

# Chapter-5:Clutches

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**Friction Clutches-** Pivot and Collar friction, Plate clutches, Cone clutch, Centrifugal clutch, Torque transmitting capacity, Clutch operating mechanisms.



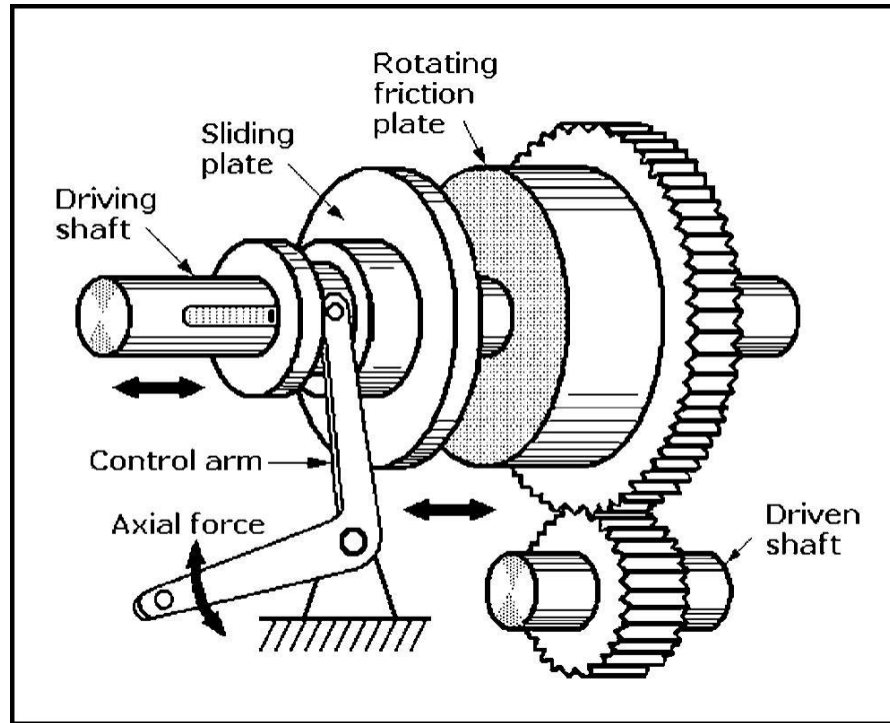
# Definition of Clutch

- **A clutch is defined as a coupling that connects and disconnects the driving and driven parts of a machine; an example is an engine and a transmission.**
- **Clutches typically contain a driving shaft and a driven shaft, and they are classed as either externally or internally controlled.**
- **Externally controlled clutches can be controlled either by friction surfaces or components that engage or mesh positively.**
- **Internally controlled clutches are controlled by internal mechanisms or devices; they are further classified as overload, overriding and centrifugal.**
- **There are many different schemes for a driving shaft to engage a driven shaft.**

# Externally Controlled Friction Clutches

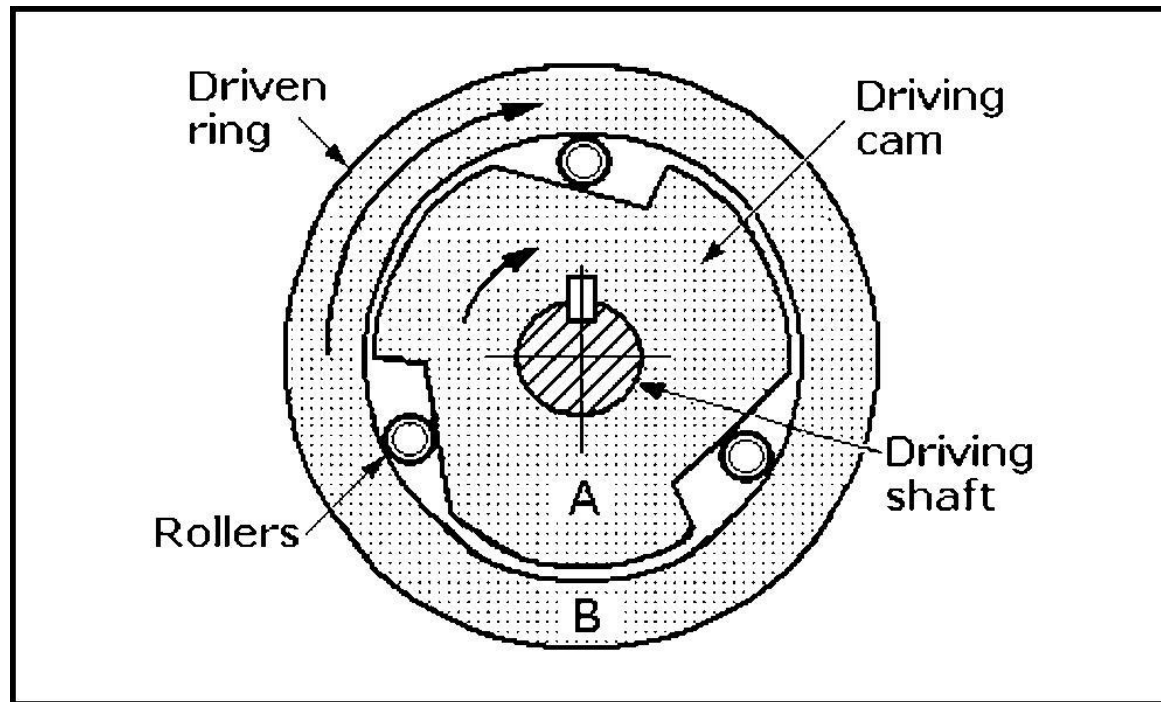
- **Friction-Plate Clutch.**
- A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded.
- The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces

