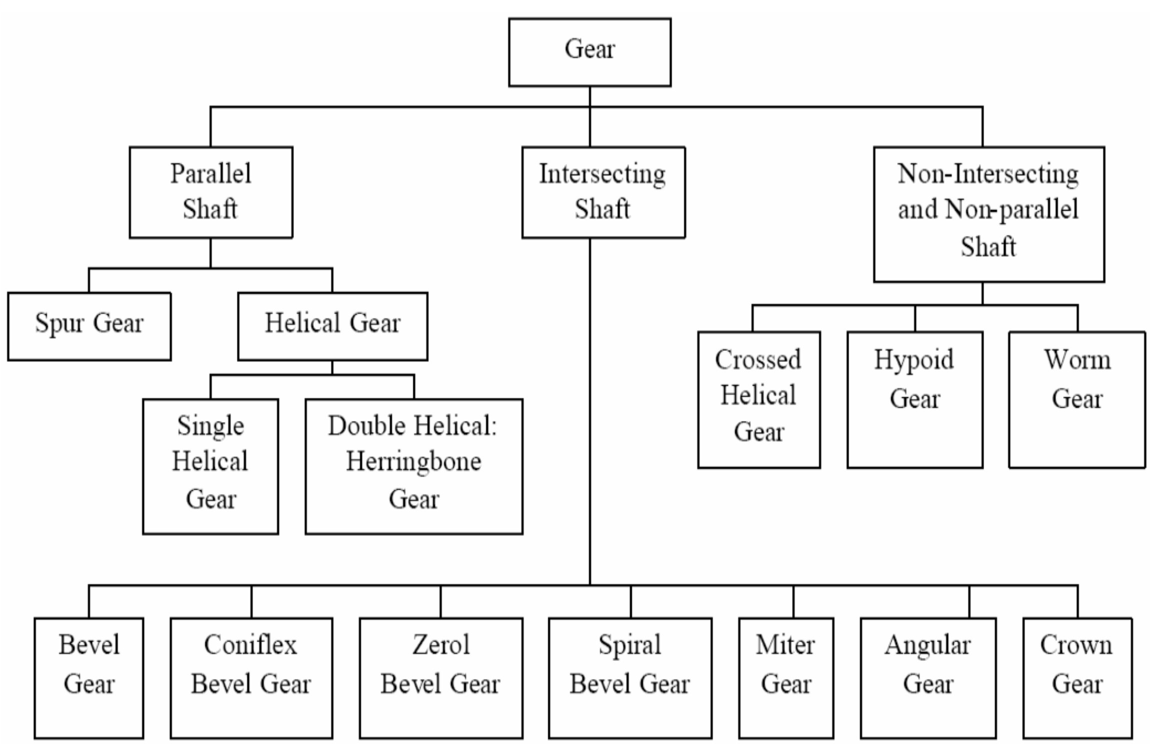
According to teeth position:

Gears are classified as straight, inclined and curved.

1. **Straight gear teeth** are those where the teeth axis is parallel to the shaft axis.
2. **Inclined gear teeth** are those where the teeth axis is at some angle.
3. **Curve gear teeth** are curved on the rim’s surface.

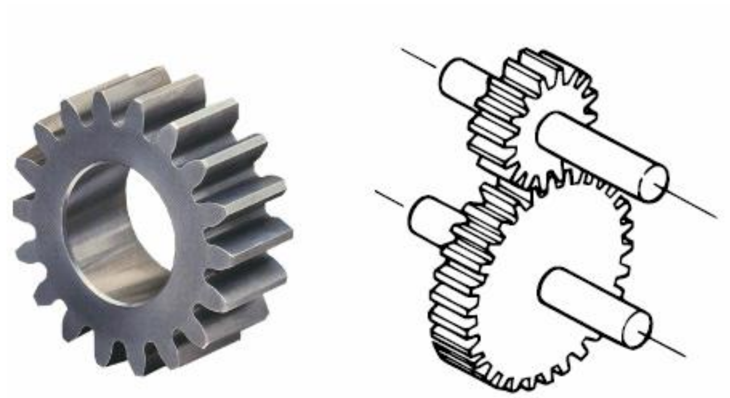
TYPE OF GEARS

Here is a brief list of the common forms.



SPUR GEARS

Spur gears are used to transmit power between two parallel shafts. The teeth on these gears are cut straight and are parallel to the shafts to which they are attached.



Spur Gears

Characteristics:

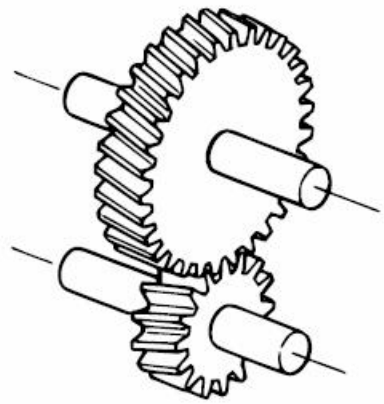
- Simplest and most economical type of gear to manufacture.
- Speed ratios of up to 8 (in extreme cases up to 20) for one step (single reduction) design; up to 45 for two step design; and up to 200 for three-step design.

Limitations:

- Not suitable when a direction change between the two shafts is required.
- Produce noise because the contact occurs over the full face width of the mating teeth instantaneously.

HELICAL GEARS

Helical gears resemble spur gears, but the teeth are cut at an angle rather than parallel to the shaft axis like on spur gears. The angle that the helical gear tooth is on is referred to as the helix angle. The angle of helix depends upon the condition of the shaft design and relative position of the shafts. To ensure that the gears run smoothly, the helix angle should be such that one end of the gear tooth remains in contact until the opposite end of the following gear tooth has found a contact. For parallel shafts, the helix angle should not exceed 20 degrees to avoid excessive end thrust.



Helical Gears

Characteristics:

The longer teeth cause helical gears to have the following differences from spur gears of the same size:

- Tooth strength is greater because the teeth are longer than the teeth of spur gear of equivalent pitch diameter.
- Can carry higher loads than can spur gears because of greater surface contact on the teeth.
- Can be used to connect parallel shafts as well as non-parallel, non-intersecting shafts.
- Quieter even at higher speed and are durable.

Limitations:

- Gears in mesh produce thrust forces in the axial directions.
- Expensive compared to spur gears.

BEVEL GEARS

A bevel gear is shaped like a section of a cone and primarily used to transfer power between intersecting shafts at right angles. The teeth of a bevel gear may be straight or spiral. Straight gear is preferred for peripheral speeds up to 1000 feet per minute; above that they tend to be noisy.

Characteristics:

- Designed for the efficient transmission of power and motion between intersecting shafts. A good example of bevel gears is seen as the main mechanism for a hand drill. As the handle of the drill is turned in a vertical direction, the bevel gears change the rotation of the chuck to a horizontal rotation.
- Permit a minor adjustment during assembly and allow for some displacement due to deflection under operating loads without concentrating the load on the end of the tooth.

MITTER GEARS

Mitter gears are identical to bevel gears with the exception that both gears always have the same number of teeth.

Characteristics:

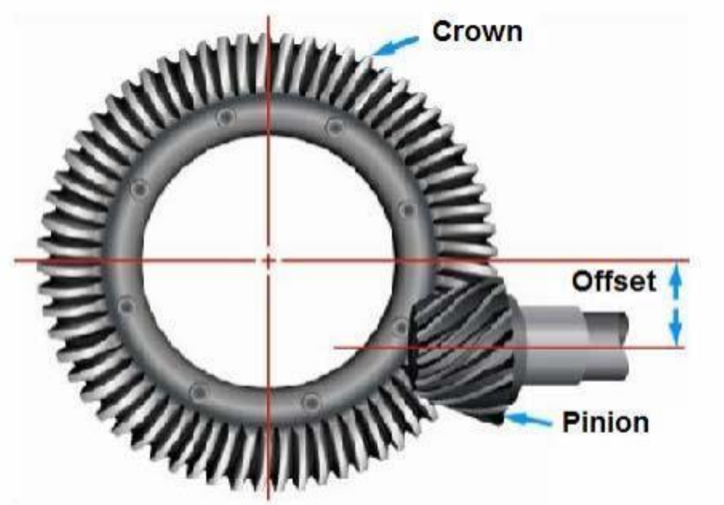
- They provide a steady ratio; other characteristics are similar to bevel gears.
- They are used as important parts of conveyors, elevators and kilns.

Limitations

- Gear ration is always 1 to 1 and therefore not used when an application calls for a change of speed.

HYPOID GEARS

Hypoid gears are a modification of the spiral bevel gear with the axis offset. The distinguishing feature of hypoid gears is that the shafts of the pinion and ring gear may continue past each other, never having their axis intersecting.



Hypoid Gears

The major advantages of the hypoid gear design are that the pinion diameter is increased, and it is stronger than a corresponding bevel gear pinion. The increased diameter size of the pinion permits the use of comparatively high gear ratios and is extremely useful for non-intersecting shaft requirements such as automotive applications where the offset permits lowering of the drive shaft.

WORM GEARS

Worm gears are used to transmit power between two shafts that are at right angles to each other and are non-intersecting.

Worm gears are special gears that resemble screws, and can be used to drive spur gears or helical gears. Worm gearing is essentially a special form of helical gearing in which the teeth have line contact and the axes of the driving and driven shafts are usually at right angles and do not intersect.

Worm Gear

Characteristics:

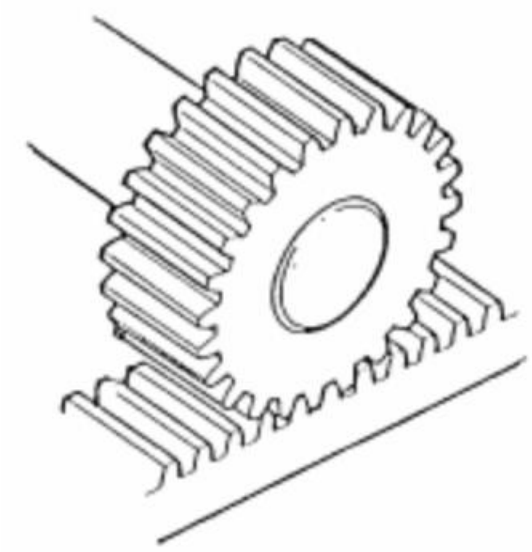
- Meshes are self-locking. Worm gears have an interesting feature that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm. This is because the angle on the worm is so shallow that when the gear tries to spin it, the friction between the gear and the worm holds the worm in place. This feature is useful for machines such as conveyor systems, in which the locking feature can act as a brake for the conveyor when the motor is not turning.
- Worm gear is always used as the input gear, i.e. the torque is applied to the input end of the worm shaft by a driven sprocket or electric motor.
- Best suited for applications where a great ratio reduction is required between the driving and driven shafts. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater.

Limitations:

- Yield low efficiency because of high sliding velocities across the teeth, thereby causing high friction losses.
- When used in high torque applications, the friction causes the wear on the gear teeth and erosion of the restraining surface.

RACKS (STRAIGHT GEARS)

The rack is a bar with a profile of the gear of infinite diameter, and when used with a meshing pinion, enables the rotary to linear movement or vice versa.



Characteristics:

- Racks with machined ends can be joined together to make any desired length.
- The most well-known application of a rack is the rack and pinion steering system used on many cars in the past. The steering wheel of a car rotates the gear that engages the rack. The rack slides right or left, when the gear turns, depending on the way we turn the wheel. Windshield wipers in cars are powered by a rack and pinion mechanism.

HERRINGBONE (DOUBLE HELICAL) GEARS

Herringbone, also known as double helical gears, are used for transmitting power between two parallel shafts. Double helical gearing offers low noise and vibration along with zero net axial thrust.

Herringbone Gears

Characteristics:

- Conduct power and motion between non-intersecting, parallel axis that may or may not have center groove with each group making two opposite helices. Action is equal in force and friction on both gears and all bearings, and free from any axial force.
- Offer reduced pulsation due to which they are highly used for specialized extrusion and polymerization. The most common application is in heavy machinery and power transmission.
- Applications include high capacity reduction drives like that of cement mills and crushers.

Limitations:

- Manufacturing difficulty makes them costlier.
- Noise level of double helical gears averaged about 4dB higher than otherwise similar single helical gears. The phenomenon is due to the axial shuttling which occurs as the double helical pinion moves to balance out the net thrust loading.

