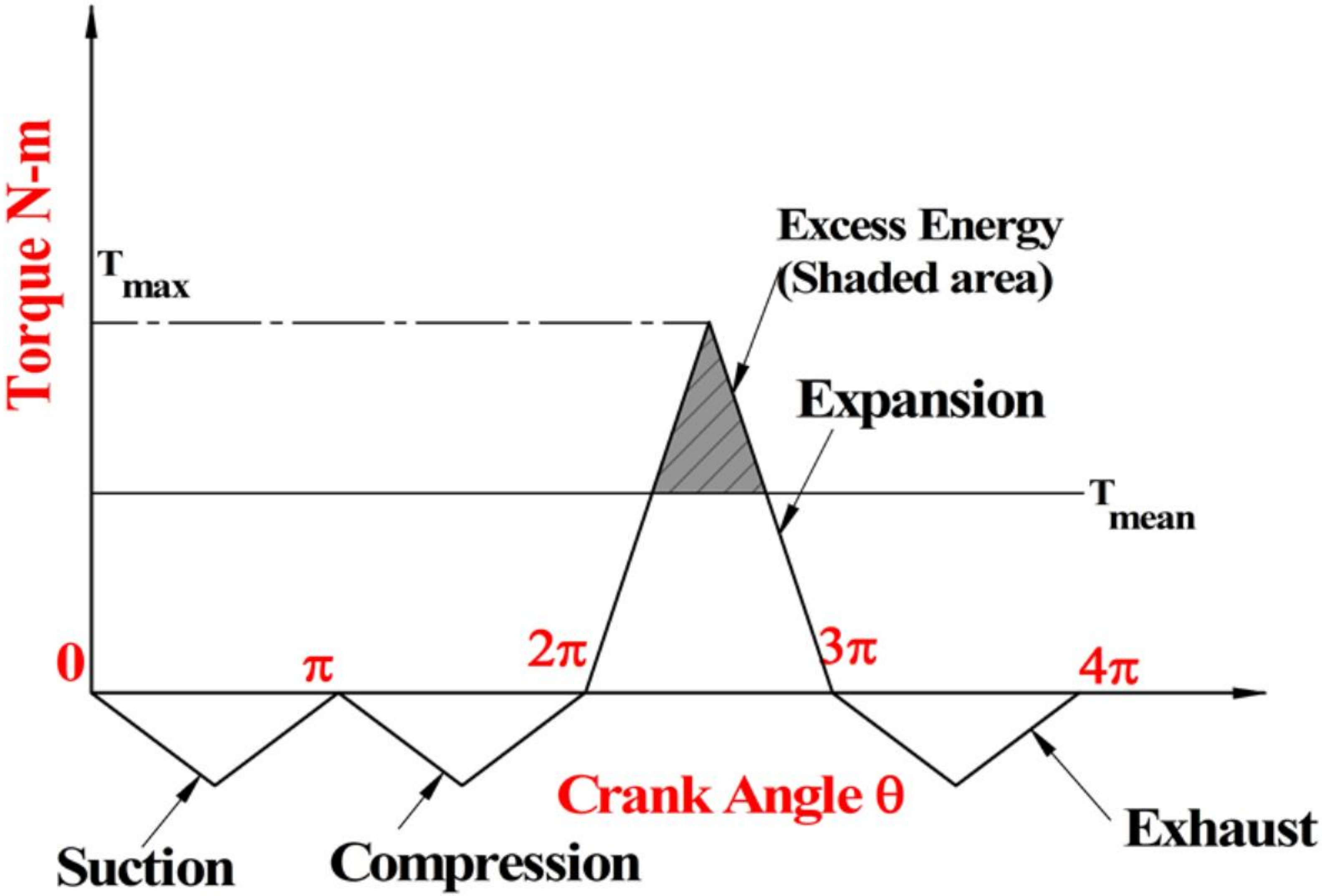
Turning Moment (Crank Effort) Diagram

For a 4-stroke IC engine



Turning Moment (Or Crank Effort) Diagram(TMD)

Turning moment diagram is a graphical representation of ***turning moment*** or ***torque*** (along Y-axis) versus crank angle (X-axis) for various positions of crank.

Uses of TMD

1. The area under the TMD gives the ***work done per cycle.***

2. The work done per cycle when divided by the crank angle per cycle gives the ***mean torque Tm.***

Uses of TMD

3. The mean torque T_m multiplied by the angular velocity of the crank gives the power consumed by the machine or developed by an engine.