# Friction-Plate Clutch.



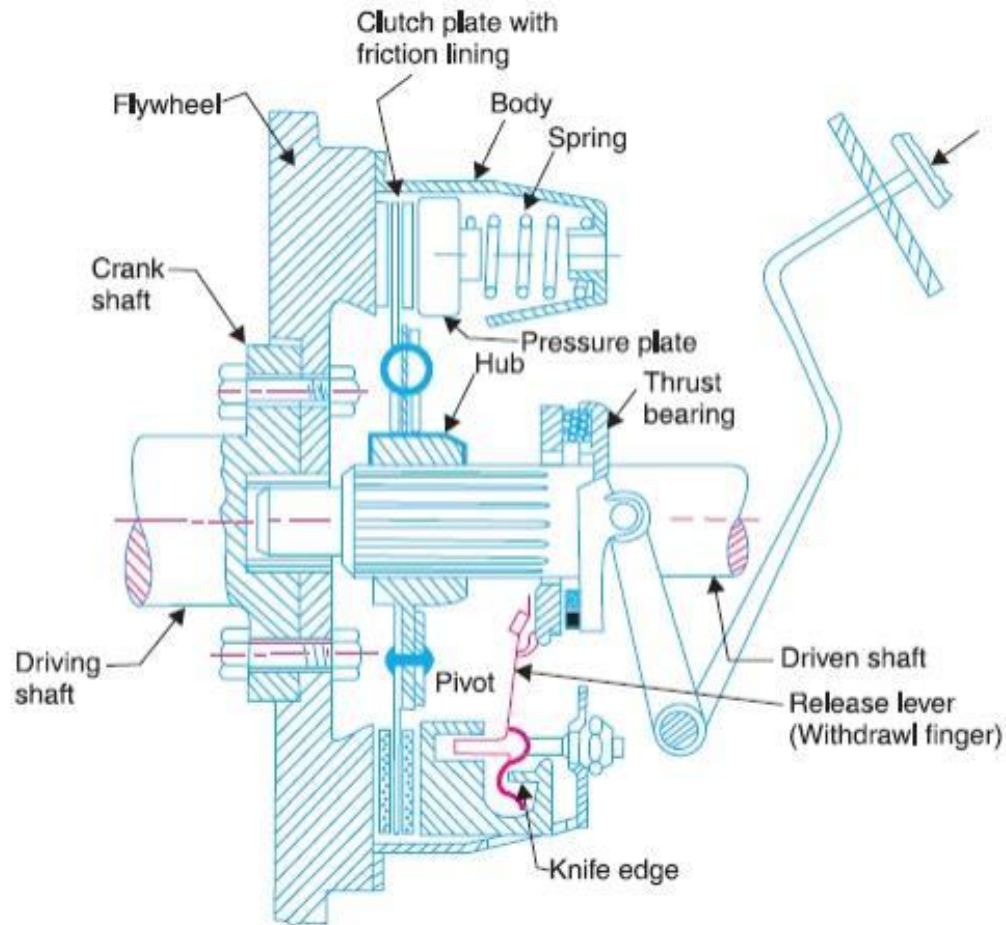
**Friction plate clutch:** When the left sliding plate on the driving shaft is clamped by the control arm against the right friction plate idling on the driving shaft, friction transfers the power of the driving shaft to the friction plate. Gear teeth on the friction plate mesh with a gear mounted on the driven shaft to complete the transfer of power to the driven mechanism. Clutch torque depends on the axial force exerted by the control arm.