INTERNAL GEAR

Internal gears have their teeth cut parallel to their shafts like spur gears, but they are cut on the inside of the gear blank. The properties and teeth shape are similar as the external gears except that the internal gears have different addendum and dedendum values modified to prevent interference in internal meshes.

Internal Gear

Characteristics:

- In the meshing of two external gears, rotation goes in the opposite direction. In the meshing of an internal gear with an external gear the rotation goes in the same direction.
- The meshing arrangement enables a greater load carrying capacity with improved safety (since meshing teeth are enclosed) compared to equivalent external gears.
- Shaft axes remain parallel and enable a compact reduction with rotation in the same sense. Internal gears are not widely available as standard.
- When they are used with the pinion, more teeth carry the load that is evenly distributed. The even distribution decreases the pressure intensity and increases the life of the gear.
- Allows compact design since the center distance is less than for external gears. Used in planetary gears to produce large reduction ratios.
- Provides good surface endurance due to a convex profile surface working against a concave surface.

Applications:

- Planetary gear drive of high reduction ratios, clutches, etc.

Limitations:

- Housing and bearing supports are more complicated because the external gear nests within the internal gear.
- Low ratios are unsuitable and in many cases impossible because of interferences.
- Fabrication is difficult and usually special tooling is required.

SUMMARY

Type	Features	Applications	Comments Regarding Precision
Spur	Parallel Shafting. Adapted to high speed applications where noise is not a concern.	Applicable to all types of trains and a wide range of velocity ratios.	Simplest tooth elements offering maximum precision. First choice, recommended for all the gear meshes, except where very high speeds and loads or special features of other types, such as right angle drive, cannot be avoided.
Helical	Parallel Shafting. Very high speeds and loads. Efficiency slightly less than spur mesh.	Most applicable to high speeds and loads; also used whenever spurs are used.	Equivalent quality to spurs, except for complication of helix angle. Recommended for all high-speed and high-load meshes. Axial thrust component must be accommodated.
Crossed Helical	Skewed shafting. Point contact. High sliding Low speeds Light loads	Relatively low velocity ratio; low speeds and light loads only. Any angle skew shafts.	Precision Rating is poor. Point contact limits capacity and precision. Suitable for right angle drives, if light load. A less expensive substitute for bevel gears. Good lubrication essential because of point of contact and high sliding action.

Internal spur	Parallel shafts High speeds High loads	Internal drives requiring high speeds and high loads; offers low sliding and high stress loading; good for high capacity, long life. Used in planetary gears to produce large reduction ratios.	Not recommended for precision meshes because of design, fabrication, and inspection limitations. Should only be used when internal feature is necessary.
Bevel	Intersecting shafts, High speeds, High loads.	Suitable for 1:1 and higher velocity ratios and for right-angle meshes (and other angles)	Good choice for right angle drive, particularly low ratios. However complicated both form and fabrication limits achievement of precision. Should be located at one of the less critical meshes of the train.
Worm mesh	Right-angle skew shafts, High velocity ratio, High speeds and loads, Low efficiency, Most designs nonreversible.	High velocity ratio Angular meshes High loads	Worm can be made to high precision, but worm gear has inherent limitations. To be considered for average precision meshes, but can be of high precision with care. Best choice for combination high velocity ratio and right-angle drive. High sliding requires excellent lubrication.

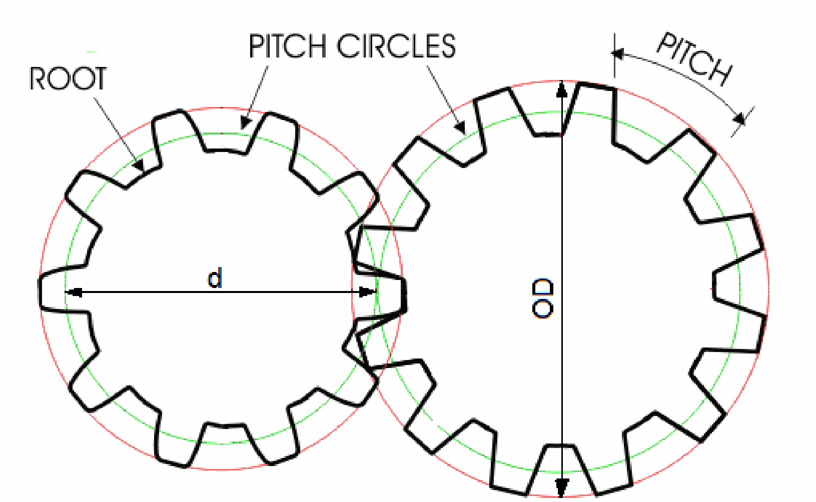
SECTION -2: GEARS FUNDAMENTALS & TERMINOLOGY

In this section, we will discuss the gear fundamentals, considering spur gears. It is the most common and the simplest form, and hence the most comprehensible. The same principles apply to spiral gears and bevel gears too.

A gear can be defined in terms of its pitch, pressure angle and number of teeth. Let's discuss few terms here:

Pitch Circle Diameter (d) - This is the diameter of a circle about which the gear tooth geometry is designed or constructed. The pitch circle is the imaginary circle found at the point where the teeth of two gears mesh. The diameter of the pitch circle is called the pitch diameter.

Outside Diameter (OD) - The outside circle is the distance around the outer edge of the gear's teeth. The diameter of the outside circle is called the outside diameter.



Root - The root is the bottom part of a gear wheel.

Pitch - Pitch is a measure of tooth spacing along the pitch circle. It is the distance between any point on one tooth and same point on the next tooth. It is expressed in the following forms:

Diametral Pitch (P_d) is the number of teeth per inch of the pitch diameter and is also an index of tooth size. It is given as:

$$P_d = \frac{Z}{d}$$

Where:

- P_d = diametral pitch
- Z = number of teeth
- d = pitch circle diameter in inches

A large diametral pitch indicates a small tooth and vice versa. Another way of saying this; larger gears have fewer teeth per inch of diametral pitch.

Important!

The use of diametral pitch is a handy reference in gear design. An important rule to remember is that a pair of gears can only mesh correctly if and when the diametral pitch (P_d) is the same, i.e.:

$$P_d = \frac{Z_{Gear}}{d_{Gear}} = \frac{Z_{Pinion}}{d_{Pinion}}$$

Module (m) is the metric equivalent of diametral pitch, i.e. the pitch diameter (in mm) divided by the number of teeth, but unlike diametral pitch, the higher number, the larger the teeth. Meshing gears must have the same module:

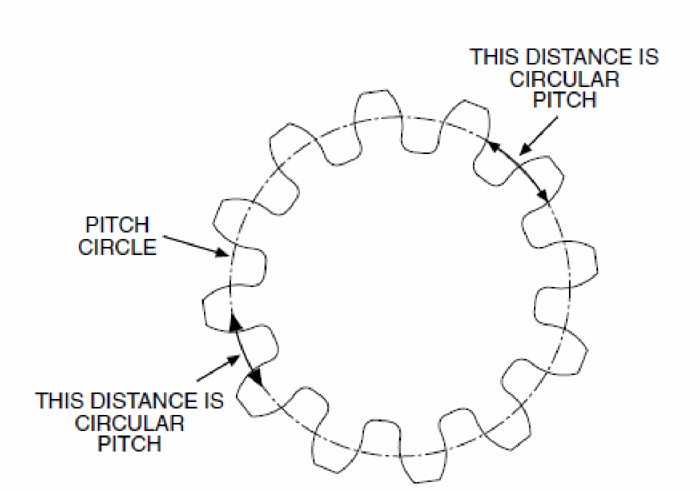
$$m = \frac{1}{P_d} = \frac{d}{Z}$$

A 1 module gear has 1 tooth for every mm of pitch circle diameter. Thus a 0.3 mod gear having 60 teeth will have a pitch circle diameter of 18 mm (0.3 *60).

Circular Pitch (Pc): is the distance from a point on one tooth to the corresponding poing on the adjacent tooth, measured along the pitch circle. Calculated in inches, the circular pitch equals the pitch circle circumference divided by the number of teeth:

$$P_c = \frac{\text{Circumfrence } (\pi d)}{\text{Number of teeth } (Z)}$$

Because the circular pitch is directly proportional to the module and inversely proportional to the diametral pitch, meshing teeth must have the same circular pitch.



Relationship between Circular Pitch and Diametral Pitch:

From $P_c = \frac{\pi d}{Z}$ and $P_d = \frac{Z}{d}$

We have,

$P_d\ P_c = \pi$

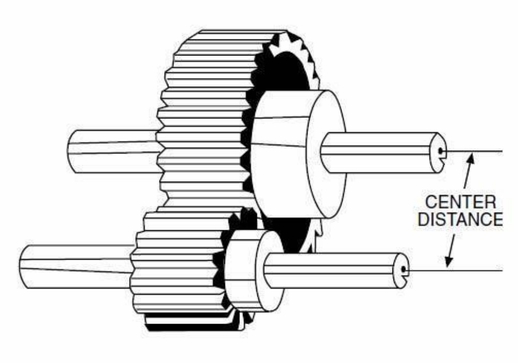
The product of the circular pitch and the diametral pitch is equal to pi (π).

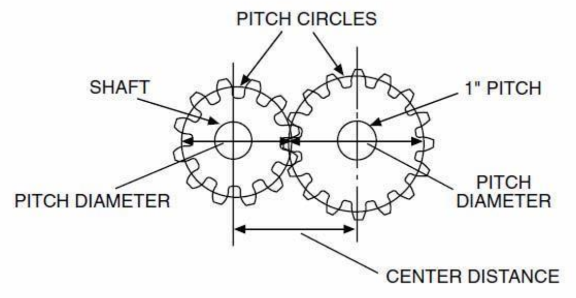
Number of Teeth (N): The number of gear teeth is related to the diametral pitch and the pitch circle diameter by equation Z= d x Pd .

Tooth Size: Diametral pitch, module and circular pitch are all indications of tooth size; ratios which determine the number of teeth in a gear for a given pitch diameter. In designing a gear set, the number of teeth in each member is of necessity. *As a rule of thumb, teeth should be large and low in number for heavily loaded gears and small and numerous for smooth operation.*

Center Distance (CD)

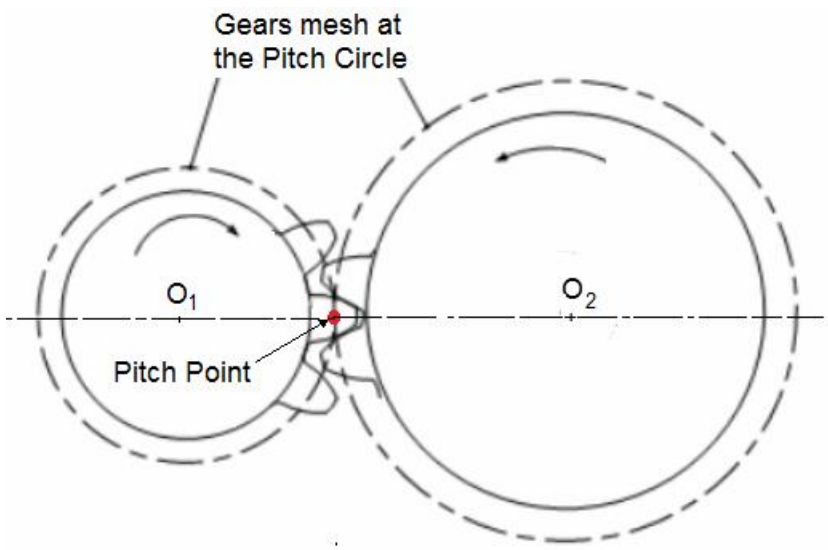
Center Distance is the distance between the centers of the shaft of one spur gear to the center of the shaft of the other spur gear. The standard center distance between two spur gears is one-half the sum of their pitch diameters.





Pitch point:

Pitch point is the point where gear teeth actually make contact with each other as they rotate. Refer to the figure below for two meshing gears. The pitch point “P” always lies at the line connecting the centers of two gears.



EXAMPLES

Example - 1:

The center distance of a 4-inch pitch diameter gear running with a 2-inch pitch diameter pinion is 3 inches. 4" + 2" ÷ 2 = 3" CD

Example -2:

A gear has 18 teeth (Z) and a diametral pitch (P_d) of 8. What is its pitch diameter (d)?