4. The area of the TMD above the mean torque line represents the excess energy that may be stored by the ***flywheel***, which helps to design the dimensions & mass of the flywheel.

FLYWHEEL

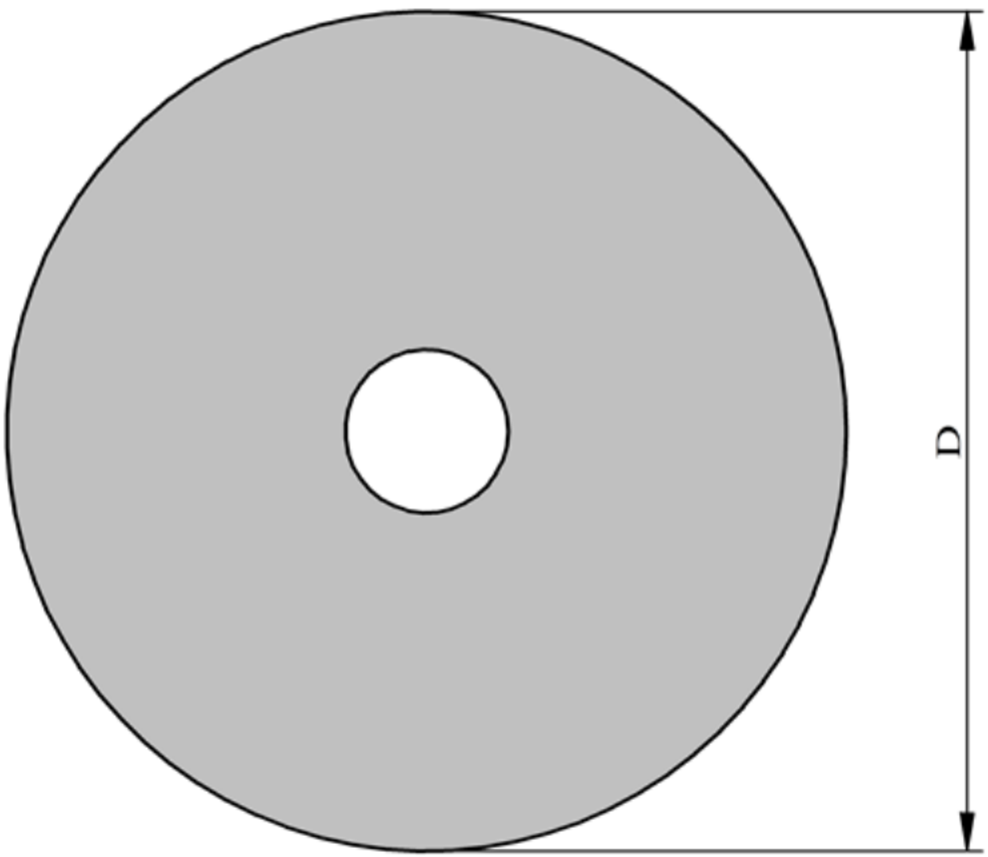
Flywheel is a device **used to store energy when available in excess & release the same when there is a shortage.**

Flywheels are used in IC engines, Pumps, Compressors & in machines performing intermittent operations such as punching, shearing, riveting etc.

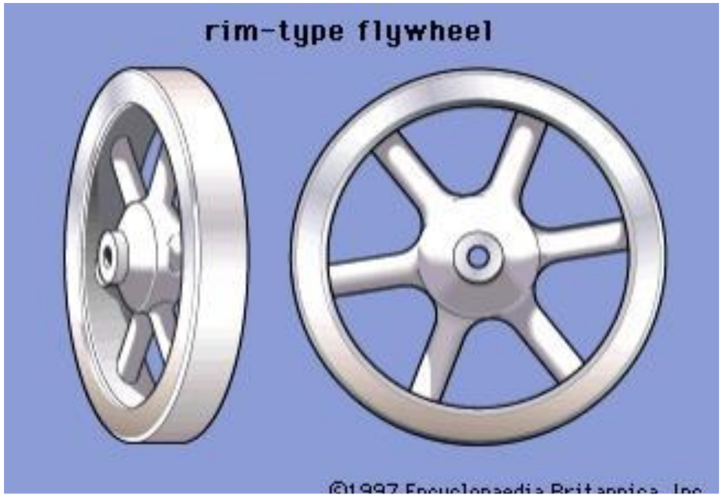
A Flywheel may be of ***Disk type*** or ***Rim Type***