# Internally Controlled Clutches

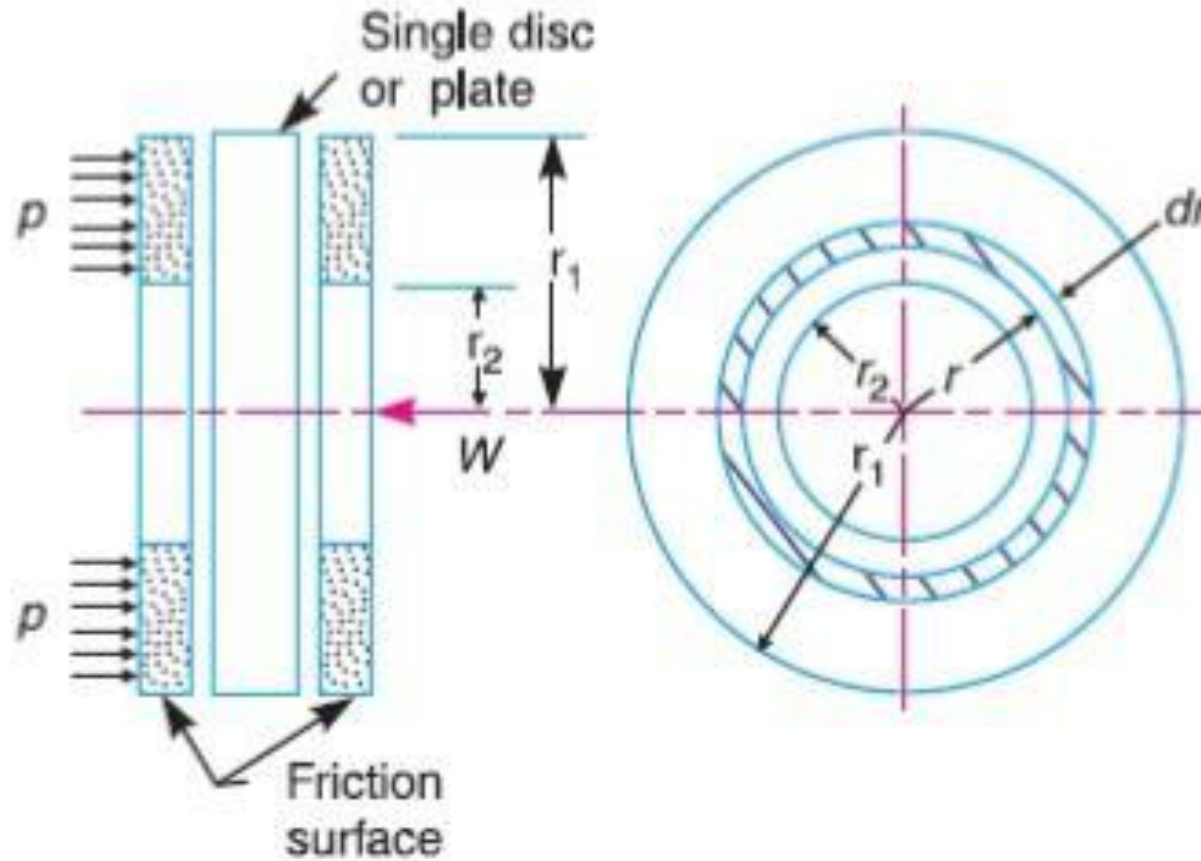


**Over running clutch:** As driving cam A revolves clockwise, the rollers in the wedge-shaped gaps between cam A and outer ring B are forced by friction into those wedges and are held there; this locks ring B to cam A and drives it clockwise. However, if ring B is turned counterclockwise, or is made to revolve clockwise faster than cam A, the rollers are freed by friction, the clutch slips, and no torque is transmitted.

# Single Discord Plate Clutch



# Mathematical representation of clutch plate

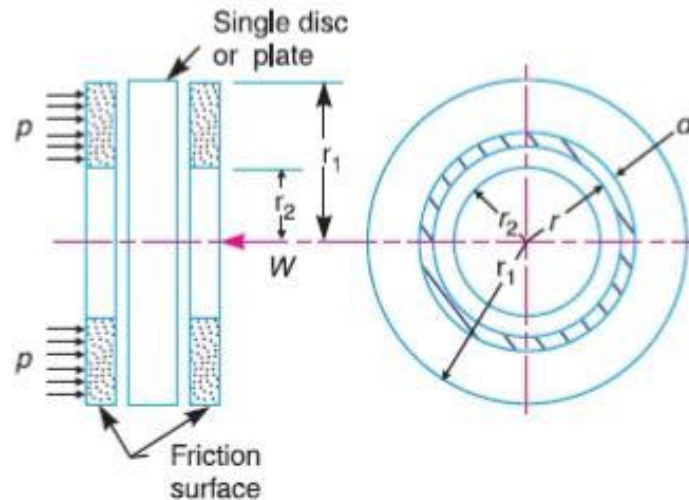




# Friction Clutch theory of Analysis

- Maximum Pressure Theory
- Maximum Wear Theory

# Single Plate Clutch Dynamic Analysis



- Let
- $T$  = Torque transmitted by the clutch,
  - $p$  = Intensity of axial pressure with which the contact surfaces are held together,
  - $r_1$  and  $r_2$  = External and internal radii of friction faces, and
  - $\mu$  = Coefficient of friction.