Answer:

d = Z/Pd = 18/8 = 2¼”

Example -3:

A gear has a pitch diameter (d) of 3.125” (3-1/8”) and a diametral pitch (P_d) of 8. How many teeth (Z) does it have?

Answer:

$Z = d \times P_d = 3.125 \times 8 = 25$ teeth

Example -4:

Calculate the center-to-center spacing for the 2 gears specified below.

Gear 1: 36 tooth, 24 P_d Drive Gear

Gear 2: 60 tooth, 24 P_d Driven Gear

Answer:

Calculate the pitch diameter for each of the two gears:

Pitch diameter of gear 1: $d_1 = Z / P_d = 36/24 = 1.5$”

Pitch diameter of gear 2: $d_2 = Z / P_d = 60/24 = 2.5$”

Obtain center to center distance by adding the two diameters and divide by 2.

Center to center distance = $(d_1 + d_2) / 2 = (1.5 + 2.5) /2 = 2$”

THE LAW OF GEARING

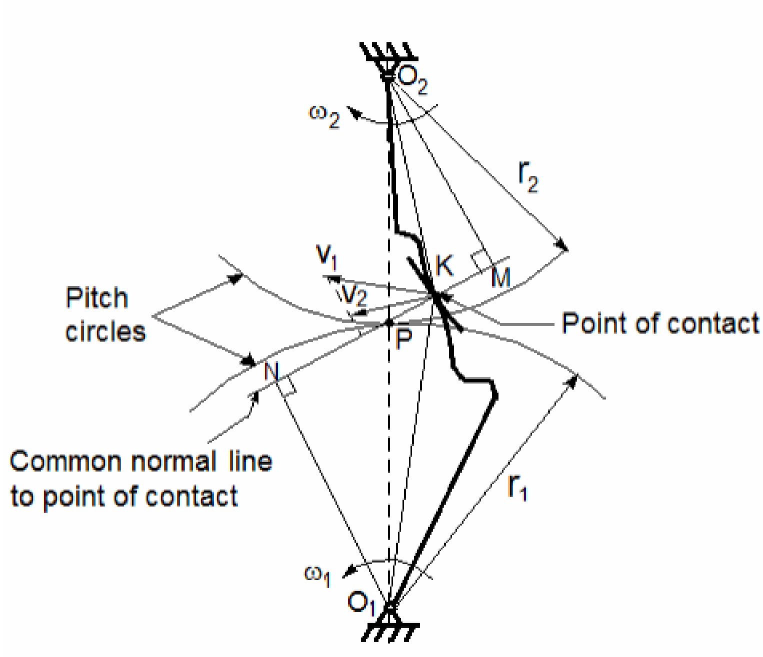
The fundamental law of gearing states that the angular velocity ratio of all gears must remain constant throughout the gear mesh. This condition is satisfied when the common normal at the point of contact between the teeth passes through a fixed point on the line of centers, known as the pitch point.

Law Governing Shape of the Teeth

The figure below shows two mating gear teeth in which:

Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point K.

- NM is the common normal of the two profiles.
- N is the foot of the perpendicular from O_1 to NM
- M is the foot of the perpendicular from O_2 to NM.



Although the two profiles have different velocities v_1 and v_2 at point K, their velocities along NM are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other.

$O_1N.\ \omega_1 = O_2M.\ \omega_2$

$\omega_1/\omega_2 = O_2M/O_1N$

We notice that the intersection of the tangency NM and the line of center O_1O_2 is point P, and $O_1N.P = O_2M.P$

Thus, the relationship between the angular velocities of the driving gear to the driven gear, or velocity ratio, of a pair of mating teeth is:

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{r_2}{r_1} = \frac{d_2}{d_1}$$

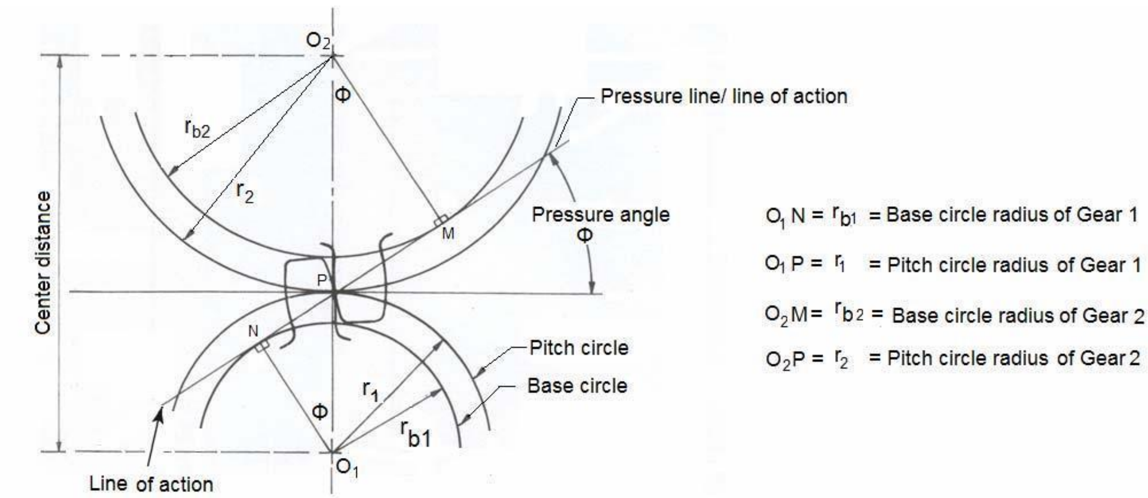
Where,

	Pinion	Gear
Radius	r_1	r_2
Diameter (inches)	d_1	d_2
Speed (rad/s)	ω_1	ω_2

Point P is very important to the velocity ratio, and it is called the pitch point. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The velocity ratio is equal to the inverse ratio of the diameters of pitch circles. This is the fundamental law of gear-tooth action.

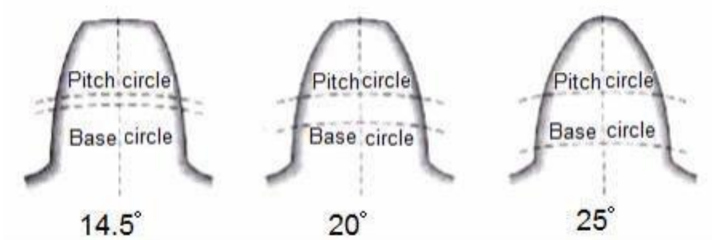
When the tooth profiles of gears are shaped so as to produce a constant angular-velocity ratio during meshing, the surfaces are said to be conjugate. **Pressure angle** defines the shape of the gear tooth and is an important criterion in gear manufacturing. Higher pressure angle results in wider base, stronger teeth and lower tendency to experience tooth tip interference, but are susceptible to noise and higher bearing loads. Low pressure angles are quieter and smoother, have lower bearing loads and lower frictional forces, but are susceptible to undercutting at low number of teeth.

Refer to the figure below:



Draw radial lines from the center of each gear O_1 & O_2 that are perpendicular to the line of action MN. The normal to the line of action up to the center of gears, O_1N and O_2M, can be used to form a circle of radius r_{b1} & r_{b2} referred to as the base circles of gear 1 and gear 2, respectively. The base circle is inscribed within the pitch circle having radius r_1 and r_2. As is evident from the geometry of the figure, the angle between the line of centers (O_1O_2) and the line segment O_1N#and O_2M is the pressure angle Φ. It is also equal to the angle between the line of action MN and the line perpendicular to the line of centers (O_1O_2) through the pitch point P.

The standard pressure angles are 14½°, 20° and 25°. The preferred angle in use today is 20º; a good compromise for power and smoothness. The increase of the pressure angle from 14½º to 25º results in a stronger tooth, because the tooth acting as a beam is wider at the base.



Gear Tooth Profile for different Pressure Angles

Important!

It is important to note that the gears must have the same pressure angles to mesh. 14½º PA tooth forms will not mesh with 20º pressure angles gears and vice versa.

Contact Ratio:

In the above description, we have considered one gear tooth in contact for simplicity. In practice, more than one tooth is actually in contact during engagement and therefore the load is partially shared with another pair of teeth. This property is called the **contact ratio**. A contact ratio between 1 and 2 means that part of the time two pairs of teeth are in contact, and during the remaining time one pair is in contact. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact.

The higher the contact ratio the more the load is shared between teeth. It is a good practice to maintain a contact ratio of 1.3 to 1.8. Under no circumstances should the ratio drop below 1.1.

GEAR PROFILES

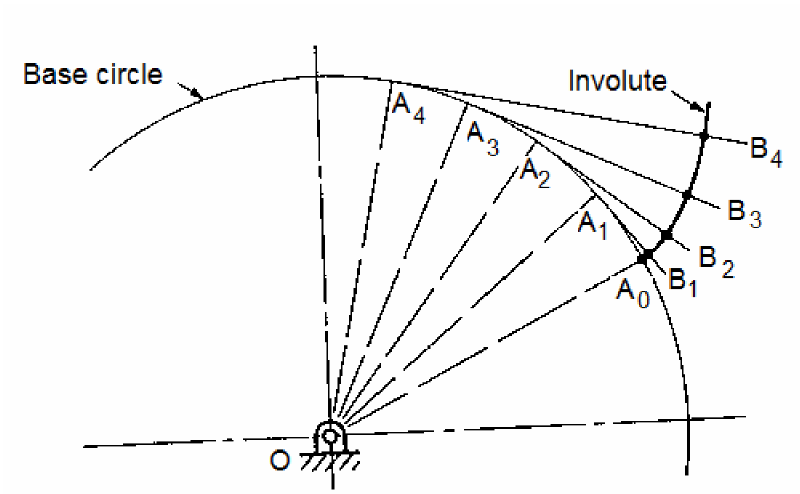
Gear profiles should satisfy the law of gearing. The profiles best suited for this law are:

1. Involute
2. Cyloidal

Involute Tooth Profile

Most modern gears use a special tooth profile called an involute. This profile has the very important property of maintaining a constant speed ratio between the two gears.

The involute profile is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the base circle.



We use the word *involute* because the contour of gear teeth curves inward. On an involute gear tooth, the contact point starts closer to one gear, and as the gear spins, the contact point moves away from that gear and towards the other. Involute gears have the invaluable ability of providing conjugate action when the gears' centre distance is varied either deliberately or involuntarily due to manufacturing and/or mounting errors.

Cycloidal Tooth Profile

Cycloidal gears have a tooth shape based on the epicycloid and hypocycloid curves, which are the curves generated by a circle rolling around the outside and inside of another circle, respectively. They are not straight and their shape depends on the radius of the generating circle. Cycloidal gears are used in pairs and are set at an angle of 180 degrees to balance the load. The input and output remains in constant mesh. Cycloidal tooth forms are used primarily in clocks for a number of reasons:

- Less sliding friction
- Less wear
- Easier to achieve higher gear ratios without tooth interference

Comparison between Involute and Cycloidal Gears

In actual practice, the involute tooth profile is the most commonly used because of following advantages:

1. The most important advantage of the involute gears is that the variations in center distance do not affect the angular velocity ratio. This is not true for cycloidal gears which require exact center distance to be maintained.
2. In involute gears, the pressure angle remains constant throughout the engagement of teeth which results in smooth running. The involute system has a standard pressure angle which is either 20° or 14½°, whereas on a cycloidal system, the pressure angle varies from zero at pitch line to a maximum at the tips of the teeth.
3. The face and flank of involute teeth are generated by a single curve, whereas in cycloidal gears, double curves (i.e. epicycloid and hypo-cycloid) are required for the face and flank, respectively.
4. Cycloidal teeth have wider flanks; therefore the cycloidal gears are stronger than the involute gears, for the same pitch. Due to this reason, the cycloidal teeth are preferred especially for cast gears used in paper mill machinery and sugar mills.
5. Cycloidal gears do not have interference.

Though there are advantages of cycloidal gears, they are outweighed by the greater simplicity and flexibility of the involute gears. It is easy to manufacture since it can be generated from a simple cutter.

The only disadvantage of the involute teeth is that the interference occurs with pinions having smaller number of teeth. This may be avoided by altering the heights of addendum and dedendum of the mating teeth, or the angle of obliquity of the teeth.