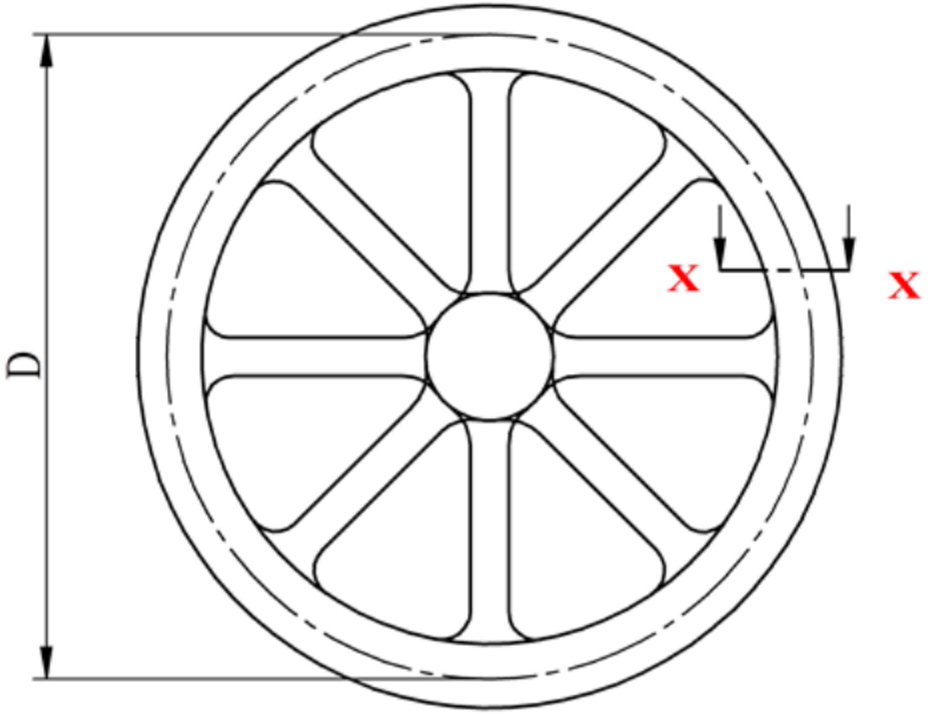
Flywheels help in **smoothening out the Fluctuations of the torque on the crank shaft & Maintain the speed within the prescribed limits**.

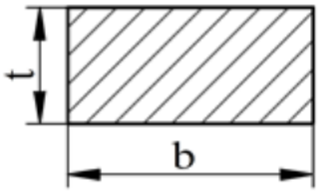
DISK TYPE FLYWHEEL



DISK TYPE FLYWHEEL







Section X-X

RIM TYPE FLYWHEEL

Comparision between Disk Type & Rim Type Flywheel :

Flywheels posess inertia due to its heavy mass.

Mass moment of inertia of a flywheel is given by

$I = mk^2$, where m=Mass of the flywheel.

k=Radius of gyration of the flywheel.

For rim type, k= $\frac{D}{2}$ where D=Mean diameter of the flyheel

For Disk type, k= $\frac{D}{2\sqrt{2}}$ where D=Outer diameter of the flywheel

Hence I= $m_{Rim}\left(\frac{D^2}{4}\right)$ and I= $m_{Disk}\left(\frac{D^2}{8}\right)$

Hence for a given diameter & inertia, the mass of the rim type flywheel is half the mass of a disk type flywheel

Important Definitions

(a) Maximum fluctuation of speed :

It is the difference between the maximum & minimum speeds in a cycle. $(=n_1 - n_2)$

(b) Coefficient of fluctuation of speed : (C_s or K_s)

It is the ratio of maximum fluctuation of speed to the mean speed.