Consider an elementary ring of radius  $r$  and thickness  $dr$  as shown in Fig. 10.22 (b).

We know that area of contact surface or friction surface,

$$= 2 \pi r . dr$$

$\therefore$  Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2 \pi r . dr$$

and the frictional force on the ring acting tangentially at radius  $r$ ,

$$F_r = \mu . \delta W = \mu . p \times 2 \pi r . dr$$

$\therefore$  Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu . p \times 2 \pi r . dr \times r = 2 \pi \times \mu . p . r^2 dr$$

We shall now consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform wear.

### 1. Considering uniform pressure

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \quad \dots(i)$$

where  $W$  = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius  $r$  and thickness  $dr$  is

$$T_r = 2 \pi \mu . p . r^2 . dr$$

Integrating this equation within the limits from  $r_2$  to  $r_1$  for the total frictional torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2 \pi \mu . p . r^2 . dr = 2 \pi \mu p \left[ \frac{r^3}{3} \right]_{r_2}^{r_1} = 2 \pi \mu p \left[ \frac{(r_1)^3 - (r_2)^3}{3} \right]$$

Substituting the value of  $p$  from equation (i),

$$\begin{aligned} T &= 2 \pi \mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3} \\ &= \frac{2}{3} \times \mu W \left[ \frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right] = \mu W R \end{aligned}$$

where

$R$  = Mean radius of friction surface

$$= \frac{2}{3} \left[ \frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

## 2. Considering uniform wear

In Fig. 10.22, let  $p$  be the normal intensity of pressure at a distance  $r$  from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r. = C \text{ (a constant) or } p = C/r \quad \dots(i)$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C dr = 2\pi C.dr$$

$\therefore$  Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.p.r^2.dr = 2\pi\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr$$

$\dots(\because p = C/r)$

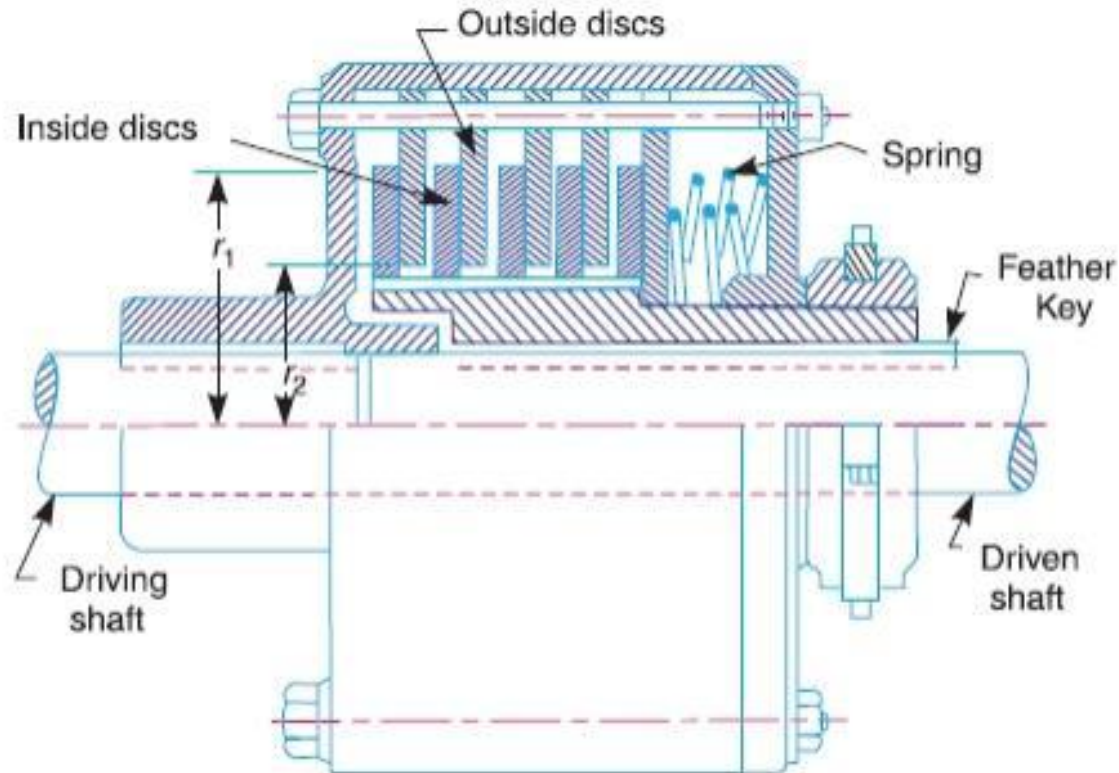
$\therefore$  Total frictional torque on the friction surface,