GEAR TOOTH NOMENCLATURE

The following terms are used when describing the dimensions of a gear tooth:

1. **Addendum:** the distance from the top of a tooth to the pitch circle. Its value is equal to one module.

2. **Dedendum:** the distance from the pitch circle to the bottom of the tooth space (root circle). It equals the addendum + the working clearance. Dedendum is bigger than addendum and is equal to Addendum + Clearance = m + 0.157m = 1.157m

3. **Whole depth:** The total depth of the space between adjacent teeth and is equal to addendum plus dedendum. Also equal to working depth plus clearance.

4. **Working depth:** Working depth is the depth of engagement of two gears; that is, the sum of their addendums.

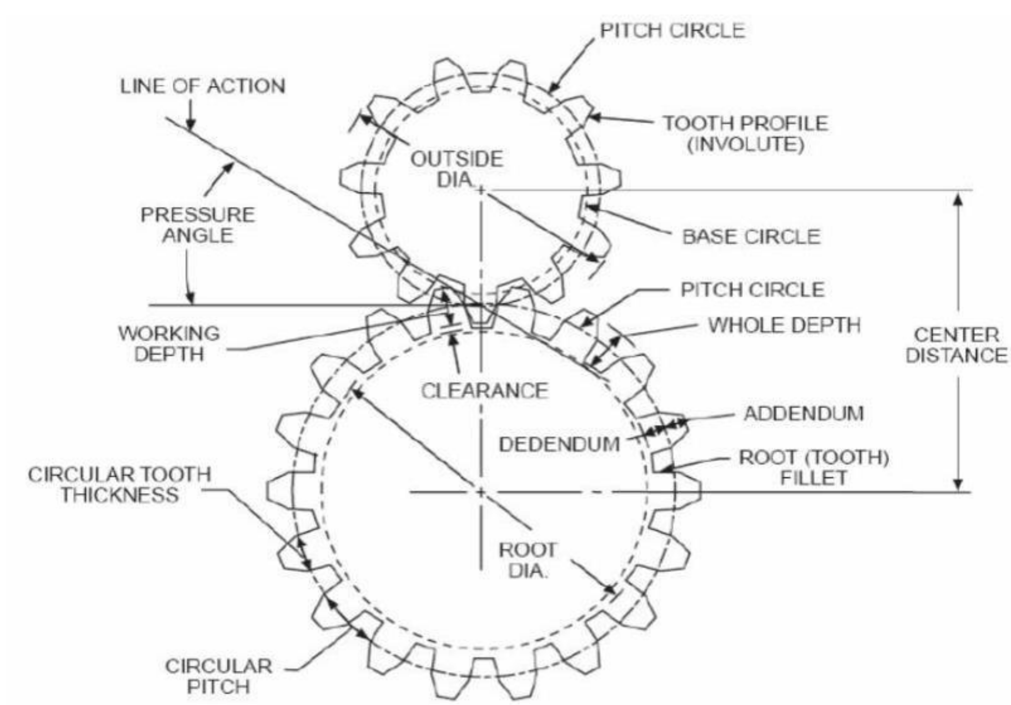
5. **Working Clearance:** This is a radial distance from the tip of a tooth to the bottom of a mating tooth space when the teeth are symmetrically engaged. Its standard value is 0.157m.

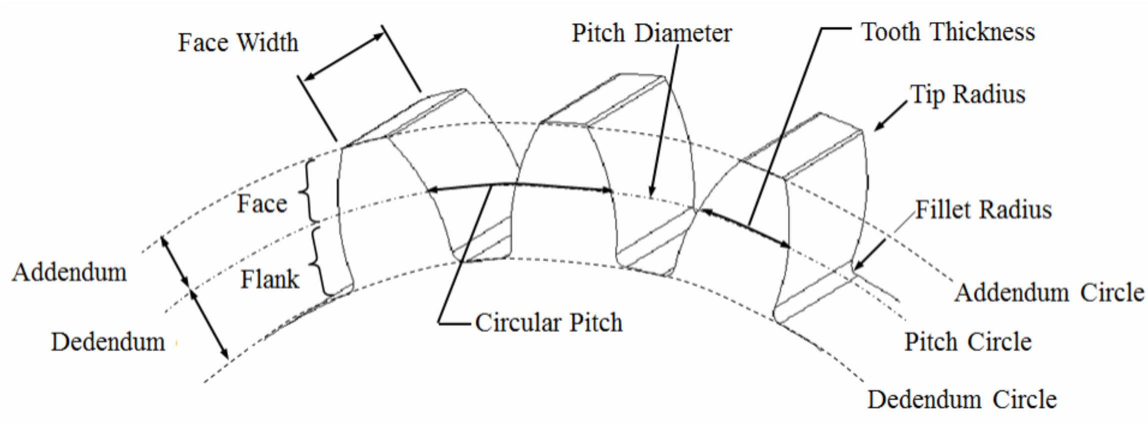
6. **Outside diameter:** The outside diameter of the gear.

7. **Base Circle diameter:** The diameter on which the involute teeth profile is based.

8. **Addendum circle:** A circle bounding the ends of the teeth in a right section of the gear.

9. **Dedendum circle:** The circle bounding the spaces between the teeth in a right section of the gear.

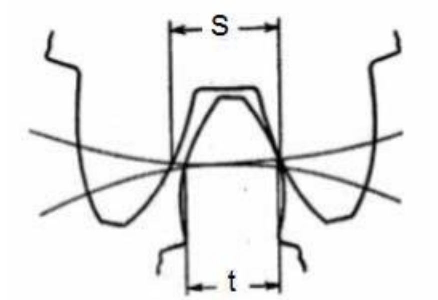




10. **Tooth space:** It is the width of space between two teeth measured on the pitch circle.
11. **Face of tooth:** It is that part of the tooth surface which is above the pitch surface.
12. **Flank of the tooth:** It is that part of the tooth surface which is lying below the pitch surface.
13. **Point of contact:** Any point at which two tooth profiles touch each other.
14. **Path of action:** The locus of successive contact points between a pair of gear teeth, during the phase of engagement.
15. **Line of action:** The line of action is the path of action for involute gears. It is the straight line passing through the pitch point and tangent to both base circles.
16. **Tooth Thickness, Space Width and Backlash**
    - **Tooth thickness,** (t) is the width of the tooth (arc distance) measured on the pitch circle.
    - **Space width,** (S) or tooth space is the arc distance between two adjacent teeth measured on the pitch circle.
    - **Backlash,** (B) is the difference between the space width and the tooth thickness.

      $B = S - t$

Standard gears are designed with a specified amount of backlash to prevent noise and excessive friction and heating of the gear teeth.



SPUR GEAR FORMULAS AND CALCULATIONS

Below is a table of formulas used in calculating spur gear information based on standard gearing practices. The spur gear formulas here are based on the "Diametral Pitch system (P_d).

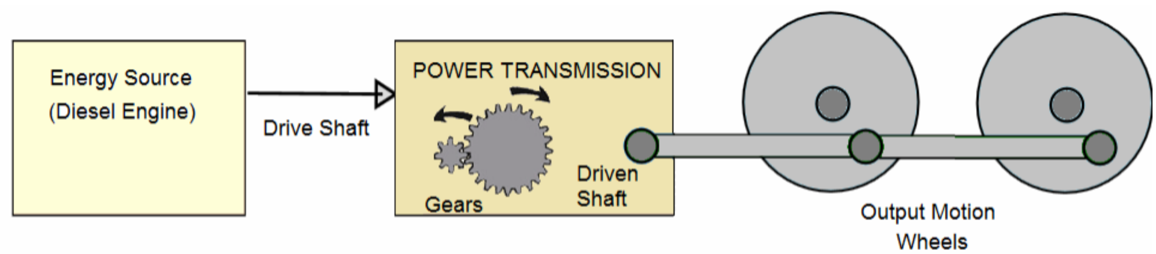
To Get	Having	Rule	Formula
Diametral Pitch	• The circular pitch	Divide Pi (3.1416) by the circular pitch	$P_d = 3.1416 / CP$
Diametral Pitch	• Pitch diameter • Number of teeth	Divide the number of teeth by the pitch diameter	$P_d = Z / d$
Diametral Pitch	• Outside diameter • Number of teeth	Divide number of teeth + 2 by the outside diameter	$P_d = (Z+2) / OD$
Diametral Pitch	• Base pitch • Pressure angle	Divide the base pitch by the cosine of the pressure angle then divide by 3.1416	$P_d = (BP / \text{Cos}\Phi) / 3.1416$
Diametral Pitch	• Module	Divide 25.4 by Module	$P_d = 25.40 / M$
Pitch Diameter	• Number of teeth • Diametral pitch	Divide the number of teeth by the diametral pitch	$d = Z / P_d$
Pitch Diameter	• Number of teeth • Outside diameter	Divide the product of the outside diameter + number of teeth by the number of teeth + 2	$d = (OD \times Z) / (Z+2)$
Pitch Diameter	• Outside diameter • Diametral pitch	Subtract 2 divided by the diametral pitch from the outside diameter	$d = OD - (2/ P_d)$

Pitch Diameter	• Addendum • Number of teeth	Multiply addendum by the number of teeth	$d = a \times Z$
Pitch Diameter	• Base diameter • Pressure angle	Divide the base diameter by the cosine of the pressure angle	$d = BD / (\text{Cos}\Phi)$
Outside Diameter	• Number of teeth • Diametral pitch	Divide number of teeth + 2 by the diametral pitch	$OD = (Z+2) / P_d$
Outside Diameter	• Pitch diameter • Diametral pitch	Two divided by the diametral pitch plus pitch diameter	$OD = (2 / P_d) + d$
Outside Diameter	• Pitch diameter • Number of teeth	Number of teeth + 2, divided by the quotient of number of teeth divided by the pitch diameter	$OD = (Z+2) / (Z / d)$
Outside Diameter	• Number of teeth • Addendum	Multiply the number of teeth + 2 by the addendum	$OD = (Z + 2) \times A$
Number Of Teeth	• Pitch diameter • Diametral pitch	Multiply the pitch diameter by the diametral pitch	$Z = d \times P_d$
Number Of Teeth	• Outside diameter • Diametral pitch	Multiply the outside diameter by the diametral pitch and subtract 2	$Z = (OD \times P_d) - 2$
Thickness Of Tooth	• Diametral pitch	Divide 1.5708 by the diametral pitch	$t = 1.5708 / P_d$
Addendum	• Diametral pitch	Divide 1 by the diametral pitch	$a = 1 / P_d$
Dedendum	• Diametral pitch	Divide 1.157 (or 1.25) by the diametral pitch	$b = 1.157 / P_d$
Working Depth	• Diametral pitch	Divide 2 by the diametral pitch	$hk = 2 / P_d$
Whole Depth	• Diametral pitch	Divide 2.157 (or 2.25) by the diametral pitch	$ht = 2.157 / P_d$

Clearance	• Diametral pitch	Divide .157 (or .250) by the diametral pitch	$c = .157 / P_d$
Clearance	• Diametral pitch	Divide thickness of tooth at the pitchline by 10	$c = t / 10$
Operating Diametral Pitch	• C.D. between both gears • Number of teeth in each	Add the number of teeth in both gears and divide by 2 then divide by the center distance	$P_{dO} = ((Z1 + Z2)/2) / C$
Center Distance	• Normal diametral pitch • Number of teeth in both gears	Add the number of teeth in both gears and divide by 2 then divide by the normal diametral pitch	$C = ((Z1 + Z2)/2) / P_{nd}$
Operating Center Distance	• Operating diametral pitch • Number of teeth in both gears	Add the number of teeth in both gears and divide by 2 then divide by the operating diametral pitch	$Co = ((Z1 + Z2)/2) / P_{od}$
Base Diameter	• Pitch diameter • Pressure angle	Multiply the pitch diameter by the cosine of the pressure angle	$BD = P_d \times (\cos\Phi)$
Pressure Angle	• Base diameter • Pitch diameter	Divide the base diameter by the pitch diameter	$\cos\Phi = BD / d$
Pressure Angle	• Base pitch • Diametral pitch	Divide Pi by the diametral pitch, then divide by the base pitch	$\cos\Phi = (3.1416 / P_d) / Pb$
Base Pitch	• Diametral pitch • Pressure angle	Divide the diametral pitch by Pi, then multiply by the cosine of the pressure angle	$Pb = (P_d / 3.1416) \times (\cos\Phi)$