It is often expressed as a % of mean speed.

$$C_s \text{ (or } K_s) = \left(\frac{n_1 - n_2}{n}\right) = \left(\frac{\omega_1 - \omega_2}{\omega}\right)$$

where ω = Angular velocity = $\left(\frac{2\pi n}{60}\right)$

Important Definitions

(c) Coefficient of fluctuation of energy : (C_e or K_e)

It is the ratio of maximum fluctuation of energy to the mean kinetc energy.

$$C_e \text{ (or } K_e) = \left(\frac{E_1 - E_2}{E}\right) = \left(\frac{\Delta E}{E}\right) = \left(\frac{e}{E}\right)$$

*** It is often expressed as the ratio of excess energy to the work done per cycle.* $C_e(\text{ or } K_e) = \left(\frac{e}{W.D / cycle}\right)$

(d) Coefficient of steadiness :

It is the reciprocal of coefficent of fluctuation of speed.

$$\therefore \text{Coefficient of steadiness} = \left(\frac{\omega}{\omega_1 - \omega_2}\right)$$

EXPRESSION FOR ENERGY STORED BY A FLYWHEEL

Let

I be the mass moment of inertia of the flywheel

ω_1 & ω_2 be the max & min speeds of the flywheel

$\omega =$ Mean speed of the flywheel

m=Mass of the flywheel, k=Radius of gyration of the flywheel

C_s=Coefficient of fluctuation of speed

The max fluctuation of energy (to be stored by the flywheel)

$$e = E_1 - E_2 = \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2 = \frac{1}{2} I\left(\omega_1^2 - \omega_2^2\right)$$

$$\Rightarrow e = \frac{1}{2} I\left(\omega_1 + \omega_2\right)\left(\omega_1 - \omega_2\right)$$

EXPRESSION FOR ENERGY STORED BY A FLYWHEEL

Putting the mean agular speed $\omega = \frac{1}{2}(\omega_1 + \omega_2)$,

We get $\boldsymbol{e = I\omega(\omega_1 - \omega_2)}$ Multiplying & dividing by ω,

$$\boldsymbol{e = I\omega^2 \frac{(\omega_1 - \omega_2)}{\omega}}$$

Also $\frac{(\omega_1 - \omega_2)}{\omega} = C_s$, the coefficient of fluctuation of speed

Hence $\boldsymbol{e = I\omega^2 C_s}$

Putting I=mk^2, we get $\boldsymbol{e = mk^2\omega^2 C_s}$

Note: 1.Alternatively, if Mean kinetic energy E=$\frac{1}{2}I\omega^2$,

$\Rightarrow I\omega^2 = 2E$, e=2E$C_s$

$\therefore \frac{e}{E} = 2C_s$ But $\frac{e}{E} = C_e$ OR $\boldsymbol{\frac{C_e}{C_s} = 2}$

EXPRESSION FOR ENERGY STORED BY A FLYWHEEL

$e = I\omega^2 C_s$, Putting mean Kinetic energy $E = \frac{1}{2} I\omega^2$

and expressing C_s as a percentage,

$$e = \frac{2EC_s}{100}$$

$$\therefore e = 0.02EC_s$$

Note: 2. Alternatively, if Mean kinetic energy $E = \frac{1}{2} mk^2\omega^2$,

$$\Rightarrow (k\omega)^2 = v^2,\ E = \frac{1}{2} mv^2$$

$$\therefore e = mv^2 c_s$$

MASS OF FLYWHEEL IN TERMS OF DENSITY & CROSSECTION AREA

We know that

mass m = Density $\rho \times$ Volume

For Disk type flywheel, Volume $= \frac{\pi D^2}{4} \times t$

For Rim type flywheel, Volume $= \pi D(A)$

where A = Cross section of the rim $= b \times t$

b = width of rim & t = thickness of the rim

Note:

(i) Velocity of the flywheel $v = \frac{\pi D n}{60} m/\sec$

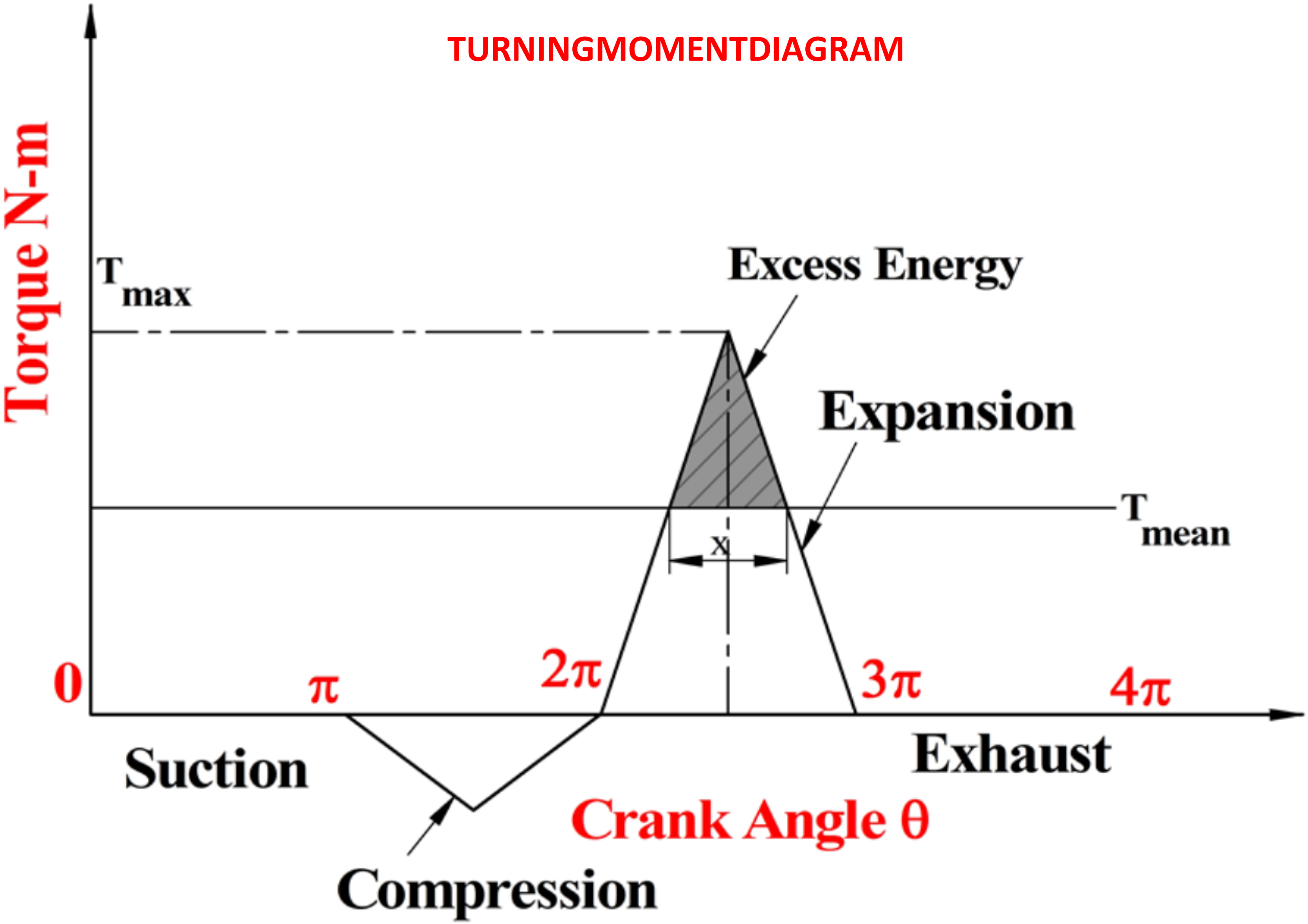
(ii) Hoop Stress (Centrifugal stress) in the flywheel

$\sigma = \rho v^2$ where ρ = density of flywheel material

Problem 1

A single cylinder 4 stroke gas engine develops 1 .4 KW at 300 rpm with work done by the gases during the expansion being 3 times the work done on the gases during compression. The work done during the suction & exhaust strokes is negligible. The total fluctuation of speed is 2 of the mean. The TMD may be assumed to be triangular in shape. Find the mass moment of inertia of the flywheel.

TURNINGMOMENTDIAGRAM



Data :

Power P=18.4 KW=18.4×10^3 W, Mean speed n=300 rpm

Work done during expansion $W_E = 3\times$ Work done during compression

$C_s = 2\% = 0.02$

Given 4-stroke cycle engine

$\Rightarrow$ Crank angle per cycle=***4π radians*(∵ *2 rev of crank shaft)***

Solution :

Angular Velocity of flywheel $\omega = \dfrac{2\pi n}{60}$

$$i.e.\ \ \omega = \frac{2\pi\times300}{60} = 31.416 \text{ rad/sec}$$

Also power $P = T_m \times \omega \Rightarrow 18.4\times10^3 = T_m \times 31.416$

$$\therefore \text{Mean torque } T_m = \frac{18.4\times10^3}{31.416} = 585.7 \text{ N-m}$$