$$\begin{aligned} T &= \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C \left[ \frac{r^2}{2} \right]_{r_2}^{r_1} = 2\pi\mu.C \left[ \frac{(r_1)^2 - (r_2)^2}{2} \right] \\ &= \pi\mu.C [(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} [(r_1)^2 - (r_2)^2] \\ &= \frac{1}{2} \times \mu.W (r_1 + r_2) = \mu.W.R \end{aligned}$$

where

$$R = \text{Mean radius of the friction surface} = \frac{r_1 + r_2}{2}$$

# Multiple Disc Clutch



Let

$n_1$  = Number of discs on the driving shaft, and

$n_2$  = Number of discs on the driven shaft.

$\therefore$  Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

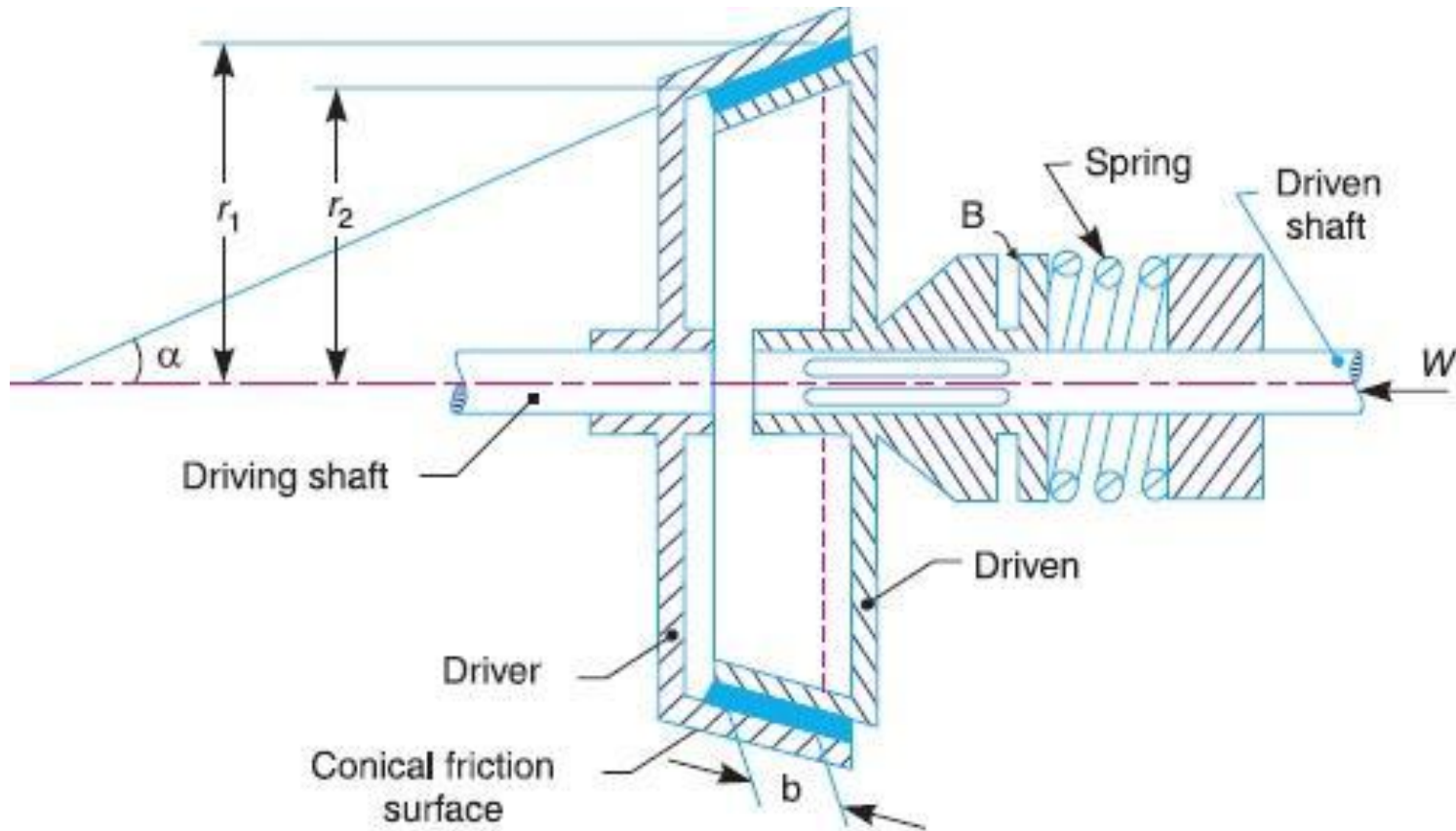
$$T = n \cdot \mu \cdot W \cdot R$$

# Cone Clutch.

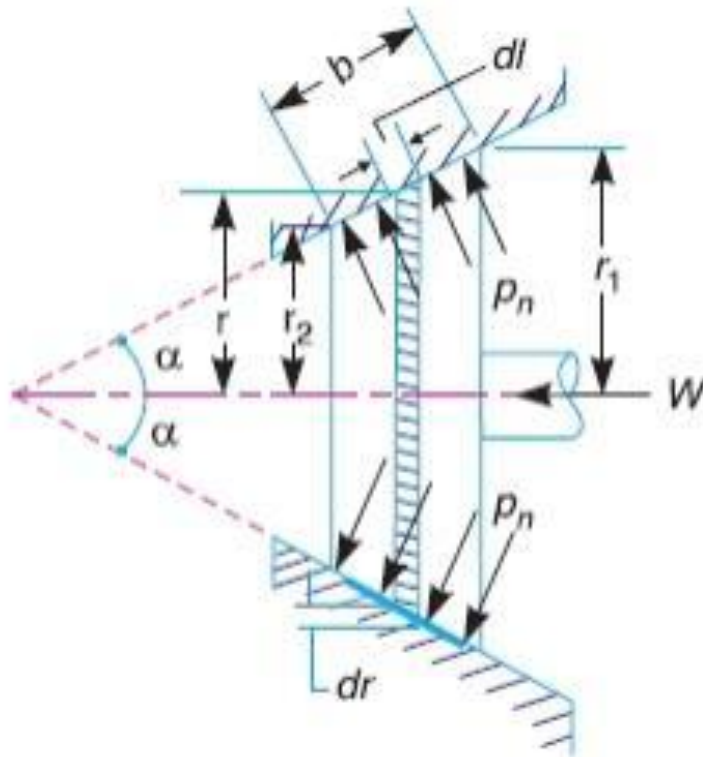