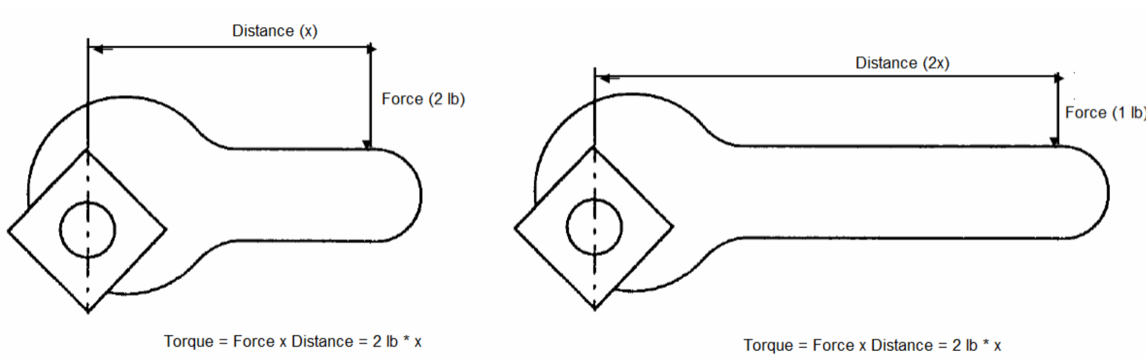
SECTION -3 POWER TRANSMISSION FUNDAMENTALS

Power transmission is the transfer of energy from its place of generation to a location where it is applied to performing useful work. Power transmission is normally accomplished by belts, ropes, chains, gears, couplings and friction clutches. Out of these, the gears are capable of transmitting force or motion without any slip and therefore are the most durable and rugged of all mechanical devices. In the schematic below, a gear transmits rotation force from prime mover (diesel engine) to another driven shaft (locomotive wheels).



Schematic of Diesel Locomotive

The most important feature of gears is that it produces a mechanical advantage, which is a measure of the force amplification. Since we do not get something for nothing, you can either achieve high velocity output or high force/torque output but not both. The model for this is the law of the lever where a smaller force acting through a greater distance produces the same output as the larger force on a smaller distance.



Energy and Power Equations

Power, torque and speed are the defining mechanical variables associated with the functional performance of rotating machinery. Let's do some analysis for gears:

- P = Power
- E = Energy
- W = Work
- F = Force
- T = torque
- d = distance of translational motion
- θ= angle of rotational motion (in radians)
- v = velocity of translational motion
- ω = angular speed (in radians per second)
- Δ = change
- Pd = Pitch diameter
- Z = number of teeth on a gear
- r = Pitch circle radius
- N = number of revolutions

Power is defined as energy per unit of time or the rate at which work is performed and thus:

$$P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t}$$

When force (F) moves a body a measured distance (Δd), the work done (W) is given by:

W (work) = Force x Distance

$W = F \times \Delta d$

This equation is true for linear motion but the corresponding definition of work for rotational power transmission is given by the Torque (T) and the angular displacement ($\Delta\theta$). Therefore, work done for rotary motion is:

$$W = T\,\Delta\theta$$

Rotation is perceived as a change in the angular position of a reference point on the body over some time interval, Δt. The power transfer in a rotary device is therefore given by:

$$P = \frac{W}{\Delta t} = \frac{T\,\Delta\theta}{\Delta t}$$ Eq. A

The rotary motion is characterized by its angular velocity (ω) and is defined as:

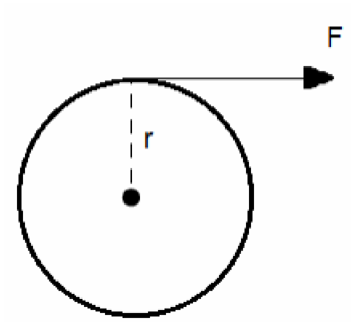
$$\omega = \frac{\Delta\theta}{\Delta t}$$

Substituting the rotary definition of work into Eq. A:

$$\mathbf{P = T\omega}$$ Eq. 1

Let's break Torque (T) and angular velocity (ω) in friendly terms.

Torque is a measure of the tendency of a force to rotate an object about some axis. In order to produce torque, the force must act at some distance from the axis or pivot point. In the following diagram, the circle represents a wheel of radius r; the dot in the center represents the axle or shaft; and the force (F) is applied tangentially at the periphery.



The amount of torque about the gear axle is:

Torque = Force x Radius

T = F x r Eq. 2

Substituting T (Eq. 2) to Eq. 1:

$$P = F * r * \omega$$ Eq. 3

Angular velocity (ω) is often referred to as rotational speed and measured in numbers of complete revolutions per minute (rpm) or per second (rps). It is usually expressed as:

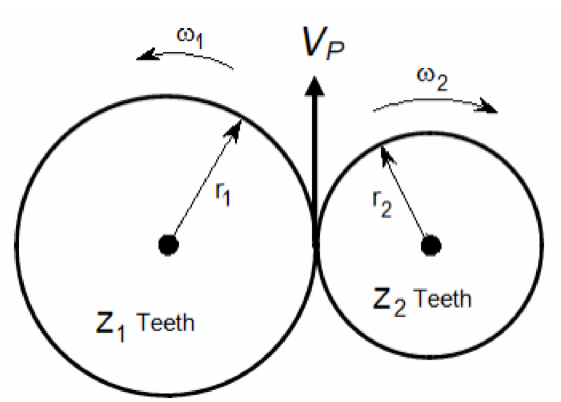
$$\omega = \frac{2\pi N}{60} \qquad \{N = \frac{\text{Number of rotations}}{\text{minute}}, \text{(RPM)}\} \qquad \text{Eq. 4}$$

Substituting ω (Eq. 4) to Eq. 3:

$$P = F * r * \frac{2\pi N}{60} \qquad \text{Eq. 5}$$

ANALYSIS FOR A GEAR PAIR

Consider two gears in a mesh. Gear 1 (driver) is turning counterclockwise at angular velocity ω_1 and has Z_1 teeth. Gear 2 (the driven gear) is turning clockwise with angular velocity ω_2 and has Z_2 teeth. The drive between the two gears is represented by plain cylinders having diameters equal to their pitch circles.



We have learnt in Section-2 that a pair of gears can only mesh correctly if and when the diametral pitch (P_d) is the same, accordingly:

$$P_d = \frac{Z_1}{d_1} = \frac{Z_2}{d_2} \qquad \text{Eq. 6}$$

The driving gear pushes the driven gear, exerting a force component perpendicular to the gear radius; and because the gear is rotating, power is transferred.

$$P_{in} = T_1 \ \omega_1$$

$$P_{out} = T_2 \ \omega_2$$

Assuming no frictional losses, the input and output power can be set equal to each other as:

$$P_{in} = P_{out}$$

or $\omega_1 T_1 = \omega_2 T_2$ Eq. 7

or $\dfrac{\omega_1}{\omega_2} = \dfrac{T_2}{T_1}$

Since, $\omega = \dfrac{2 \pi N}{60}$ $\{N = \dfrac{\text{Number of rotations}}{\text{minute}}, (\text{RPM})\}$

$\dfrac{\omega_1}{\omega_2} = \dfrac{N_1}{N_2} = \dfrac{T_2}{T_1}$ Eq. 8

It follows that torque and speed are inversely proportional. If a high torque is desired, then the speed must be sacrificed. When speed increases, the torque decreases proportionally.

We now consider the relative velocity of the two gears. The point of contact of the two pitch surfaces shall have velocity along the common tangent. The velocity of a point on a rotating object is given by r ω. Because there is no slip, definite motion of gear 1 can be transmitted to gear 2, therefore:

$$v = \omega_1 r_1 = \omega_2 r_2$$

Where r_1 and r_2 are pitch circle radii of gears 1 and 2, respectively.

$\omega_1 r_1 = \omega_2 r_2$ Eq. 9

or $\dfrac{2\pi N_1}{60} r_1 = \dfrac{2\pi N_2}{60} r_2$

or $N_1 r_1 = N_2 r_2$

or $\dfrac{N_1}{N_2} = \dfrac{r_2}{r_1}$

Putting it in terms of diameter, r = d/2, it implies:

$$N_1 \frac{d_1}{2} = N_2 \frac{d_2}{2}$$

or $$N_1 d_1 = N_2 d_2$$ Eq. 10

or $$\frac{N_1}{N_2} = \frac{d_2}{d_1}$$

It follows that speed and diameter are inversely proportional. If a high speed is desired, then the diameter of driven gear must be lower than the driving gear.

Since, pitch circle radius of a gear is proportional to its number of teeth (Z):

$$N_1 \frac{Z_1}{P_d} = N_2 \frac{Z_2}{P_d}$$

or $$N_1 Z_1 = N_2 Z_2$$

or $$\frac{N_1}{N_2} = \frac{Z_2}{Z_1}$$ Eq. 11

It follows that the velocity ratio of a pair of gears is the inverse ratio of their number of teeth, i.e. the gear with the greater number of teeth will always revolve slower than the gear with the smaller number of teeth.

We can now combine the torque equation (Eq. 8), dia. Equation (Eq. 10) and the velocity equation (Eq. 11) to get the relationship with the gear teeth ratio.

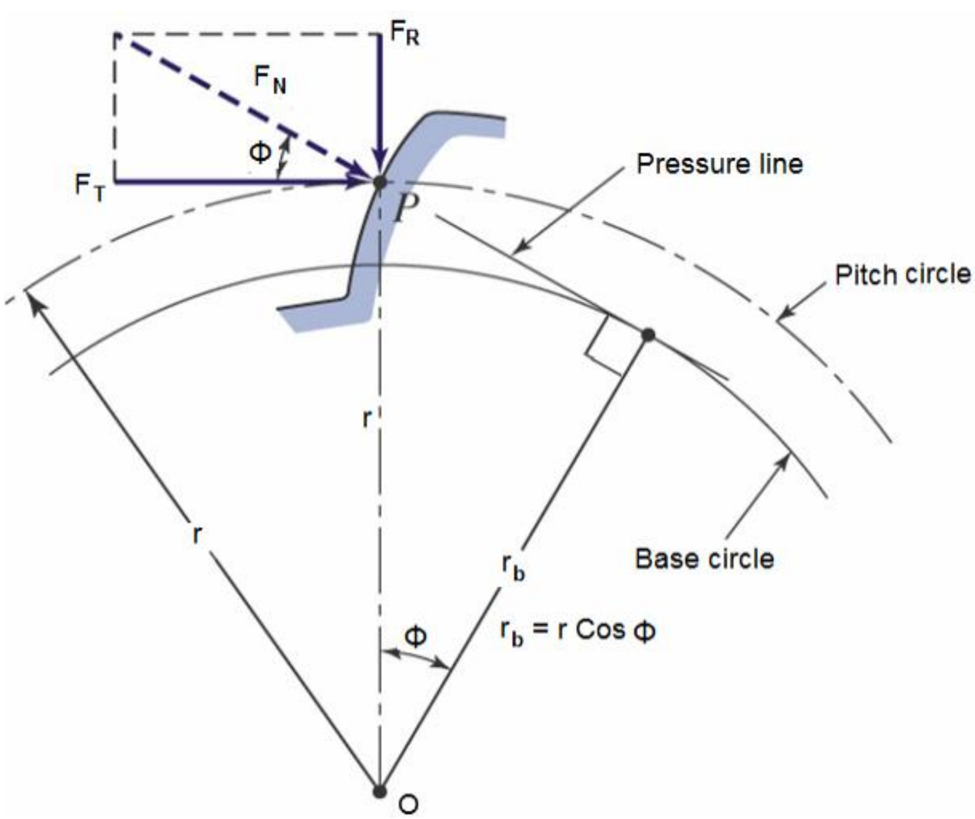
$$\frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{T_2}{T_1} = \frac{d_2}{d_1} = \frac{Z_2}{Z_1}$$

Gear ratio is defined as the ratio of diameters (teeth) of output to the input gear. When the input gear is smaller than the output gear, the output torque is higher than the input torque and the output speed is lower than the input speed or in other words "a higher gear ratio equates to high torque and lower speed".