Work done per cycle

Work done per cycle=$T_m \times$ Crank angle per cycle

i.e. W.D/Cycle =$T_m \times 4\pi = 585.7 \times 31.416$

$\therefore$ W.D/Cycle = 7360 N-m

W.D/Cycle = W.D during expansion – W.D during compression

(As the W.D during suction & compression are neglected)

$\Rightarrow 7360 = (W_E - W_C)$

Given $W_E = 3W_C$ Or $W_C = \frac{W_E}{3}$, we can write

$$7360 = \left(W_E - \frac{W_E}{3} \right) = \frac{2}{3} W_E \Rightarrow W_E = 11040 \text{ N-m}$$

This work represents the area under triangle for expansion stroke

$$i.e.\ 11040 = \frac{1}{2} \times \pi \times T_{max}$$

$\Rightarrow$ Max torque $T_{max} = 7028.3$ N-m

Excess energy stored by the flywheel

The shaded area represents the excess energy.

i.e. excess energy stored by flywheel $e = \frac{1}{2} \times x \times (T_{\max} - T_{mean})$

where x is the base of shaded triangle, given by

$$\frac{x}{\pi} = \frac{(T_{\max} - T_{mean})}{T_{\max}}$$

$$\Rightarrow x = \frac{(T_{\max} - T_{mean})}{T_{\max}} \times \pi = \frac{(7028.3 - 585.7)}{7028.3} \times \pi = 2.88 rad$$

Hence $e = \frac{1}{2} \times 2.88 \times (7028.3 - 585.7) =$ 9276.67 N-m

We know that excess energy is given by $e = I.\omega^2 C_s$
$= I x (31.416)^2 x 0.02 = 9276.67$

i.e. $I = 470$ kg-m^2

i.e. Mass moment of inertia of thje flywheel = 470 kg-m^2

Problem 2

A single cylinder internal combustion engine working on 4-stroke cycle develops 75 KW at 3 0 rpm. The fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle. If the fluctuation of speed is not to exceed 1   and the maximum centrifugal stress in the flywheel is to be 5.5 MN/m^2, estimate the diameter and the cross sectional area of the rim. The material of the rim has a density 7.2 Mg/m^3.

Data :

Power P=75 KW=75×10^3W, Mean speed n=360 rpm

Fluctuation of energy e=$0.9\times$ W.D/cycle

4 stroke cycle $\Rightarrow$ Crank angle per cycle=***4π radians***

Density ρ=7.2 Mg/m^3 = 7200 Kg/m^3, Hoop stress σ=5.5 MPa

Solution :

Angular Velocity of flywheel $\omega=\dfrac{2\pi n}{60}$

$$i.e.\ \ \omega=\frac{2\pi\times360}{60}=37.7\ \text{rad/sec}$$

Also power $P=T_m\times\omega\Rightarrow75\times10^3=T_m\times37.7$

$$\therefore \text{Mean torque } T_m=\frac{75\times10^3}{37.7}=1989.4\ \text{N-m}$$

Work done per cycle :

Work done per cycle $= T_m \times$ Crank angle per cycle

i.e. W.D/Cycle $= T_m \times 4\pi = 1989.4 \times 4\pi$

$\therefore$ W.D/Cycle = 25000 N-m

Also given $\boldsymbol{e = 0.9 \times W.D / cycle = 22500\ N-m}$

Diameter of the flywheel :

Hoop stress $\sigma = \rho v^2 \Rightarrow 5.5 \times 10^6 = 7200 \times (v^2)$

Hence, $\boldsymbol{velocity of\ flywheel\ v = 27.64 m / sec}$

$$\boldsymbol{Also\ v} = \frac{\pi D n}{60} \Rightarrow \boldsymbol{27.64} = \frac{\pi \times D \times 360}{60}$$

$\therefore$ ***Diameter of the flywheel = 1.466 m***

Flywheel for Punch press

If 'd' is the diameter of the hole to be punched in a metal plate of thickness 't' , the shearing area

$$\boldsymbol{A=\pi dt}\ mm^2$$

If the energy or work done /sheared area is given, the work done per hole =W.D/mm^2 x Sheared area per hole.

As one hole is punched in every revolution, WD/min=WD/hole x No of holes punched /min

Power of motor required P=WD per min/ 0