- **Cone Clutch.** A clutch operating on the same principle as the friction-plate clutch except that the control arm advances a cone on the driving shaft to engage a mating rotating friction cone on the same shaft; this motion also engages any associated gearing that drives the driven shaft.
- The friction surface can be on either cone but is typically only on the sliding cone.



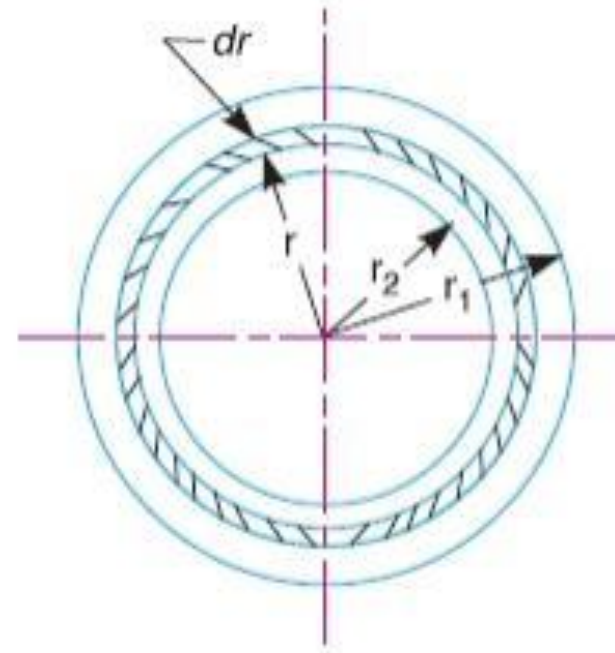
# Schematic of Cone Clutch.



# Mathematical formulation of CC



(a)



(b)

Let

$p_n$  = Intensity of pressure with which the conical friction surfaces are held together (i.e. normal pressure between contact surfaces),

$r_1$  and  $r_2$  = Outer and inner radius of friction surfaces respectively.