Let's compare this analogy to a car engine gearbox. In top gears, one turn of the engine crankshaft results in one turn of the drive wheels. Lower gears require more turns of the engine to provide single turn of the drive wheels, producing more torque at the drive wheel. For example, if the driving gear has 10 teeth and the driven gear has 20 teeth, the gear ratio is 2 to 1. Every revolution of the driving gear will cause the driven gear to revolve through only half a turn. Thus, if the engine is operating at 2,000 rpm, the speed of the driven gear will be only 1,000 rpm; the speed ratio is then 2 to 1. This arrangement doubles the torque on the shaft of the driven unit. The speed of the driven unit, however, is only half that of the engine. On the other hand, if the driving gear has 20 teeth and the driven gear has 10 teeth, the speed ratio is 1 to 2, and the speed of the driven gear is doubled. The rule applies equally well when an odd number of teeth is involved. If the ratio of the teeth is 37 to 15, the speed ratio is slightly less than 2.47 to 1. In other words, the driving gear will turn through almost two and a half revolutions while the driven gear makes one revolution.

POWER FLOW THROUGH GEAR PAIR

With a pair of gears, power is transmitted by the force developed between contacting teeth. According to fundamental law of gear this resultant force always acts along the pressure line (or the line of action). To investigate how the forces are typically transmitted between a pair of gears, refer to the figure below.



Power Flow through a Gear Pair

(View showing tangential and radial forces on the gear)

The force transmitted along the line of action results in a torque generated at the base circle. The torque can be calculated by:

$T = F_N r_b$ Eq. 12

From the figure above, the relationship between the base circle and pitch circle radius can be stated as:

$r_b = r \cos\Phi$

Or

$T = F_N r \cos\Phi$ Eq. 13

Where,

- F_N is the force action on the line of action
- Φ is the pressure angle
- r_b is the radius of base circle
- r is the pitch circle radius of the gear

This resultant force F_N can be resolved into two components: tangential component F_T and radial components F_R at the pitch point.

- F_T - the tangential force component acting at the radius of the pitch circle. It determines the magnitude of the torque and consequently the power transmitted. The Tangential component is expressed as: **$F_T = F_N \cos\Phi$**
- F_R - the radial or normal force directed towards the center of the gear. F_R serves to separate the shafts connected to the gears and for this reason F_R is sometimes referred to as the separating force. The radial component is expressed as: **$F_R = F_N \sin\Phi$ or $F_T \tan\Phi$**

Torque exerted on the gear shaft in terms of the pitch circle radius can be found by substituting $F_T = F_N \cos\Phi$ in Eq. 13. The resultant expression is:

$T = F_T \times r$

Or

$T = F_T \times \frac{d}{2}$ Eq. 14

The maximum force (F_T) is exerted along the common normal through the pitch point which is line perpendicular to the line of centers. After determining F_T, the magnitude of the other force components and/or their directions can be readily determined.

Recall the following relationship existing between speed, torque and power:

$$P = T\omega$$

$$T = \frac{P}{\omega}$$

Since, $\omega = \frac{2 \pi N}{60}$

The torque transmitted by the gear is given by:

$$T = P \times \frac{60}{2 \pi N}$$

Where,

- T = Torque transmitted gears (N- m)
- P = Power transmitted by gears (kW)
- N = Speed of rotation (RPM)

Alternatively in US units:

$$T = P \times \frac{63000}{N} \qquad \text{Eq. 15}$$

Where,

- P = Power, HP
- T = Torque in-lbs
- N = RPM

Substituting Eq. 14 into Eq. 15

$$F_T \times \frac{d}{2} = P \times \frac{63000}{N}$$

Or

$$F_T = P \times \frac{63000}{N} \times \frac{2}{d}$$

Or

$$F_T = P \times \frac{126000}{N\ d}$$

Power can also be expressed in terms of the pitch line velocity v.

$P = F_T\ v$

- P = Power in watts
- F_T = force, N
- v = velocity, m/s

Or in terms of customized units

$$P = F_T\ v \left[\frac{1m}{60s}\right] \left[\frac{1\ H.P}{550\ ft\text{-}lbs/s}\right] = \frac{F_T\ v}{33{,}000}$$

- v = velocity in ft/min
- F_T = force in lbs
- P = Power in HP

The above expression can be rearranged to solve for F_T;

$$F_T = \frac{33{,}000\ P}{v}$$

The velocity v:

v = .262 X d X N

Where,

- d = Pitch diameter
- N = Revolutions per minute, RPM

Important!

If the forces transmitted between the teeth of meshing gears are transmitted along the line of action at every point of contact, then regardless of the angular position of the gears, the forces transmitted between the gears maintain a fixed orientation in space. Maintaining a fixed orientation in space for the forces to be transmitted between gears enables the power transmitted between the gears to be independent of the angular position of the gears. This is a very desirable characteristic for gears.

Example

20-tooth, 8 pitch, 1-inch-wide, 20° pinion transmits 5 HP at 1725 rpm to a 60-tooth gear. Determine driving force, separating force, maximum force and surface speed that would act on mounting shafts.

Solution:

$$T = \frac{63000 \times P}{N}$$

$$T = \frac{63000 \times 5}{1725} = 183 \text{ in-lb}$$

Find pitch circle

$$d = \frac{Z}{P_d}$$

$$d = \frac{20 \text{ teeth}}{8 \text{ teeth/in diameter}} = 2.5 \text{ in}$$

Find transmitted force

$$F_T = \frac{2T}{d}$$

$$F_T = \frac{2 \times 183 \text{ in-lb}}{2.5 \text{ in}} = 146 \text{ lb}$$

Find separating force

$F_R = F_T \tan\Phi$

FR = 146 tan 20°

$F_R = 53$ lb

Find maximum force

$$F_N = \frac{F_T}{\cos\Phi}$$

$$F_N = \frac{146 \text{ lb}}{\cos 20^\circ}$$

$F_N = 155$ lb

Find pitch line velocity or surface speed

v = .262 X D X RPM

v = 0.262 x 2.5 x 1725 = 1129 ft/min

Summarizing…….

The fundamental equations for a gear pair are:

$T_{in}\,\omega_{in} = T_{out}\,\omega_{out}$ --------- (Power equality)

$\frac{\omega_{out}}{\omega_{in}} = \frac{r_{in}}{r_{out}}$ ---------- (Velocity relationship in terms of radiuses)

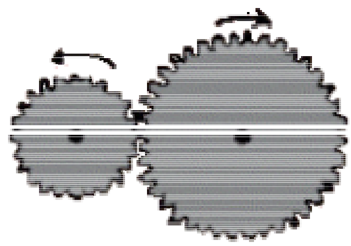
$\frac{\omega_{out}}{\omega_{in}} = \frac{Z_{in}}{Z_{out}}$ ---------- (Velocity relationship in terms of number of teeth)

$\frac{T_{out}}{T_{in}} = \frac{r_{out}}{r_{in}}$ ---------- (Torque relationship in terms of radiuses)

$\frac{T_{out}}{T_{in}} = \frac{Z_{out}}{Z_{in}}$ ---------- (Torque relationship in term of number of teeth)

SECTION -3 GEAR TRAINS

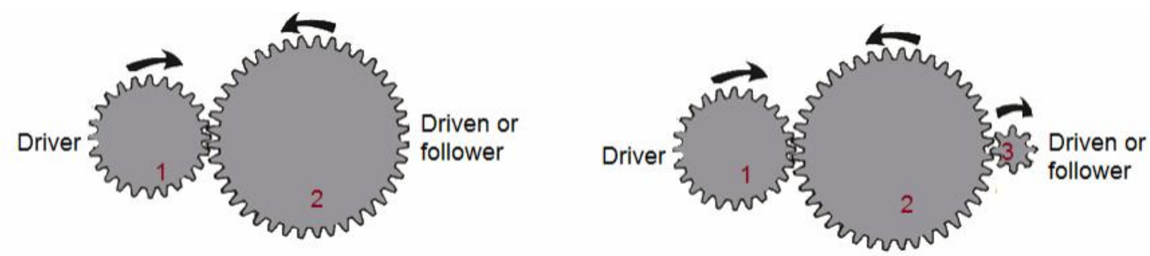
A gear train is a power transmission system made up of two or more gears. The gear to which the force is first applied is called the driver and the final gear on the train to which the force is transmitted is called the driven gear. Any gears between the driver and the driven gears are called the idlers. Conventionally, the smaller gear is the **Pinion** and the larger one is the **Gea**r. In most applications, the pinion is the driver; this reduces speed but increases torque.



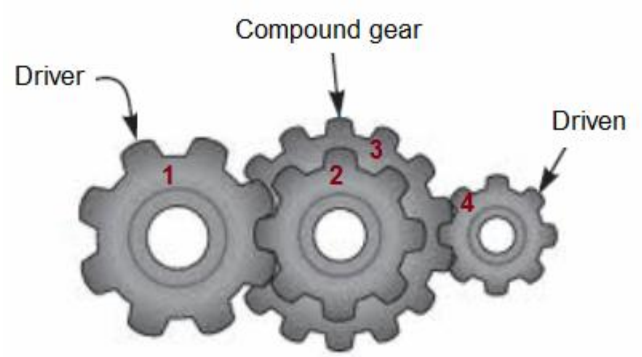
Types of gear trains

1. Simple gear train
2. Compound gear train
3. Planetary gear train

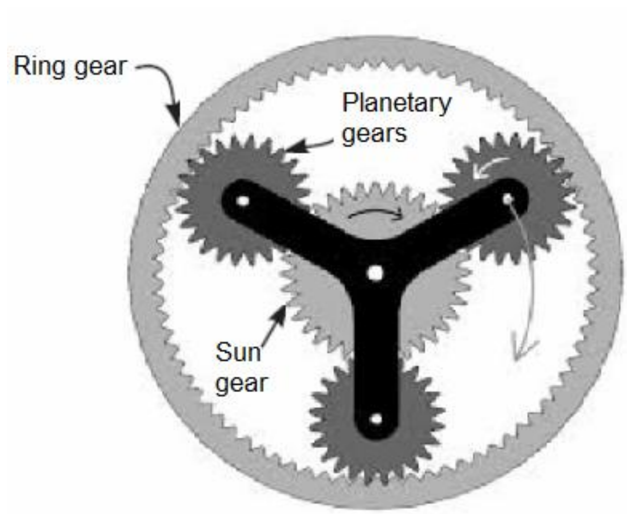
Simple Gear Train - Simple gear trains have only one gear per shaft. The simple gear train is used where there is a large distance to be covered between the input shaft and the output shaft.



Compound Gear Train - In a compound gear train at least one of the shafts in the train must hold two gears. Compound gear trains are used when large changes in speed or power output are needed and there is only a small space between the input and output shafts.



Planetary Gear Train - A planetary transmission system (or Epicyclic system as it is also known), consists normally of a centrally pivoted sun gear, a ring gear and several planet gears which rotate between these. This assembly concept explains the term planetary transmission, as the planet gears rotate around the sun gear as in the astronomical sense the planets rotate around our sun.



Planetary gearing or epicyclic gearing provides an efficient means to transfer high torques utilizing a compact design.

GEAR RATIO

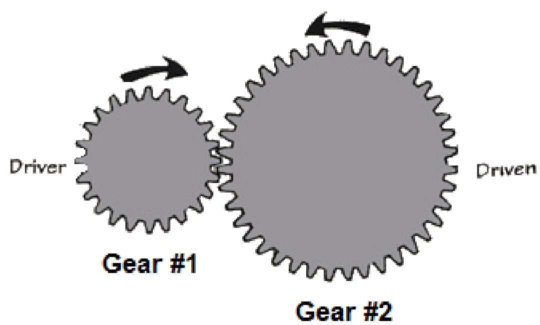
We have learnt in the previous section that ***"If two gears are in mesh, then the product of speed (revolutions) and teeth must be conserved".*** Let's check this simple rule with a help of an example.

If you turn a gear with 6 teeth 3 times and is meshed with a second gear having 18 teeth, than the driving gear 18 teeth (6 x 3) will move through the meshed area. This means that the 18 teeth from the second gear also move through the meshed area. If the second gear has 18 teeth, then it only has to rotate once because 18 x 1=18.