$R$  = Mean radius of the friction surface =  $\frac{r_1 + r_2}{2}$ ,

$\alpha$  = Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

$\mu$  = Coefficient of friction between contact surfaces, and

$b$  = Width of the contact surfaces (also known as face width or clutch face).



$$dl = dr \cdot \operatorname{cosec} \alpha$$

∴ Area of the ring,

$$A = 2\pi r \cdot dl = 2\pi r \cdot dr \operatorname{cosec} \alpha$$

### 1. Considering uniform pressure

We know that normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\pi$$

and the axial load acting on the ring,

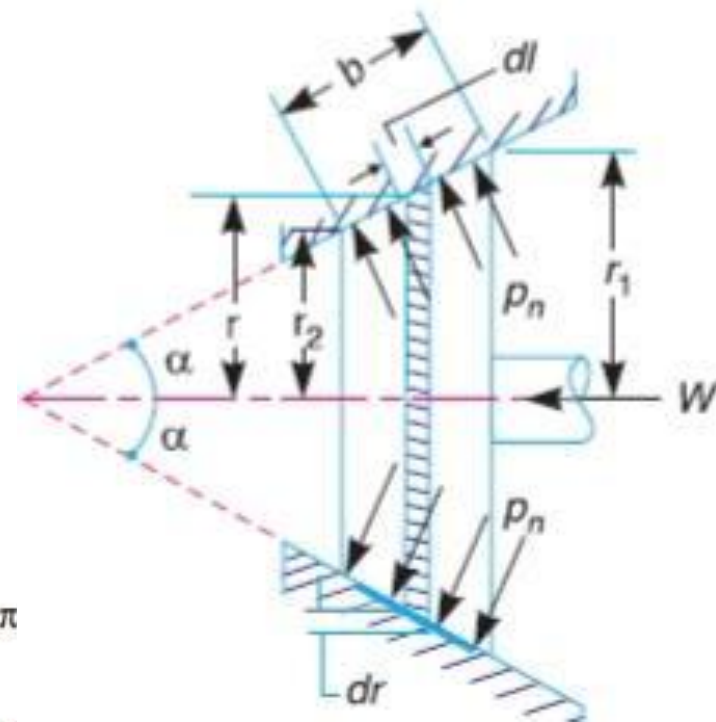
$$\begin{aligned} \delta W &= \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W) \\ &= \delta W_n \times \sin \alpha = p_n \times 2\pi r \cdot dr \cdot \operatorname{cosec} \alpha \times \sin \alpha = 2\pi \times p_n \cdot r \cdot dr \end{aligned}$$

∴ Total axial load transmitted to the clutch or the axial spring force required,

$$\begin{aligned} W &= \int_{r_2}^{r_1} 2\pi p_n r \cdot dr = 2\pi p_n \left[ \frac{r^2}{2} \right]_{r_2}^{r_1} = 2\pi p_n \left[ \frac{(r_1)^2 - (r_2)^2}{2} \right] \\ &= \pi p_n [(r_1)^2 - (r_2)^2] \end{aligned}$$

$$\therefore p_n = \frac{W}{\pi [(r_1)^2 - (r_2)^2]}$$

...(i)



We know that frictional force on the ring acting tangentially at radius  $r$ ,

$$F_r = \mu \cdot \delta W_n = \mu \cdot p_n \times 2 \pi r \cdot dr \cdot \operatorname{cosec} \alpha$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu \cdot p_n \times 2 \pi r \cdot dr \cdot \operatorname{cosec} \alpha \cdot r = 2 \pi \mu \cdot p_n \cdot \operatorname{cosec} \alpha \cdot r^2 \cdot dr$$

Integrating this expression within the limits from  $r_2$  to  $r_1$  for the total frictional torque on the clutch.

∴ Total frictional torque,

$$\begin{aligned} T &= \int_{r_2}^{r_1} 2 \pi \mu \cdot p_n \cdot \operatorname{cosec} \alpha \cdot r^2 \cdot dr = 2 \pi \mu \cdot p_n \cdot \operatorname{cosec} \alpha \left[ \frac{r^3}{3} \right]_{r_2}^{r_1} \\ &= 2 \pi \mu \cdot p_n \cdot \operatorname{cosec} \alpha \left[ \frac{(r_1)^3 - (r_2)^3}{3} \right] \end{aligned}$$

Substituting the value of  $p_n$  from equation (i), we get

$$\begin{aligned} T &= 2 \pi \mu \times \frac{W}{\pi [(r_1)^2 - (r_2)^2]} \times \operatorname{cosec} \alpha \left[ \frac{(r_1)^3 - (r_2)^3}{3} \right] \\ &= \frac{2}{3} \times \mu \cdot W \cdot \operatorname{cosec} \alpha \left[ \frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right] \quad \dots(ii) \end{aligned}$$

## 2. Considering uniform wear

In Fig. 10.25, let  $p_r$  be the normal intensity of pressure at a distance  $r$  from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$$\therefore p_r \cdot r = C \text{ (a constant) or } p_r = C/r$$

We know that the normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r \cdot dr \operatorname{cosec} \alpha$$

and the axial load acting on the ring ,

$$\delta W = \delta W_n \times \sin \alpha = p_r \cdot 2\pi r \cdot dr \cdot \operatorname{cosec} \alpha \cdot \sin \alpha = p_r \times 2\pi r \cdot dr$$

$$= \frac{C}{r} \times 2\pi r \cdot dr = 2\pi C \cdot dr \quad \dots(\because p_r = C/r)$$

$\therefore$  Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C \cdot dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C (r_1 - r_2)$$

or 
$$C = \frac{W}{2\pi(r_1 - r_2)} \quad \dots(iii)$$

We know that frictional force acting on the ring,

$$F_r = \mu \cdot \delta W_n = \mu \cdot p_r \times 2\pi r \times dr \operatorname{cosec} \alpha$$

and frictional torque acting on the ring,

$$T_r = F_r \times r = \mu \cdot p_r \times 2\pi r \cdot dr \cdot \operatorname{cosec} \alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi r^2 \cdot dr \cdot \operatorname{cosec} \alpha = 2\pi \mu \cdot C \operatorname{cosec} \alpha \times r \cdot dr$$

∴ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu C \operatorname{cosec} \alpha r dr = 2\pi\mu C \operatorname{cosec} \alpha \left[ \frac{r^2}{2} \right]_{r_2}^{r_1}$$
$$= 2\pi\mu C \operatorname{cosec} \alpha \left[ \frac{(r_1)^2 - (r_2)^2}{2} \right]$$

Substituting the value of  $C$  from equation (i), we have

$$T = 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \operatorname{cosec} \alpha \left[ \frac{(r_1)^2 - (r_2)^2}{2} \right]$$
$$= \mu W \operatorname{cosec} \alpha \left( \frac{r_1 + r_2}{2} \right) = \mu W R \operatorname{cosec} \alpha \quad \dots(iv)$$

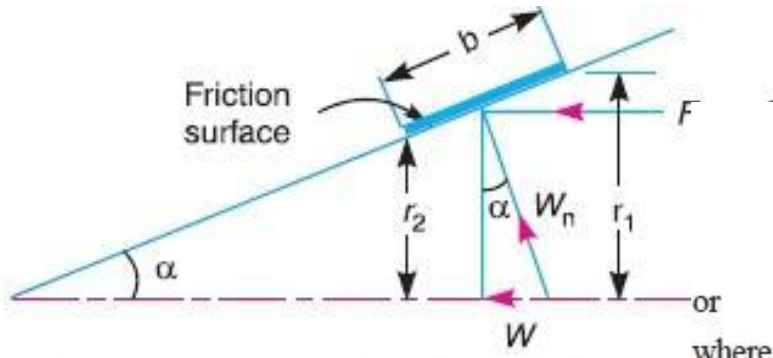
where

$$R = \frac{r_1 + r_2}{2} = \text{Mean radius of friction surface}$$

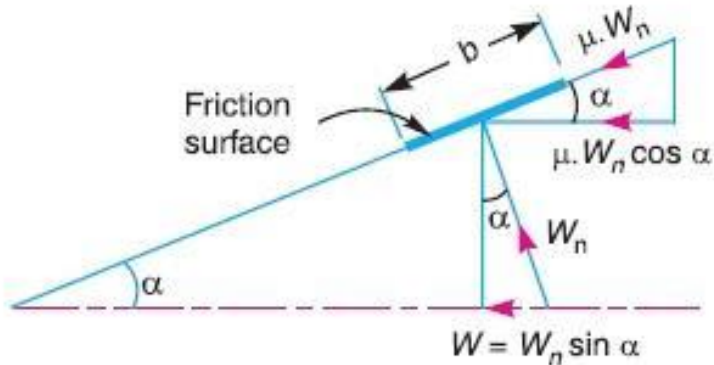
Since the normal force acting on the friction surface,  $W_n = W/\sin \alpha$ , therefore the equation (iv) may be written as

$$T = \mu W_n R \quad \dots(v)$$

# Operation Variables of Cone Clutch



(a) For steady operation of the clutch.



(b) During engagement of the clutch.

$\therefore$  From equation, (i), normal pressure acting on the friction surface,

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{W}{\pi(r_1 + r_2)(r_1 - r_2)} = \frac{W}{2\pi R b \sin \alpha}$$

$$W = p_n \times 2\pi R b \sin \alpha = W_n \sin \alpha$$

$$W_n = \text{Normal load acting on the friction surface} = p_n \times 2\pi R b$$

Now the equation (iv) may be written as,

$$T = \mu(p_n \times 2\pi R b \sin \alpha) R \operatorname{cosec} \alpha = 2\pi \mu p_n R^2 b$$

Axial force required for engaging the clutch,

$$\begin{aligned} W_e &= W + \mu W_n \cos \alpha = W_n \sin \alpha + \mu W_n \cos \alpha \\ &= W_n (\sin \alpha + \mu \cos \alpha) \end{aligned}$$