Also, the second gear will be turning slower than the first because it is larger, and larger gears turn slower than smaller gears because they have more teeth.

Gear Ratio for Simple Gear Train

Consider a simple gear train shown below. Notice that the arrows show how the gears are turning. When the driver is turning clockwise the driven gear is anti-clockwise.



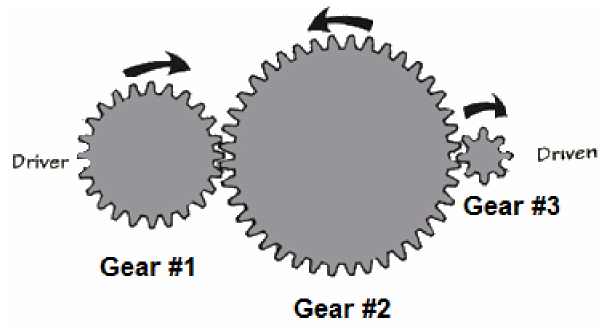
Further assume driver gear #1 has 20 teeth and rotating at 100 rpm. Find the speed of driven gear #2 having 60 teeth.

- N_1 = 100 rpm
- Z_1 = 30 teeth
- N_2 =?
- Z_2 = 60 teeth

Solving the equation above for N_2, we have:

$N_2 = (Z_1/Z_2) * N_1 = (30/60) * 100 = 50$ rpm

Let's add a third gear to the train. Assume gear 2 drives gear 3 and gear 3 has Z_3 = 20 teeth. Here the driver is gear #1 and the final driven element is gear #3. Gear #2 in between is called the **idler** gear. Find the speed of driven gear #3?



Well, since gears 2 and 3 are in mesh, our conservation law says that:

$N_2 * Z_2 = N_3 * Z_3$

We could do the arithmetic ($N_3 = (Z_2/Z_3) * N_2 = (60/20) * 50 = 150$ rpm) to find N_3. Or, we could note that, since both N_1*Z_1 and N_3*Z_3 are equal to N_2*Z_2, they must be equal to each other.

$N_1 * Z_1 = N_3 * Z_3$

So,

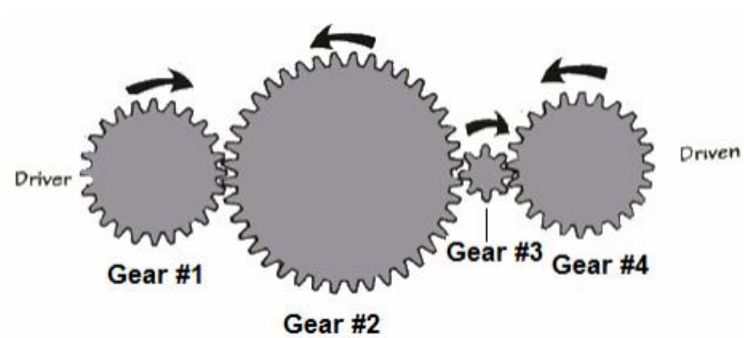
$N_3 = (Z_1/Z_3) * N_1 = (30/20) * 100 = 150$ rpm.

What does this prove?

"An idler gear between a driver and driven gear has NO effect on the overall gear ratio, regardless of how many teeth it has".

(Note that Z_2 never entered into our computation in the last equation.)

Suppose now that we add a fourth gear with $Z_4 = 40$ teeth to our developing gear train.



Its speed must be $N_4 = (Z_3/Z_4) * N_3 = (20/40) * 150 = 75$ rpm. Again, by using the conservation principle, we have:

$N_4 = (Z_1/Z_4) * N_1 = (30/40) * 100 = 75$ rpm.

We can continue like this indefinitely, but the two fundamental learning objectives here are:

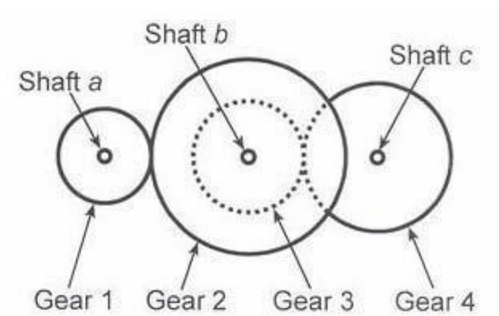
1. The number of teeth on the intermediate gears does not affect the overall velocity ratio, which is governed purely by the number of teeth on the first and last gear.
2. If the train contains an odd number of gears, the output gear will rotate in the same direction as the input gear, but if the train contains an even number of gears, the output gear will rotate opposite that of the input gear. If it is desired that the two gears and shafts rotate in the same direction, a third idler gear must be inserted between the driving gear and the driven gear. The idler revolves in a direction opposite that of the driving gear.

Major Caveat:

Note that everything said to this point assumes a simple gear train where each of the gears in the gear train is on its own, separate shaft. Sometimes gears are 'ganged' by keying or otherwise welding them together and both gears turn as a unit on the same shaft. This arrangement is known as compound gear train and it complicates the computation of the gear ratio, to some extent.

Compound Gear Train

The figure below shows a set of compound gears with the two gears, 2 and 3, mounted on the middle shaft b. Both of these gears will turn at the same speed because they are fastened together, i.e. $N_b = N_2 = N_3$



When gear 1 and gear 2 are in mesh:

$N_1 * Z_1 = N_2 * Z_2$

It’s still true that:

$N_1 * Z_1 = N_b * Z_2$

$N_b = (Z_1/Z_2) * N_1$

If gears 3 and 4 are in mesh:

$N_b * Z_3 = N_4 * Z_4$

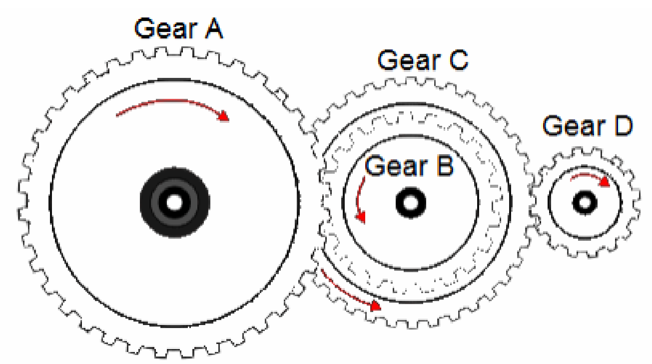
Therefore,

$N_4 = (Z_3/Z_4) * N_b = (Z_3/Z_4)*(Z1/Z_2) * N_1$

So the end-to-end gear ratio is $(Z_1*Z_3)/ (Z_2*Z_4)$ and it does depend on the intermediate gears; unlike the previous case when each gear could turn on its own separate axis. Note that the resultant gear ratio is just the product of the two separate gear ratios: $(Z_1/Z_2)*(Z_3/Z_4)$.

Example:

In the figure below, Gears B and C represent a compound gear and have the following details:



Gear A - 120 teeth

Gear B - 40 teeth

Gear C - 80 teeth

Gear D - 20 teeth

What is the output in revs/min at D, and what is the direction of rotation if Gear A rotates in a clockwise direction at 30 revs/min?

Solution:

When answering a question like this, split it into two parts. Treat Gears A and B as the first part of the question. Treat Gears C and D as the second part.

Gear ratio AB = driven/driving = 40/120 = 1/3

Gear ratio CD = driven /driving = 20/80 = 1/4

Since the driving Gear A rotates 30 RPM and the Gear B is smaller than Gear A, we can conclude that the RPMs for Gear B is 30*3 = 90 RPM

Since Gears B and C represent a compound gear, they have the same rotational speed. Therefore, Gear D speed is obtained by multiplying 4 to Gear C speed of 90 RPM.

Thus, Gear D moves at 90*4 = 360 rev/min

OR

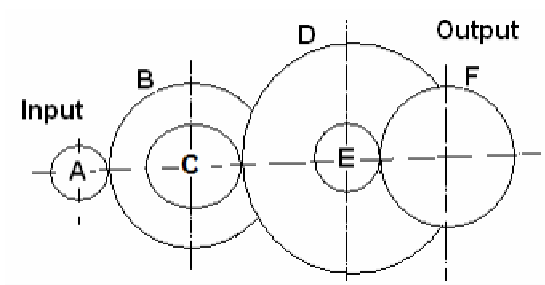
1/3 x ¼ = 1/12

Since Gear A moves at 30 RPM and Gear D is smaller, we multiply by 12:

30 x 12 = 360 RPM

Example:

Calculate the gear ratio for the compound chain shown below. If the input gear rotates clockwise, in which direction does the output rotate?



Gear A has 20 teeth

Gear B has 100 teeth

Gear C has 40 teeth

Gear D has 100 teeth

Gear E has 10 teeth

Gear F has 100 teeth

Solution:

The driving teeth are A, C and E

The driven teeth are B, D and F

Gear ratio = (100 x 100 x 100) / (20 x 40 x 10) = 125

Alternatively we can say there are three simple gear trains as follows:

First gear GR = 100 / 20 = 5

Second chain GR = 100 / 40 = 2.5

Third chain GR = 100 / 10 = 10

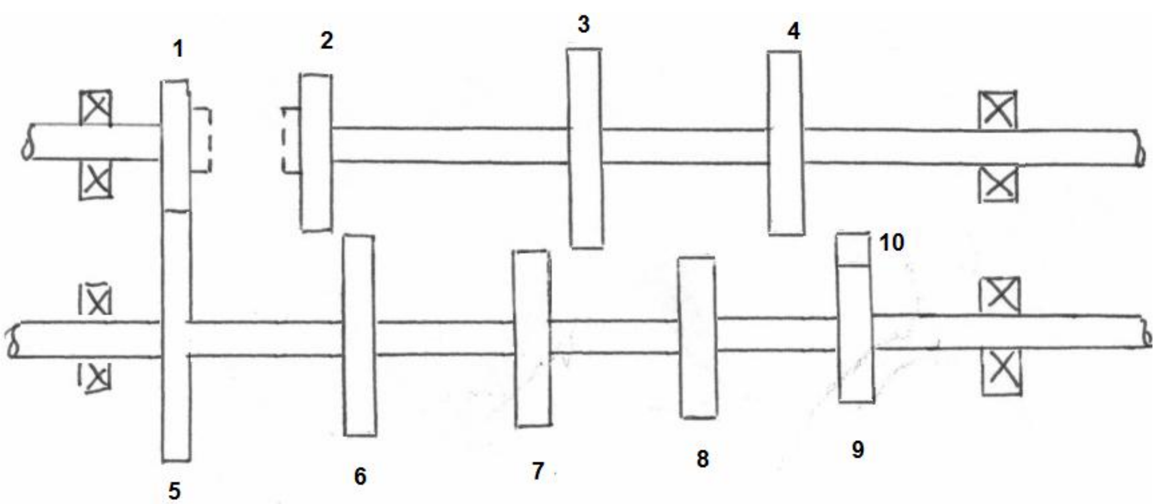
The overall ratio = 5 x 2.5 x 10 = 125

Each chain reverses the direction of rotation so if A is clockwise, B and C rotate anti-clockwise, so D and E rotate clockwise. The output gear F hence rotates anti-clockwise.

More complex compound gear trains can achieve high and low gear ratios in a restricted space by coupling large and small gears on the same axle. In this way gear ratios of adjacent gears can be multiplied through the gear train.

Example:

Suppose we look at a standard four-speed car gearbox with a reverse gear.



The following table shows the number of teeth for each gear:

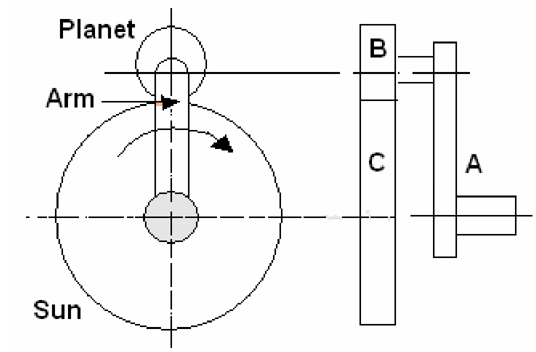
Gear	1	2	3	4	5	6	7	8	9	10
N	12	15	25	20	38	35	25	30	12	12

The table below shows the speed ratio (SR) calculations for each gear selection possible.

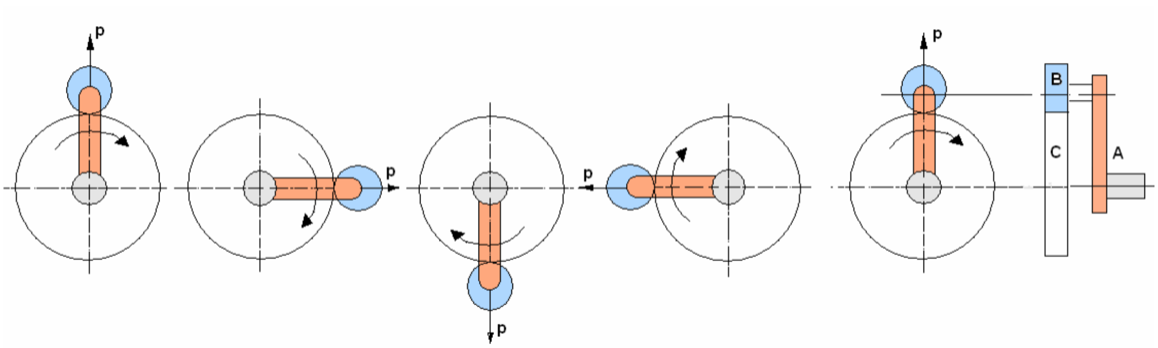
Selection	Gear Train	SR Formula	SR Formula	SR Value
1st Gear	1,5,7,3	$\frac{N_5 \times N_3}{N_1 \times N_7}$	$\frac{38 \times 25}{12 \times 25}$	3.16
2nd Gear	1,5,8,4	$\frac{N_5 \times N_4}{N_1 \times N_8}$	$\frac{38 \times 20}{12 \times 30}$	2.11
3rd Gear	1,5,6,2	$\frac{N_5 \times N_2}{N_1 \times N_6}$	$\frac{38 \times 15}{12 \times 35}$	1.36
4th Gear	1 locked with 2	None	None	1.00
Reverse	1,5, 9,10,4	$\frac{N_5 \times N_4 \times N10}{N_1 \times N_9 \times N10}$	$\frac{38 \times 12 \times 12}{12 \times 12 \times 12}$	-3.16

EPICYCLIC GEAR TRAIN

In epicyclic gear train, the axis of rotation of one or more of the wheels is carried on an arm which is free to revolve about the axis of rotation of one of the other wheels in the train. The diagram shows a Gear B on the end of an arm A. Gear B meshes with Gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun.



Now let's see what happens when the planet gear orbits the sun gear.



Observe point p and you will see that Gear B also revolves once on its own axis. Any object orbiting around a center must rotate once. Now consider that B is free to rotate on its shaft and meshes with C. Suppose the arm is held stationary and Gear C is rotated once. B spins about its own center and the number of revolutions it makes is the ratio N_C / N_B. B will rotate by this number for every complete revolution of C.

Now consider that C is unable to rotate and the Arm A is revolved once. Gear B will revolve $1 + (N_C / N_B)$ because of the orbit. It is the extra rotation that causes confusion. One way to get around this is to imagine that the whole system is revolved once. Then identify the gear that is fixed and revolve it back one revolution. Work out the revolutions of the other gears and add them up. The following tabular method makes it easy.

Method 1

Suppose Gear C is fixed and the Arm A makes one revolution. Determine how many revolutions the planet Gear B makes.

Step 1 is to revolve everything once about the centre.

Step 2 is to identify that C should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions for B.

Step 3 is to simply add them up and find that the total revs of C is zero and the arm is 1.

Step	Action	A	B	C
1	Revolve all once	1	1	1
2	Revolve C by -1 rev	0	$+ N_C / N_B$	-1
3	Add	1	$1 + N_C / N_B$	0

The number of revolutions made by B is $(1+ t_C /t_B)$. Note that if C revolves -1, then the direction of B is opposite so $+ t_C / t_B$

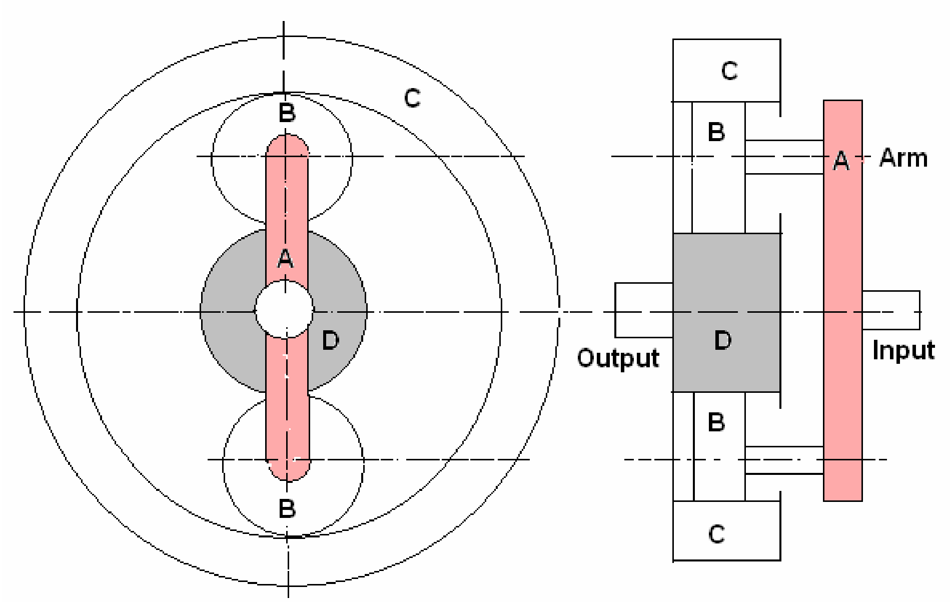
Example:

A simple epicyclic gear has a fixed sun gear with 100 teeth and a planet gear with 50 teeth. If the arm is revolved once, how many times does the planet gear revolve?

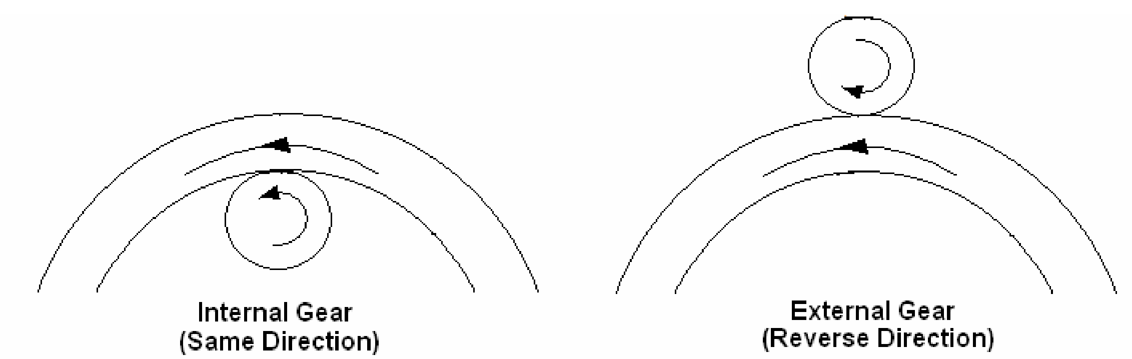
Solution:

Step	Action	A	B	C
1	Revolve all once	1	1	1
2	Revolve C by -1 rev	0	+ 100 / 50	-1
3	Add	1	3	0

The design considered so far has no identifiable input and output. We need a design that puts an input and output shaft on the same axis. This can be done in several ways.

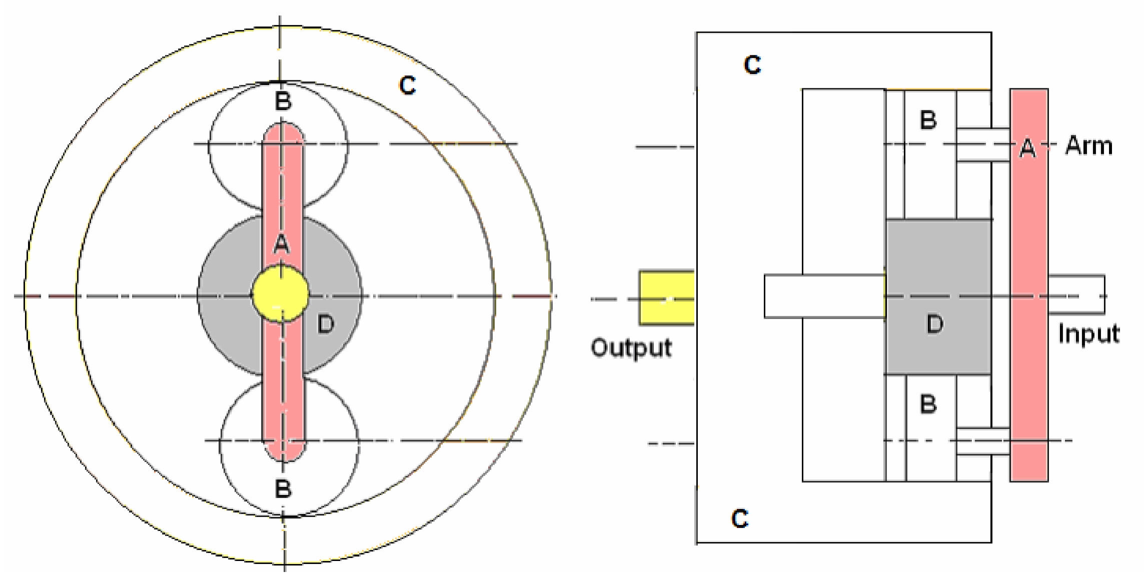


The arm is the input and Gear D is the output. Gear C is a fixed internal gear and is normally part of the outer casing of the gear box. There are normally four planet gears and the arm takes the form of a cage carrying the shafts of the planet gears. Note that the planet gear and the internal gear both rotate in the same direction.



Method 2

In this case the sun Gear D is fixed and the internal Gear C is made into the output.



Example:

An epicyclic gear box has a fixed sun Gear D and the internal Gear C is the output with 300 teeth. The planet Gears B have 30 teeth. The input is the arm/cage A. Calculate the number of teeth on the sun gear and the ratio of the gear box.

Solution:

$N_C = N_D + 2\ N_B$

$300 = N_D + 2 \times 30$

$N_D = 300 – 60 = 240$

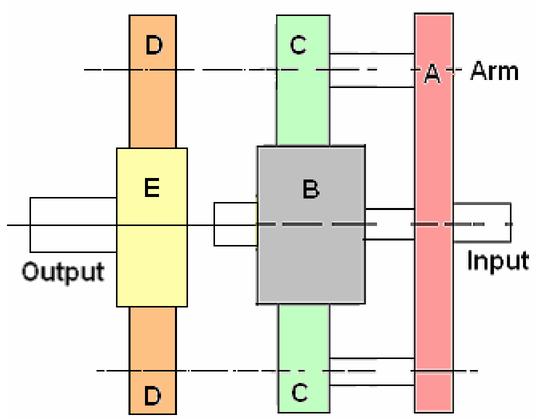
Identify that Gear D is fixed and the arm must do one revolution, so it must be D that is rotated back one revolution holding he arm stationary.

Step	Action	A	B	C	D
1	Revolve all once	1	1	1	1
2	Revolve D by -1 rev	0	240 / 30	240/300	-1
3	Add	1	9	1.8	0

The ratio A/C is then 1:1.8 and this is the gear ratio. Note that the solution would be the same if the input and output are reversed but the ratio would be 1.8.

Method 3

In this design a compound Gear C and D is introduced. Gear B is fixed and Gears C rotate upon it and around it. Gears C are rigidly attached to gears D and they all rotate at the same speed. Gears D mesh with the output Gear E.



Example:

An epicylic gear box is shown above. Gear C has 100 teeth, B has 50, D has 50 and E has 100. Calculate the ratio of the gear box.

Solution

Identify that Gear B is fixed and that A must do one revolution, so it must be B that is rotated back one revolution holding A stationary.

Step	**Action**	**A**	**B**	**C/D**	**E**
1	Revolve all once	1	1	1	1
2	Revolve B by -1 rev	0	-1	½	-¼
3	Add	1	0	1½	¾

The ratio A/E is then ¾:1 or 3:4

Note that the input and output may be reversed but the solution would be the same with ratio of 4:3 instead of 3:4.